Proton stability as a probe of grand unified theories

Complements of particle physics, Università degli Studi di Perugia

Carlo Tasillo

October 30, 2019

RWTH AACHEN UNIVERSITY

Grand Unified Theory (*GUT*):

- *QFT* with unified strong and electroweak force
- $G \supset \mathrm{SU}(3)_C \times \mathrm{SU}(2)_I \times \mathrm{U}(1)_Y$
- G could be $\mathrm{SU}(5)$, $\mathrm{SO}(10)$, E_6
- Intermediate step towards *TOE*

Motivation:

- Gauge coupling unification
- Explanation for charge quantization
- Reduce number of *dof* of *SM*

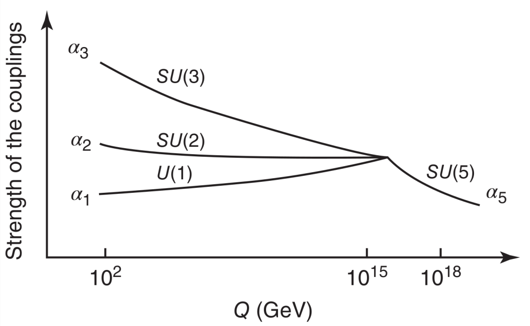


Figure 1: Running couplings of the *SM* meet at the *GUT* scale.

Why question proton stability?

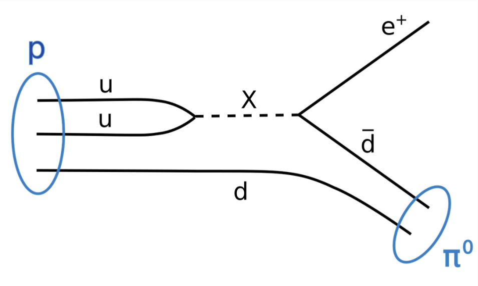


Figure 2: Feynman graph for $p \to e^+\pi^0$. X is a *GUT* gauge boson.

Why do we care about proton stability?

- Predicted by most GUT candidates
- One of the few accessible GUT tests

Other possibilities to test GUTs?

- Electric dipole moments
- Neutrino properties
- Magnetic monopoles, e.g. in Pati-Salam model $\mathrm{SU}(4)_C \times \mathrm{SU}(2)_L \times \mathrm{SU}(2)_R$ which predicts stable proton

Current stability limits (2017)

- Super-Kamiokande: $\tau_p > 1.6 \cdot 10^{34}$ yr
- Georgi-Glashow, minimal SU(5), excluded since 1996
- SUSY SU(5), SUSY SO(10) and flipped SU(5) still possible

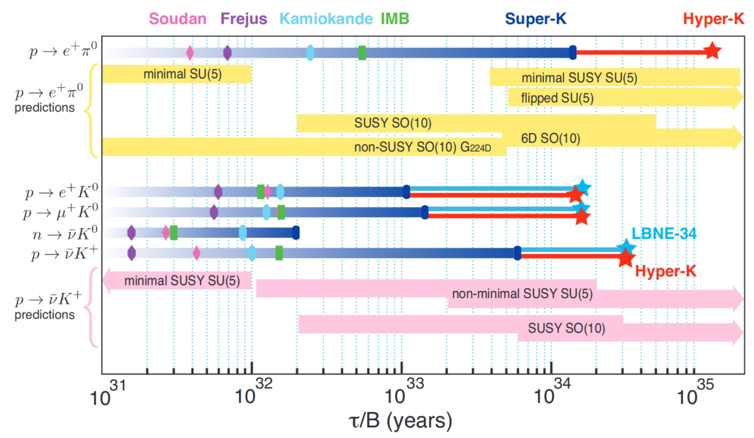


Figure 3: Current and future stability limits and excluded models.

Two different approaches:

Recoil method

Measure recoil of remaining nucleus

+ Observe all decay channels

− Higher background

Decay products method

Detect decay products

+ Lower background

+ Much bigger quantity of matter observable

(−) Specialized on decay channel

Historical example: Homestake Mine, South Dakota, 1980

Figure 4: Homestake Mine in 1900.

- Nobel prize 2002 for solar neutrinos
- Čerenkov detectors, $1600\,\text{m}$ deep, $150\,\text{t}$ water
- Search for up-moving muons from

$$p \to e^+\pi^0,\ \bar{\nu}K^+,\ \ldots$$
$$\pi^0, K^+, \ldots \to \mu^\pm X$$

- For SU(5): BR$(p \to \mu^\pm X) \approx 0.27$
- Limit in 1980: $\tau_p > 2 \cdot 10^{30}\,\text{yr}$

Expected count rate

Water contains about $3 \cdot 10^{26}$ protons per liter.

$\rightsquigarrow \quad R = 3 \cdot 10^{29} \, \frac{\text{protons}}{\text{ton water}} \cdot \frac{\text{BR}(p \to e^+ X)}{\tau_p} \cdot \varepsilon$

where ε is the detection efficiency of the experiment

Example:

$\varepsilon = 75\%$, $\text{BR}(p \to e^+ X) = 75\%$, $\tau_p = 10^{31}\,\text{yr}$

$\Rightarrow \quad R \approx 0.02\,\text{t}^{-1}\,\text{yr}^{-1}$

Events are very rare, **large tanks** and **long observations** are necessary. **Background** subtraction gets very important.

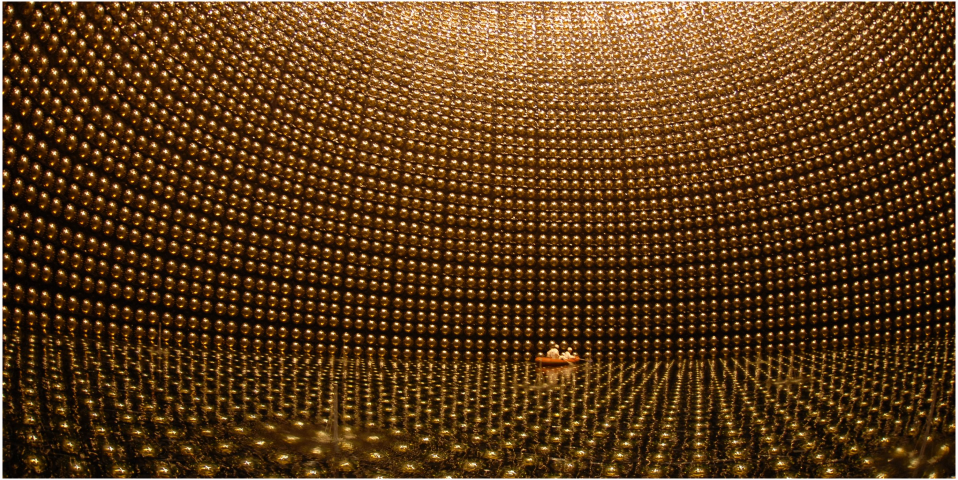
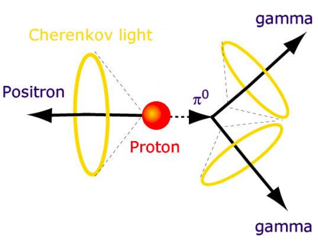


Figure 5: Inside Super-Kamiokande: 55 kt of ultrapure water, 13000 photo multipliers.



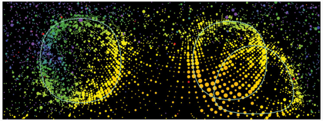


Figure 6: Expected Čerenkov emission and corresponding event display.

Event selection:

- Fully contained
- 2 or 3 rings
- All rings are EM showers
- $m_{\pi^0} = 85 - 185\,\text{MeV}$
- No μ-decay electrons
- $m_{\text{tot}} = 800 - 1050\,\text{MeV}$
- $p_{\text{tot}} < 250\,\text{MeV}$

$$\text{RGE} \rightsquigarrow \quad \frac{1}{\alpha_j(Q^2)} = \frac{1}{\alpha_j(M_W^2)} - \frac{\beta_j}{4\pi} \ln \frac{M_X^2}{Q^2}$$

$$j = 1,2;\ n_G = 3;\ \mathrm{SU}(5) \rightsquigarrow \quad \sin^2\theta_W = \frac{3}{8}\left[1 - \frac{\alpha}{4\pi}\frac{110 - n_H}{9} \ln \frac{M_X^2}{Q^2}\right]$$

$$j = 1,3;\ n_G = 3;\ \mathrm{SU}(5) \rightsquigarrow \quad \frac{\alpha}{\alpha_\mathsf{s}} = \frac{3}{8}\left[1 - \frac{\alpha}{2\pi}\left(11 + \frac{n_H}{6}\right) \ln \frac{M_X^2}{Q^2}\right]$$

Naive estimate:

$\alpha_\mathsf{s} = \frac{12\pi}{25 \ln Q^2/\Lambda^2}$ $\qquad \Lambda = 300\,\text{MeV}$ $\qquad n_H = 0$ $\qquad \alpha(Q^2) = 1/137.04$

$\Rightarrow M_X = 3.7 \cdot 10^{16}\,\text{GeV}$ $\qquad \sin^2\theta_W(M_W^2) = 0.20$ $\qquad \alpha_5(M_X^2) = \frac{(e/\sin^2\theta_W)^2}{4\pi} = 0.022$

$\Rightarrow \tau_p \simeq \frac{M_X^4}{\alpha_5^2 m_p^5} = 10^{38}\,\text{yr}$ $\lightning$

Corrections to calculation of M_X

M_X should be scaled down by two orders of magnitude:

- $\alpha(M_W^2) \simeq \frac{1}{128}$: factor of ≈ 10
- Two-loop corrections of RGE: factor of ≈ 4
- Renormalization schemes MOM or $\overline{\text{MS}}$ up to order g^3: factor of ≈ 3
- (corrections due to additional heavy or light particles in extended models)

$$\Longrightarrow \quad M_X = 6 \cdot (1.5)^{\pm}\, 10^{14}\,\text{GeV}$$

where $(1.5)^{\pm}$ is due to uncertainty in Λ. $\alpha_5 = 0.0244 \pm 0.0002$ nearly unchanged.

Constructing the effective theory

Assume only one family, neglect mixing and go over to effective 4-fermion Fermi theory:

$$-\mathcal{L}_{\text{eff}}^{\text{SU}(5)} = \frac{4G}{\sqrt{2}}\left[\left(\epsilon_{\alpha\beta\gamma}\bar{u}_L^{c\gamma}\gamma^\mu u_L^\beta\right)\left(2\bar{e}_L^+\gamma_\mu d_L^\alpha + \bar{e}_R^+\gamma_\mu d_R^\alpha\right) - \left(\epsilon_{\alpha\beta\gamma}\bar{u}_L^{c\gamma}\gamma^\mu d_L^\beta\right)\left(\bar{\nu}_R^c\gamma_\mu d_R^\alpha\right)\right] + \text{h.c.}$$

where $G/\sqrt{2} = g_5^2/(8M_X^2)$. It can be directly reasoned that:

- $\not{B}$ and $\not{L}$ but $B - L$ conserved
- $\Delta S = 0$ or $\Delta S = -\Delta B$

$\Rightarrow$ e.g. $p \to \bar{\nu}\pi^+$ allowed, but $n \to e^+K^-$ forbidden.

Comparison with SO(10)

Decompose Lagrangian into operators:

$$
\begin{aligned}
-\mathcal{L}_{\text{eff}}^{\mathsf{SU}(5)} &= \frac{4G}{\sqrt{2}}\left[2\mathcal{O}_{e_L^+} + \mathcal{O}_{e_R^+} + \mathcal{O}_{\nu_R^c}\right] + \text{h.c.} \\
-\mathcal{L}_{\text{eff}}^{\mathsf{SO}(10)} &= -\mathcal{L}_{\text{eff}}^{\mathsf{SU}(5)} + \frac{4G'}{\sqrt{2}}\left[2\mathcal{O}_{\nu_L^+} + \mathcal{O}_{e_R^+} + \mathcal{O}_{\nu_R^c}\right] + \text{h.c.}
\end{aligned}
$$

The operators $\mathcal{O}$ have certain symmetries, e.g. strong isospin and parity.

$$
\rightsquigarrow \quad \Gamma(p \to e^+ X) \geq \frac{1+r^2}{2}\,\Gamma(p \to \nu^c X) \quad \text{where} \quad r = \frac{2/M_X^2}{1/M_X^2 + 1/M_{X'}^2}
$$

SU(5): $M_{X'} \to \infty \Leftrightarrow r = 2$. Most protons decay into positrons.

SO(10): $r < 2$. Decay into neutrinos gets more frequent.

Full calculation of proton lifetime

General formula for proton decay width:

$$\Gamma \simeq \frac{g_5^4}{M_X^4} |\psi(0)|^2 \left(\frac{2m_p}{3}\right)^2 |A|^2 \lambda$$

where $|\psi(0)|^2 \simeq 2.0 \cdot 10^{-3}\,\text{GeV}$ and the anomalous dimension $|A|^2 \simeq 10$ is due to the tree-level effective Lagrangian. Values for the phase space λ are model dependent due to unknown quark masses.

$$\Rightarrow \qquad \tau_p = \frac{1}{\Gamma} = 1.2 \cdot 10^{31\pm 2}\,\text{yr}$$

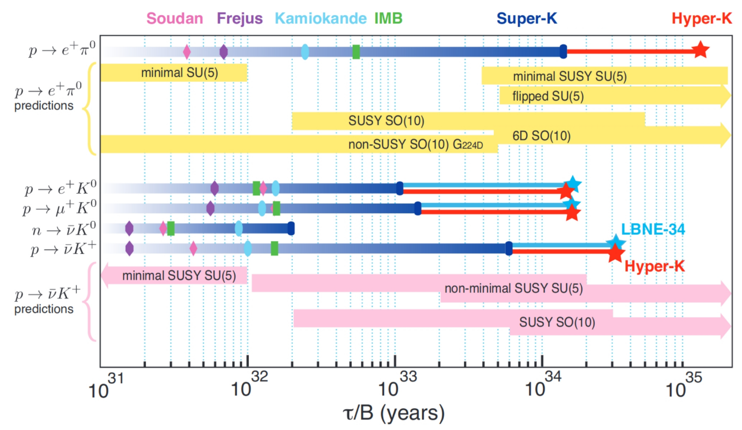


Figure 7: Current and future stability limits and excluded models.

Future experiments to measure m_ν, CP violation, new ν, etc.:

Hyper-Kamiokande

Successor of Super-Kamiokande

- $m_{\mathrm{H_2O}}^{\mathrm{HK}} = 20\, m_{\mathrm{H_2O}}^{\mathrm{SK}} = 1000\,\mathrm{kt}$
- $650\,\mathrm{m}$ underground
- Well suited for $p \to e^+\pi^0$ which is dominant in non-SUSY GUTs
- Start: 2025

DUNE (formerly LBNE)

Deep Underground Neutrino Experiment

- $m_{\mathrm{Ar}}^{\mathrm{DUNE}} = 34\,\mathrm{kt}$, LArTPC
- $1500\,\mathrm{m}$ underground
- Well suited for $p \to K^+\bar{\nu}$ which is dominant in SUSY GUTs
- Start: 2024

Thank you for your attention.

Literature

- *Beyond the Standard Model of Elementary Particle Physics*, Y. Nagashima, 2014
- *Grand Unification Theories and Proton Decay*, P. Langacker, 1980
- *The Homestake Mine proton decay experiment*, R. Steinberg, 1980
- *Planning the Future of U.S. Particle Physics*, J. L. Hewett, 2013, arXiv: 1401.6077v1
- *The Deep Underground Neutrino Experiment – DUNE: the precision era of neutrino physics*, E. Kemp, 2017, arXiv: 1709.09385v2
- *Review of Particle Physics*, Particle Data Group, 2019