Final presentation of the master's thesis

Listening to hot dark sector phase transitions

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Presented findings based on work with
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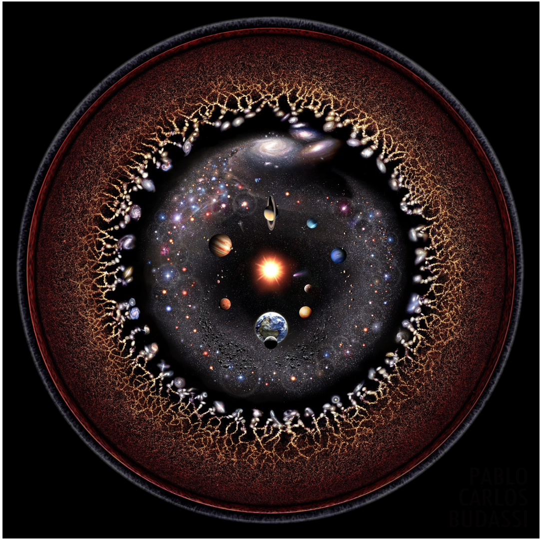
Outline of this talk

1. Motivation and background
2. Dark sectors and their thermal evolution after a phase transition
3. The dark photon model
4. Conclusion

[Camille Flammarion, 1888]

Motivation and background

What do we know about our Universe?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[Pablo Carlos Budassi, 2020]

From CMB anisotropies: ΛCDM model

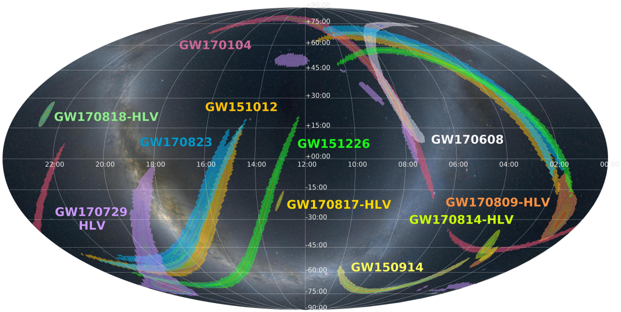
- Isotropic and homogeneous
- 13.8 billion years old
- Expands with a rate of $68\,\mathrm{km\,s^{-1}\,Mpc^{-1}}$
- 95 % of today's energy content is dark!?

⇝ **What lies beyond the surface of last scattering?**

Gravitational waves as a “new” messenger

- Observed > 50 compact binary mergers in five years
- Sensitivity will increase considerably with start of LISA, Einstein Telescope, etc.

⇝ **What will the stochastic gravitational wave background look like?**



[LIGO, Virgo & KAGRA Collaboration, 2020]

Sources of gravitational radiation

Extreme
events!

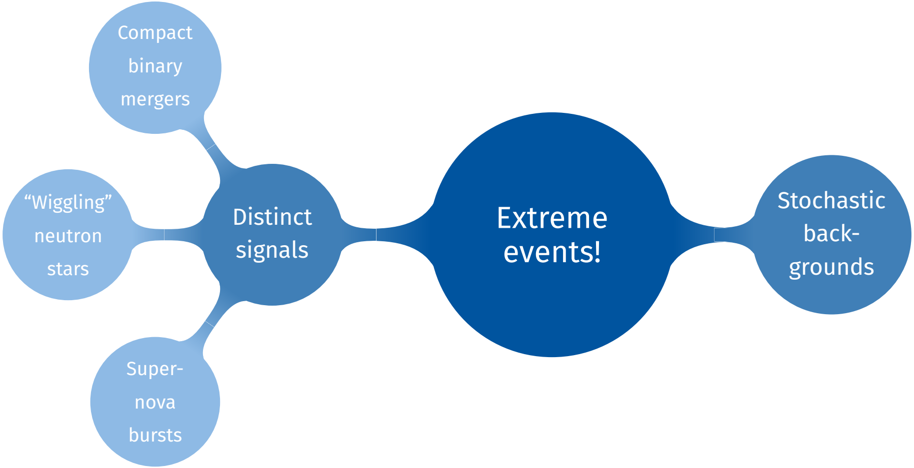
Sources of gravitational radiation

Distinct signals

Extreme events!

Stochastic backgrounds

Sources of gravitational radiation



Extreme events!

Distinct signals

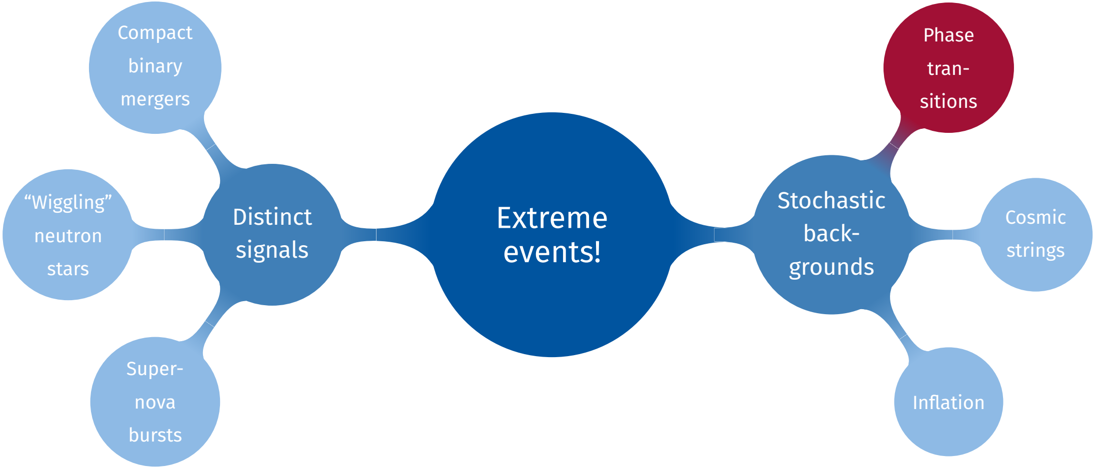
Compact binary mergers

“Wiggling” neutron stars

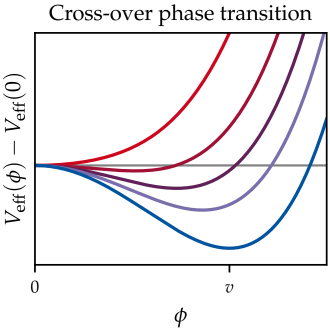
Supernova bursts

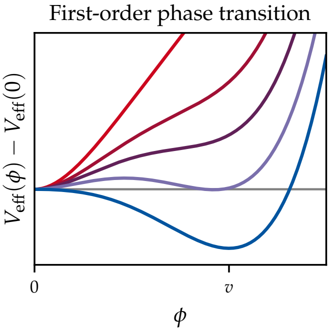
Stochastic back-grounds

Sources of gravitational radiation



Cross-over and first-order phase transitions



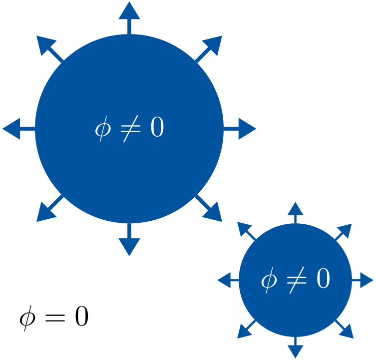


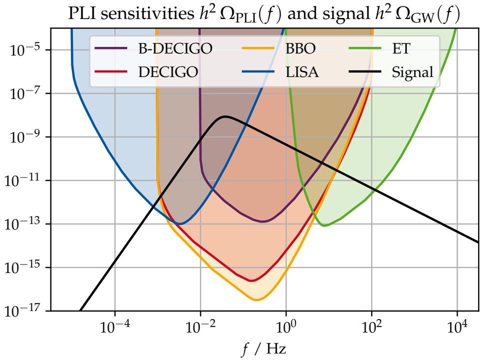
The scalar field “rolls down” from $\phi = 0$ to $\phi = v$, when the bath cools from high temperatures to low temperatures.

The scalar field tunnels to the true potential minimum ($\phi \neq 0$) to minimize its action ($\sim$ free energy).

Gravitational waves from first-order phase transitions

Bubbles of the new phase nucleate and eventually collide...





... giving rise to a stochastic gravitational wave background.

Objective of my thesis

We have seen that

- Primordial GWs could be used for “listening” beyond the CMB
- First-order phase transitions emit gravitational wave signals
- Majority of our Universe is “dark”

⇝ What kind of **dark sector** could produce observable GW signals?

Dark sector: particle bath without thermal contact to SM particles:

$$T_{\text{DS}} = \xi \, T_{\text{SM}}$$

Previous work by Breitbach et al. showed that cold $(\xi < 1)$ dark sectors produce weak signals…

Objective of my thesis

We have seen that

- Primordial C
  “listening” b
- First-order p
  gravitationa
- Majority of (

⇝ What kind o
produce observable GW signals?

th without
articles:

$_{\mathrm{SM}}$

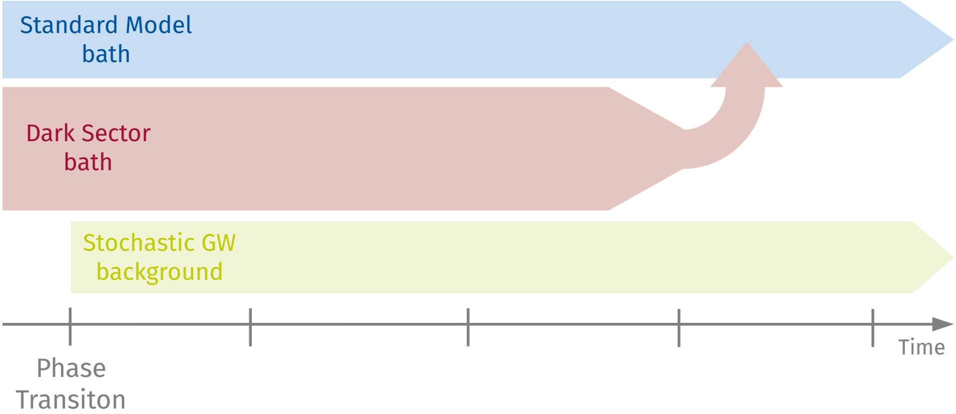
ach et al.
) dark sectors

Can **hot ($\xi > 1$)** dark sector phase transitions emit observable GW signals?

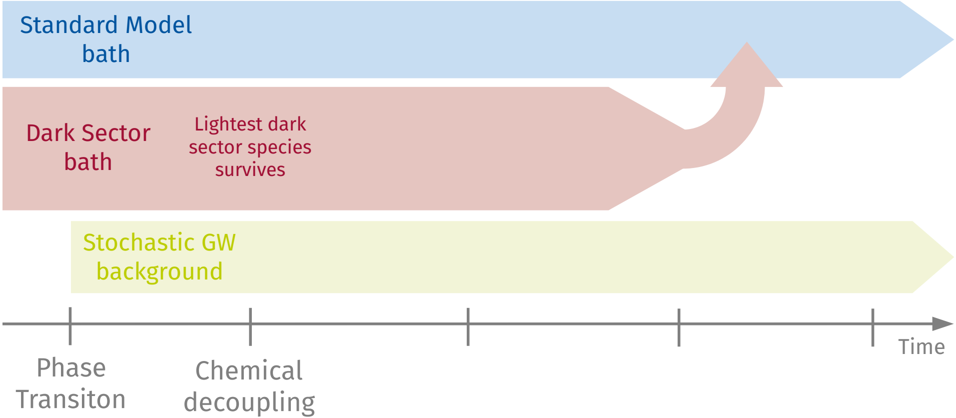
What happens when the dark sector finally decays to SM particles?

The thermal history of a dark sector

Long-lived dark sector evolution after a phase transition



Long-lived dark sector evolution after a phase transition



Standard Model bath

Dark Sector bath

Lightest dark sector species survives

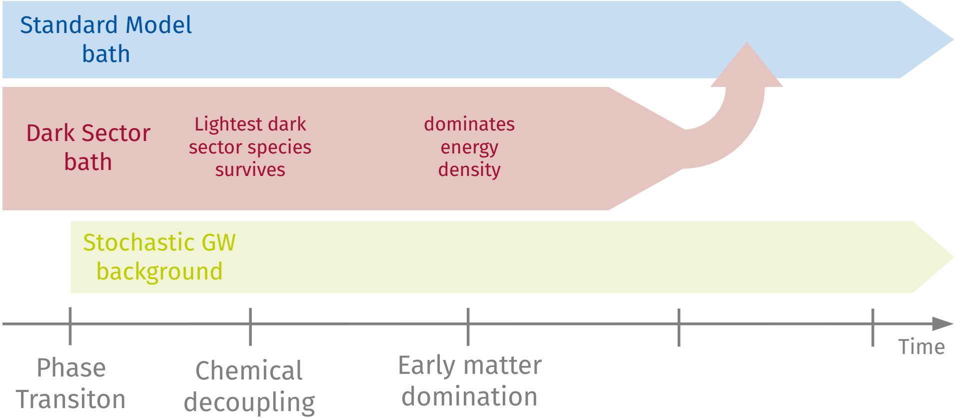
Stochastic GW background

Time

Phase Transiton

Chemical decoupling

Long-lived dark sector evolution after a phase transition



Standard Model bath

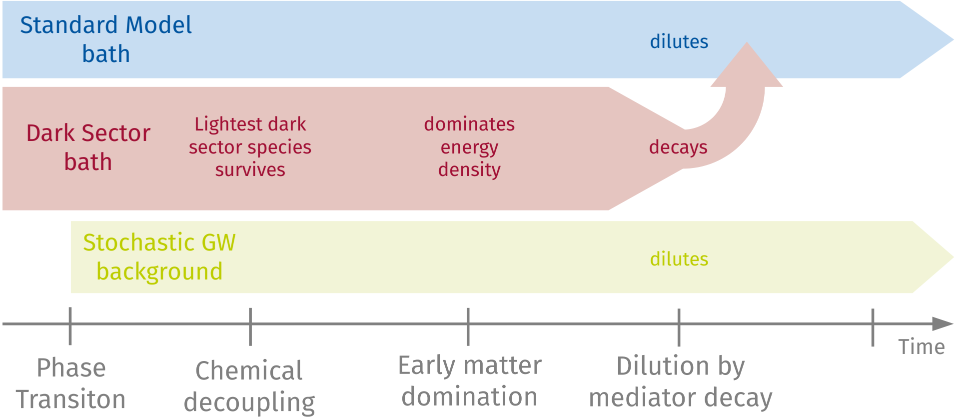
Dark Sector bath

Lightest dark sector species survives

dominates energy density

Stochastic GW background

Long-lived dark sector evolution after a phase transition



Standard Model bath

dilutes

Dark Sector bath

Lightest dark sector species survives

dominates energy density

decays

Stochastic GW background

dilutes

Time

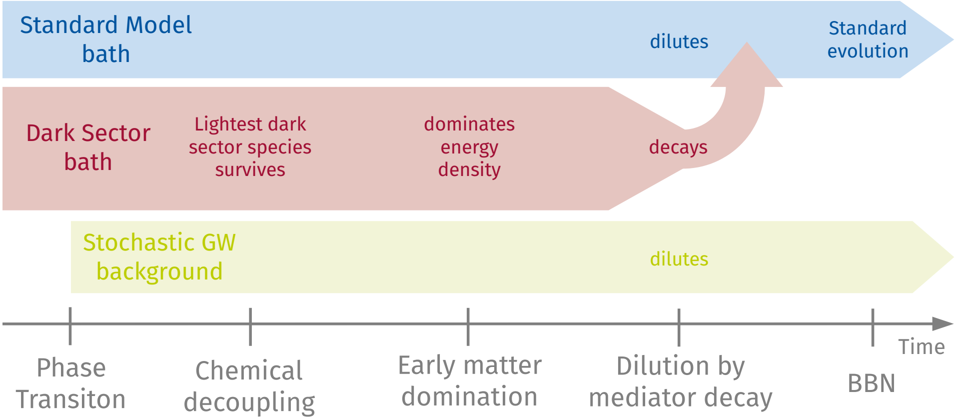
Phase Transiton

Chemical decoupling

Early matter domination

Dilution by mediator decay

Long-lived dark sector evolution after a phase transition



Standard Model bath

dilutes

Standard evolution

Dark Sector bath

Lightest dark sector species survives

dominates energy density

decays

Stochastic GW background

dilutes

Time

Phase Transiton

Chemical decoupling

Early matter domination

Dilution by mediator decay

BBN

Describing the dark sector in equilibrium

For several dark sector species in thermal equilibrium: can define effective DOFs

$$\rho_{\text{tot}}(T_{\text{SM}}) = \left[g_{\text{eff},\rho}^{\text{SM}}(T_{\text{SM}}) + g_{\text{eff},\rho}^{\text{DS}}(T_{\text{SM}})\,\xi^4(T_{\text{SM}})\right] \frac{\pi^2}{30}\, T_{\text{SM}}^4$$

$$s_{\text{tot}}(T_{\text{SM}}) = \left[g_{\text{eff},s}^{\text{SM}}(T_{\text{SM}}) + g_{\text{eff},s}^{\text{DS}}(T_{\text{SM}})\,\xi^3(T_{\text{SM}})\right] \frac{2\,\pi^2}{45}\, T_{\text{SM}}^3$$

Describing the dark sector in equilibrium

For several dark sector species in thermal equilibrium: can define effective DOFs

$$\rho_{\text{tot}}(T_{\text{SM}}) = \left[g^{\text{SM}}_{\text{eff},\rho}(T_{\text{SM}}) + g^{\text{DS}}_{\text{eff},\rho}(T_{\text{SM}})\,\xi^4(T_{\text{SM}})\right] \frac{\pi^2}{30}\, T^4_{\text{SM}}$$

$$s_{\text{tot}}(T_{\text{SM}}) = \left[g^{\text{SM}}_{\text{eff},s}(T_{\text{SM}}) + g^{\text{DS}}_{\text{eff},s}(T_{\text{SM}})\,\xi^3(T_{\text{SM}})\right] \frac{2\,\pi^2}{45}\, T^3_{\text{SM}}$$

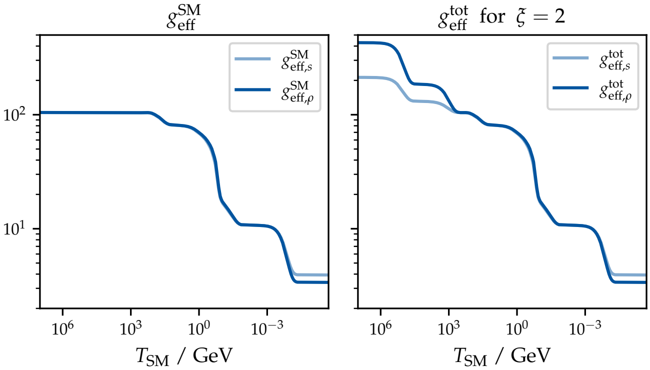
As entropy is conserved separately in the two baths, the temperature ratio follows

$$\xi(T_{\text{SM}}) = \tilde{\xi} \left(\frac{g^{\text{SM}}_{\text{eff},s}}{\tilde{g}^{\text{SM}}_{\text{eff},s}}\right)^{1/3} \left(\frac{\tilde{g}^{\text{DS}}_{\text{eff},s}}{g^{\text{DS}}_{\text{eff},s}}\right)^{1/3}$$

When SM particles annihilate, ξ decreases.

When dark sector DOF decrease, ξ increases.

Describing the dark sector in equilibrium



Example: Thermal evolution of a hot ($\xi = 2$) dark sector consisting of a dark photon ($m_{\text{DP}} = 10^6$ GeV) and a dark Higgs boson ($m_{\text{DH}} = 10^4$ GeV).

The out-of-equilibrium decay of a dark mediator

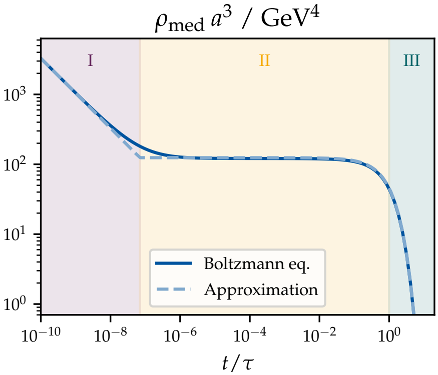
Evolution of the lightest dark sector state ("mediator") after chemical decoupling:

$$\dot{\rho}_{\mathrm{med}} \simeq -3\,\zeta\,H\,\rho_{\mathrm{med}} - \Gamma\,\rho_{\mathrm{med}}$$

with

$$\zeta = 1 + \frac{P_{\mathrm{med}}}{\rho_{\mathrm{med}}} = \begin{cases} 4/3 & \text{rel.} \\ 1 & \text{non-rel.} \end{cases}$$

Three phases: Relativistic, non-relativistic and decaying mediator



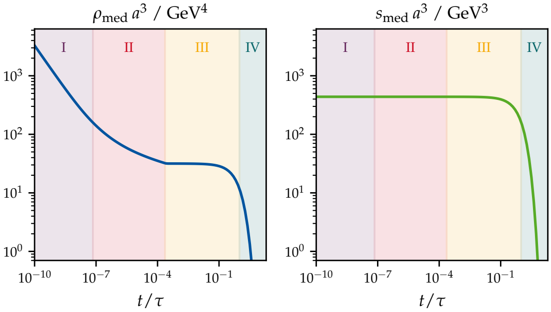
The out-of-equilibrium decay of a dark mediator

Number-changing processes of the mediator lead to a "cannibalistic" phase with $\mu_{\text{med}} = 0$. Therefore, the unique function $\rho_{\text{med}}(s_{\text{med}})$ exists. We found:

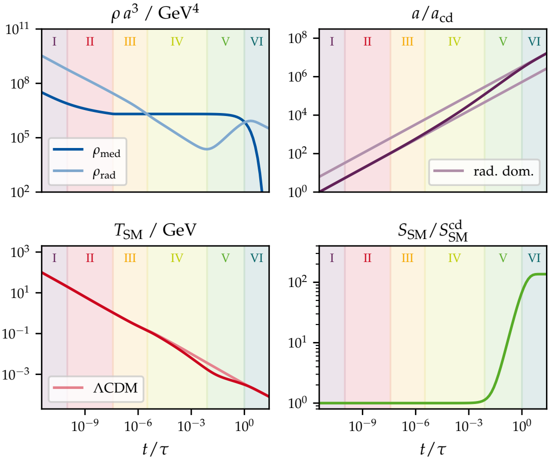
$$\zeta = \begin{cases} \dfrac{\mathrm{d}\ln \rho_{\text{med}}}{\mathrm{d}\ln s_{\text{med}}} & 3 \to 2 \text{ efficient} \\ 1 & 3 \to 2 \text{ inefficient} \end{cases}$$

During cannibalism, ζ goes smoothly from $4/3$ to 1.

Four phases: Relativistic, cannibalistic, non-relativistic and decaying mediator



The out-of-equilibrium decay of a dark mediator



The Friedmann equation, $\dot{\rho}_{\text{med}}(t)$, $g_{\text{eff},s/\rho}^{\text{SM}}(T_{\text{SM}})$, $\rho_{\text{rad}}(T_{\text{SM}})$, and $T_{\text{SM}}(t)$ provide a set of coupled ODEs.

$\rightsquigarrow$ Six phases:

I Relativistic mediator
II Cannibalistic mediator
III Non-relativistic mediator
IV Early matter domination
V Entropy injection
VI Mediator decay

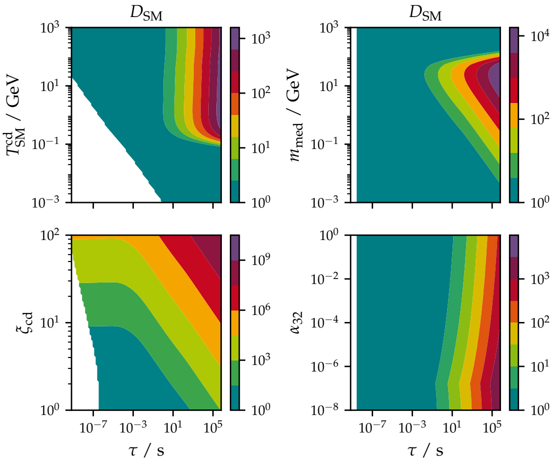
The out-of-equilibrium decay of a dark mediator

Dark sector parameters:

- SM temperature $T_{\text{SM}}^{\text{cd}}$ at chemical decoupling
- Mediator mass m_{med}
- Temperature ratio ξ_{cd} at chemical decoupling
- Effective $3 \to 2$ coupling α_{32}

Define dilution factor:

$$D_{\text{SM}} = \frac{S_{\text{SM}}^{\text{after decay}}}{S_{\text{SM}}^{\text{before decay}}}$$



Parametrization of the GW signal

Assuming strong[1] phase transitions, the GW spectrum can be parameterized by

$$h^2\,\Omega_{\mathrm{GW}}(f) \simeq \frac{\mathcal{O}(10^{-6})}{D^{4/3}} \left(\frac{\alpha}{1+\alpha}\right)^2 \left(\frac{\beta}{H}\right)^{-2} \frac{3.8\,(f/f_{\mathrm{p}})^{2.8}}{1+2.8\,(f/f_{\mathrm{p}})^{3.8}}\,, \quad \text{where}$$

$$D \equiv \frac{g_{\mathrm{eff},s}^{\mathrm{SM,n}}}{g_{\mathrm{eff},s}^{\mathrm{tot,n}}}\, D_{\mathrm{SM}} \qquad \text{and} \qquad f_{\mathrm{p}} \simeq \frac{\mathcal{O}(\mu\mathrm{Hz})}{D^{1/3}} \left(\frac{\beta}{H}\right) \left(\frac{T_{\mathrm{SM}}^{\mathrm{n}}}{100\ \mathrm{GeV}}\right)$$

⇝ GW spectrum fixed by the transition strength α, the inverse time scale β/H, the nucleation temperature $T_{\mathrm{SM}}^{\mathrm{n}}$ and the dilution factor D

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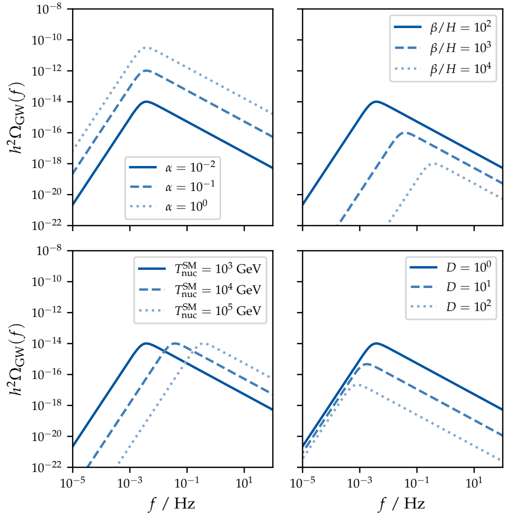
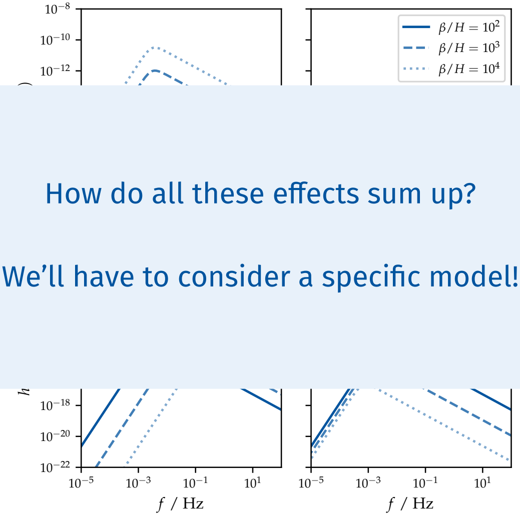
[1] This is only to get an intuition, the actually performed calculations are more involved

Parametrization of the GW signal



Parametrization of the GW signal

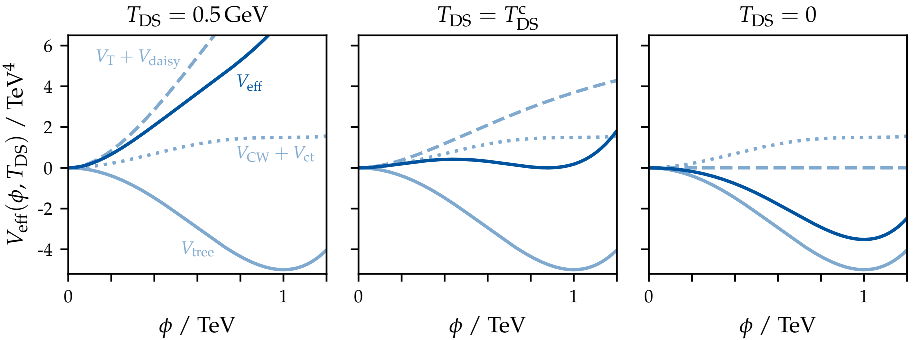


How do all these effects sum up?

We'll have to consider a specific model!

The dark photon model

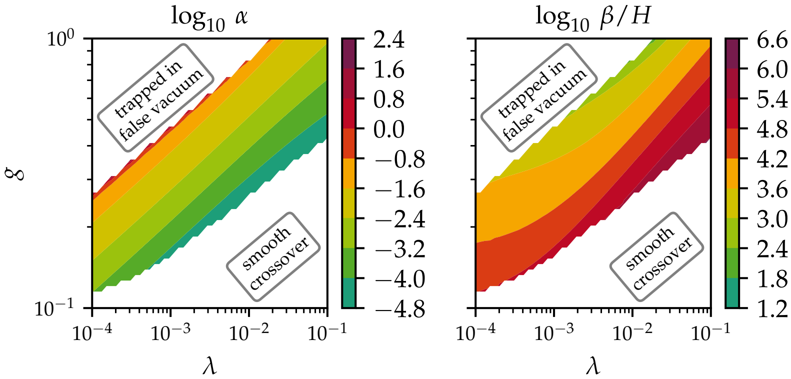
The dark photon model



Add a $\mathrm{U}(1)_{\mathrm{D}}$ to the SM gauge groups. Its gauge boson, the “dark photon”, gets massive when a “dark Higgs” obtains $\phi \neq 0$. Effective potential controlled by the tree-level VEV v, dark Higgs quartic coupling λ and gauge coupling g.

Strength and time scale of the transition

Analyze the phase structure and determine the strength α and inverse time scale β/H. Vary quartic coupling λ and gauge coupling g to identify region of strong and slow transitions.



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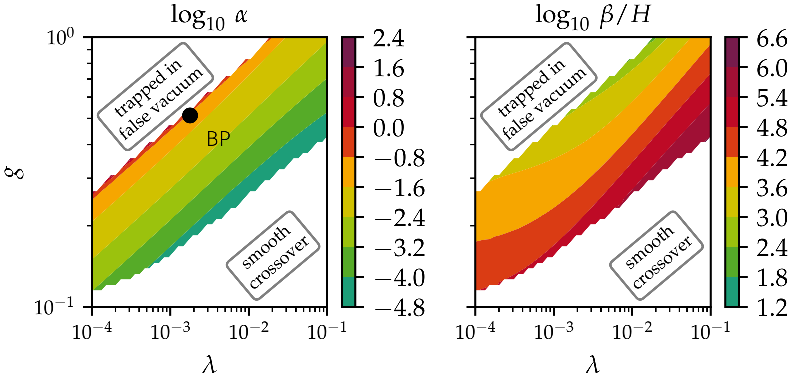
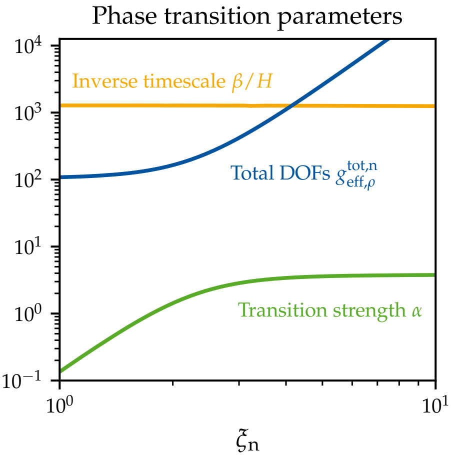
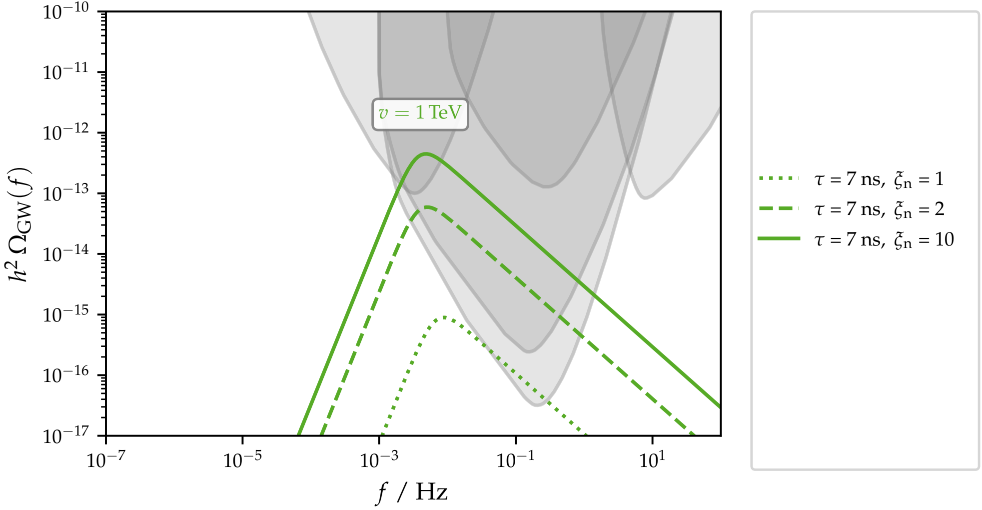


The temperature ratio's impact on α and β/H

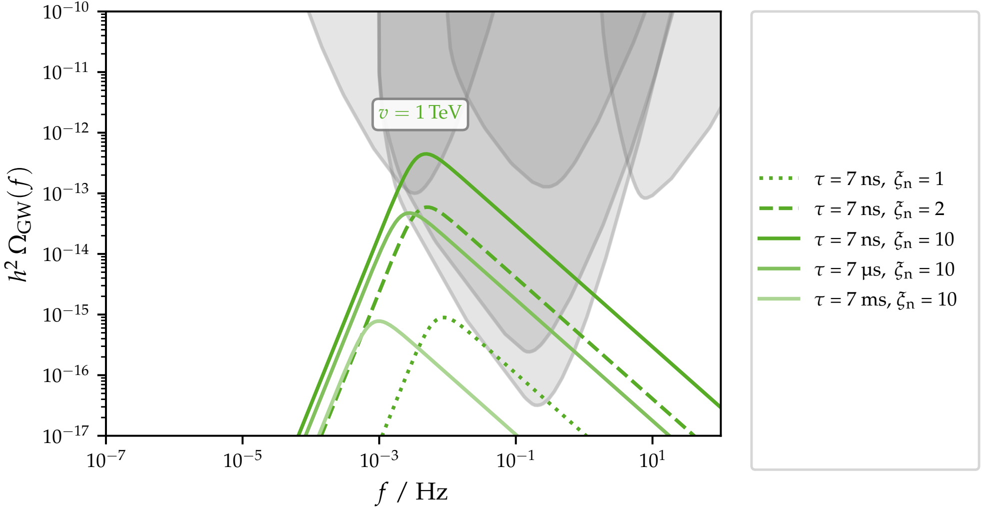


The transition strength α increases $\propto \xi_\mathrm{n}^4$, but only until the Universe is completely dominated by the dark sector. Then, the relative temperature difference becomes irrelevant. The inverse timescale is virtually independent of ξ_n.

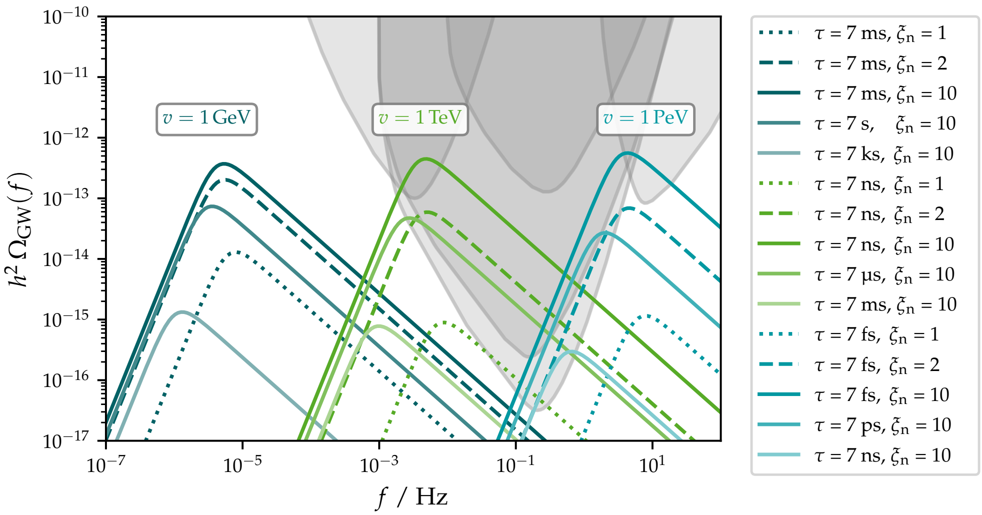
The temperature ratio’s impact on the GW signal



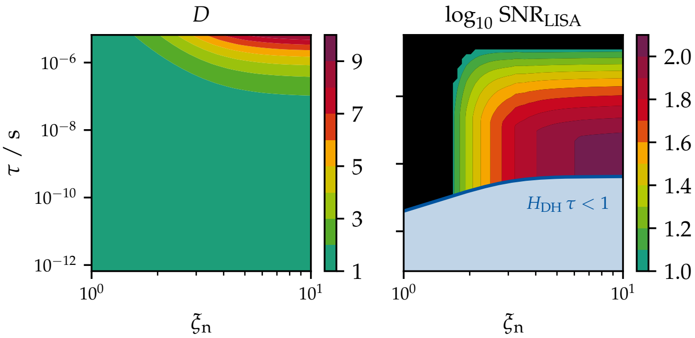
The dark Higgs lifetime's impact on the GW signal



The vacuum expectation value's impact on the GW signal

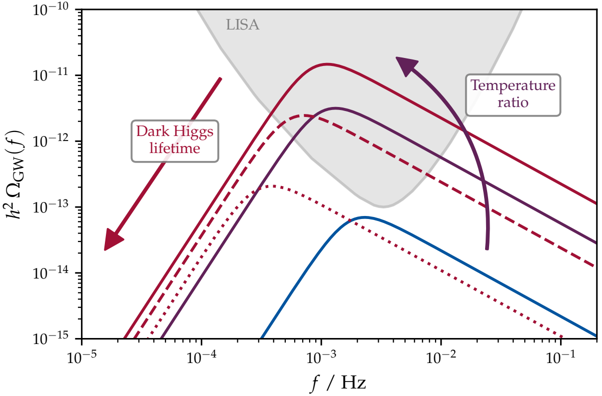


LISA benchmark point, $v = 2\,\text{TeV}$



Signals observable for $\xi_{\mathrm{n}} > 2$, if dark sector not too long-lived, that is $\tau \lesssim 1\,\mu\mathrm{s}$. Otherwise, signals too weak/diluted. For $\tau \lesssim 1\,\mathrm{ns}$, assumption of non-relativistic decays breaks and inverse decays become relevant, thermalizing the two baths.

Summary



- Hot dark sectors can produce strong first-order phase transitions
- Parts of the $\mathrm{U}(1)_{\mathrm{D}}$ model parameter space will be testable by LISA (and ET)
- If the mediator species becomes too long-lived, its out-of-equilibrium decay will dilute the signal

Conclusions

Conclusions

- Could answer the question of how the **dark sector temperature** alters the produced gravitational wave signals of a first-order phase transition!
- **Extension and improvement of literature** on dilution through out-of-equilibrium decays after possible mediator **cannibalism**
- **Publication** in ca. one month, **including code** for model parameter scans, calculating effective potentials, dilution factors and signal-to-noise ratios
- Ignored **bubble wall effects** by assuming sufficiently strong transitions $\rightsquigarrow$ further work necessary. Why not add a DM candidate to the model?
- The first GW detection was only five years ago, last year NANOGrav detected hints for GW background. **We live in exciting times!**

Thank you very much for your attention!

Please feel free to ask me about anything, if you have questions.

Literature

- **Dark, Cold, and Noisy: Constraining Secluded Hidden Sectors with Gravitational Waves**, M. Breitbach *et al.*, 2014, arXiv: 1811.11175v2
- **CosmoTransitions: Computing Cosmological Phase Transition Temperatures and Bubble Profiles with Multiple Fields**, C.L. Wainwright, 2011, arXiv: 1109.4189v1
- **Gravitational Waves from Cosmological Phase Transitions**, M. Breitbach, Master Thesis, 2018
- **Finite Temperature Field Theory and Phase Transitions**, M. Quirós, 1999, arXiv: 9901312v1
- **Homeopathic Dark Matter or how diluted heavy substances produce high energy cosmic rays**, M. Cirelli *et al.*, 2019, arXiv: 1811.03608v3

Backup slides

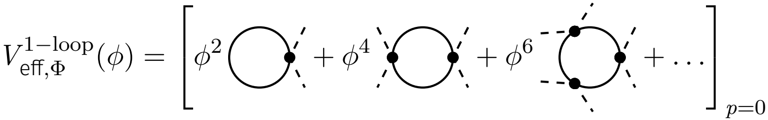
First-order phase transitions in thermal field theory

To demonstrate construction of $V_{\text{eff}}(\phi, T)$, take the toy-model Lagrangian...

$$\mathcal{L} = \frac{1}{2}\left(\partial_\mu \phi\right)\left(\partial^\mu \phi\right) - V_{\text{tree}}(\phi)$$

$$\text{with} \quad V_{\text{tree}}(\phi) = -\frac{1}{2}\mu^2\phi^2 + \frac{\lambda}{4}\phi^4$$

... and consider all 1-loop 1-PI graphs:



First-order phase transitions in thermal field theory

And calculate 1-loop effective potential with $m^2(\phi) = \partial_\phi^2 V_{\mathsf{tree}}(\phi) = -\mu^2 + 3\lambda\phi^2$

$$\begin{aligned} V_{\mathsf{eff}}(\phi, T) = & \quad \frac{1}{2}\int \frac{\mathrm{d}^4 k_{\mathsf{E}}}{(2\pi)^4} \log\left[k_{\mathsf{E}}^2 + m^2(\phi)\right] && \text{with } k_E^0 \text{ being } \frac{2\pi}{T}\text{-periodic} \\ = & \quad \frac{T}{2}\sum_n \int_{\mathbf{k}} \log\left[\left(\frac{2\pi n}{T}\right)^2 + E_k^2\right] && \text{with } E_k = \sqrt{k^2 + m^2(\phi)} \\ = & \quad \int_{\mathbf{k}} \left[\frac{E_k}{2} + T\log\left\{1 - e^{-E_k/T}\right\}\right] \\ = & \quad V_{\mathsf{CW}}(\phi) + V_{\mathsf{T}}(\phi, T) \end{aligned}$$

Interpretation: V_{tree} is the classical energy density contained in a background field ϕ, $V_{\mathsf{CW}}(+V_{\mathsf{T}})$ is the vacuum energy density of a quantum field living in this background, which is completely analogous to the zero-point energy of a harmonic oscillator (in a thermal bath)

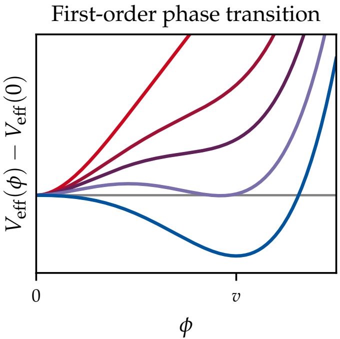
First-order phase transitions in thermal field theory

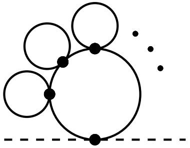
$$V_{\mathsf{T}} = \int_{\mathbf{k}} T \log\left\{1 - e^{-E_k/T}\right\}$$
$$= -\frac{\pi^2 T^4}{90} + \frac{T^2 m^2(\phi)}{24} - \frac{T m^3(\phi)}{12\pi} + \ldots$$

However, around T_c, V_{eff} is dominated by $>$ 1-loop effects. “Daisies” dominate:

$$V_{\text{daisy}} = -\frac{T}{12\pi}\left[\left(m^2(\phi) + \Pi(T)\right)^{3/2} - m^3(\phi)\right]$$

And cancel the potential barrier in V_{eff}. But: Transversal gauge boson component doesn’t acquire $\Pi(T)$. ⇝ Gauge bosons can save potential barrier and thus FOPTs.





First-order phase transitions in thermal field theory

Summary:

$$V_{\text{eff}}^{\text{1-loop}}(\phi, T) = V_{\text{tree}}(\phi) + V_{\text{CW}}(\phi) + V_{\text{ct}}(\phi) + V_{\text{T}}(\phi, T) + V_{\text{daisy}}(\phi, T)$$

Coleman-Weinberg potential and its counter-terms

1-loop thermal corrections

Daisy corrections, dominate at T_{c}

How to get a thermal FOPT?

- Need scalar charged under gauge group with massive gauge bosons
- Dominant $V_{\text{tree}} + V_{\text{CW}}$ contributions can always destroy potential barrier, though $\rightsquigarrow$ as in SM with too high m_h forbidding FOPT

First-order phase transitions in thermal field theory

$$V_{\text{eff}}^{1-\text{loop}}(\phi,\,T) = V_{\text{tree}} + V_{\text{CW}} + V_{\text{ct}} + V_T + V_{\text{daisy}}$$

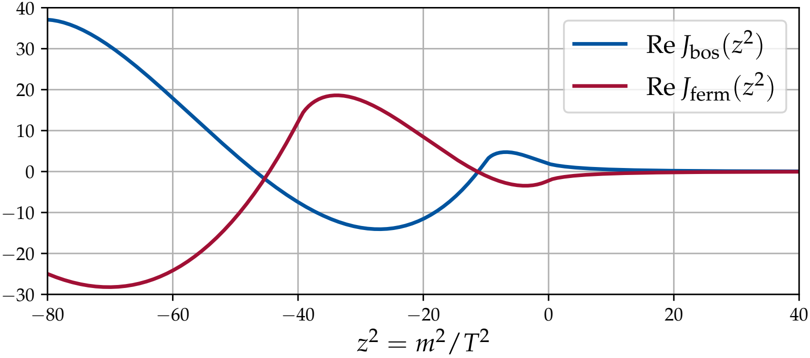
has the individual contributions

$$V_{\text{CW}}(\phi) = \sum_x \eta_x\, n_x\, \frac{m_x^4(\phi)}{64\,\pi^2}\left[\ln\frac{m_x^2(\phi)}{\Lambda^2} - C_a\right] \;,$$

$$V_T(\phi,\,T) = \frac{T^4}{2\,\pi^2}\sum_x \eta_x\, n_x\, J_{\eta_x}\left(\frac{m_x^2(\phi)}{T^2}\right) \;,$$

$$V_{\text{daisy}}(\phi,\,T) = -\frac{T}{12\,\pi}\sum_{\text{b}} n_b^{\text{L}}\left[\left(m^2(\phi) + \Pi(T)\right)_b^{3/2} - \left(m^2(\phi)\right)_b^{3/2}\right]$$

Thermal functions



Bubble expansion

Euclidean action of scalar field

$$S\left[\phi\right] = \int \mathrm{d}^4 x_{\mathsf{E}} \left[\frac{1}{2}\left(\frac{\partial\phi}{\partial\tau}\right)^2 + \frac{\left(\nabla\phi\right)^2}{2} + V_{\mathsf{eff}}(\phi)\right]$$

Minimizing for O(4)-case gives

$$\frac{\mathrm{d}^2\phi}{\mathrm{d}\rho^2} + \frac{3}{\rho}\frac{\mathrm{d}\phi}{\mathrm{d}\rho} = V'_{\mathsf{eff}}(\phi)$$

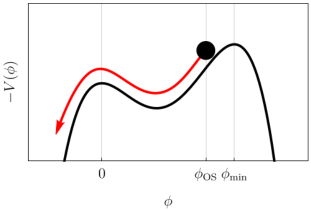
At finite T and in real space:

$$\frac{\mathrm{d}^2\phi}{\mathrm{d}r^2} + \frac{2}{r}\frac{\mathrm{d}\phi}{\mathrm{d}r} = V'_{\mathsf{eff}}(\phi, T)$$

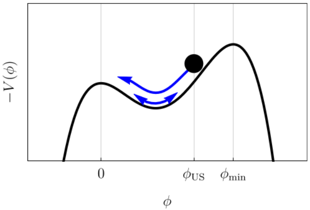
Can be solved by overshoot-undershoot method

Over-undershoot method

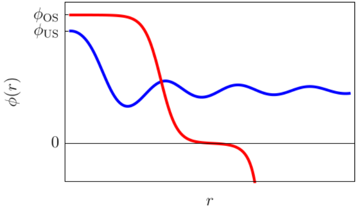
It is possible to transform the boundary value problem into an initial value problem. Adjust the starting point ϕ_0 until $\phi(r \to \infty) = 0$ and ϕ comes to halt exactly at the local maximum of $-V_{\text{eff}}$ at $\phi = 0$



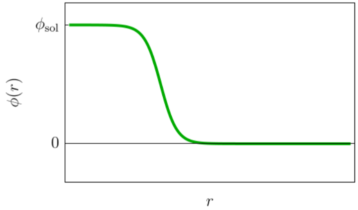
(a) Undershooting



(b) Overshooting



(a) Under-/overshooting solutions



(b) Solution of the bounce equation

Bubble formation and thermal tunneling

Nucleation rate: $\Gamma = \mathcal{A} e^{-S_4}$ with

$$S_4 = \int \frac{1}{2} \left(\frac{\mathrm{d}\phi}{\mathrm{d}\tau} \right)^2 + \frac{1}{2} (\nabla \phi)^2 + V_{\text{eff}}(\phi)\, \mathrm{d}^4 x_{\mathrm{E}}$$

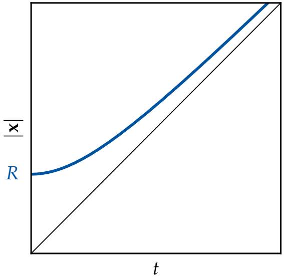
and $\mathcal{A} \sim T^4$. Extremalization yields KG equation with classical potential source:

$$\frac{\mathrm{d}^2\phi}{\mathrm{d}\tau^2} + \Delta\phi = \frac{\mathrm{d}\, V_{\text{eff}}}{\mathrm{d}\phi}$$

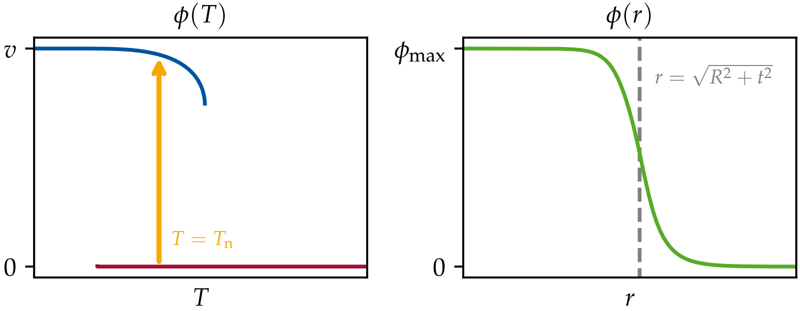
with b.c. $\phi(\rho \to \infty) \to 0$ and $\phi'(\rho = 0) = 0$ where $\rho \equiv \sqrt{\tau^2 + |\mathbf{x}|^2}$.
Solutions typically $O(4)$ symmetric:

$$\frac{\mathrm{d}^2\phi}{\mathrm{d}\rho^2} + \frac{3}{\rho}\frac{\mathrm{d}\phi}{\mathrm{d}\rho} = \frac{\mathrm{d}\, V_{\text{eff}}}{\mathrm{d}\phi}$$

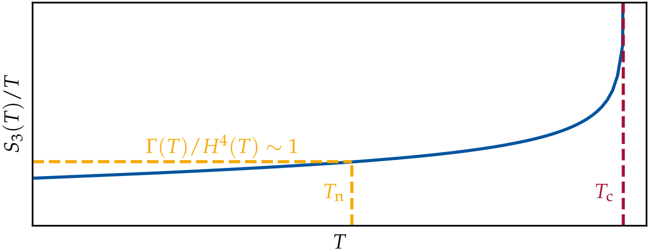
In 3-space: $r = |\mathbf{x}| = \sqrt{\rho^2 + c^2 t^2} \rightsquigarrow$
Nucleation and expansion with $v \to c$



Temperature dependence of potential minima and bubble profile



Nucleation criterion



The nucleation condition $\Gamma(T_\mathrm{n})\, H^{-4}(T_\mathrm{n}) = 1$ gives

$$\left.\frac{S_3(T)}{T}\right|_{T=T_\mathrm{n}} \sim 146 - 2\ \ln\left(\frac{g^\mathrm{tot}_{\mathrm{eff},\rho}(T_\mathrm{n})}{100}\right) - 4\ \ln\left(\frac{T_\mathrm{n}}{100\ \mathrm{GeV}}\right)$$

Can be solved by repeated evaluation of S_3/T and subsequent minimization.

GW parameter calculation

Radiation energy density at nucleation

$$\rho_{\mathsf{R}} = \frac{\pi^2}{30}\left(g_{\mathsf{eff},\rho}^{\mathsf{SM,n}} + g_{\mathsf{eff},\rho}^{\mathsf{DS,n}}\,\xi^4\right)\left(T_{\mathsf{SM}}^{\mathsf{n}}\right)^4$$

Transition strength

$$\alpha = \frac{1}{\rho_{\mathsf{R}}}\left(-\Delta V + T_{\mathsf{DS}}^{\mathsf{n}}\,\frac{\partial \Delta V}{\partial T}\bigg|_{T_{\mathsf{DS}}^{\mathsf{n}}}\right)$$

Inverse time scale

$$\frac{\beta}{H} = T_{\mathsf{DS}}^{\mathsf{n}}\,\frac{\mathrm{d}S_{\mathsf{E}}(T)}{\mathrm{d}T}\bigg|_{T_{\mathsf{DS}}^{\mathsf{n}}}$$

Critical transition strength for runaway bubbles

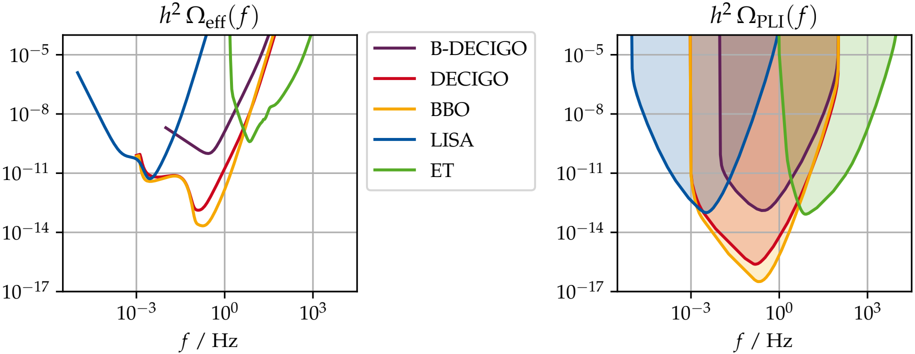
$$\alpha_\infty = \frac{\left(T_{\mathsf{DS}}^{\mathsf{n}}\right)^2}{\rho_{\mathsf{R}}}\left(\sum_{i=\mathsf{bos}} n_i\frac{\Delta m_i^2}{24} + \sum_{i=\mathsf{fer}} n_i\frac{\Delta m_i^2}{48}\right)$$

SGWB model

$$\Omega_{\mathsf{GW}}(f) = \frac{1}{\rho_c}\frac{\mathrm{d}\rho_{\mathsf{GW}}(f)}{\mathrm{d}\log f} \simeq \sum \mathcal{N}\,\Delta\left(\frac{\kappa\,\alpha}{1+\alpha}\right)^p \left(\frac{H}{\beta}\right)^q s(f)$$

	Scalar field Ω_ϕ	Sound waves Ω_{sw}	Turbulence Ω_{turb}
$\mathcal{N}$	1	$1.59\cdot 10^{-1}$	$2.01\cdot 10^{1}$
κ	κ_ϕ	κ_{sw}	$\varepsilon_{\mathsf{turb}}\kappa_{\mathsf{sw}}$
p	2	2	$\frac{3}{2}$
q	2	1	1
Δ	$\frac{0.11v_w^3}{0.42+v_w^2}$	v_w	v_w
f_{p}	$\frac{0.62\beta}{1.8-0.1v_w+v_w^2}$	$\frac{2\beta}{\sqrt{3}v_w}$	$\frac{3.5\beta}{2v_w}$
$s(f)$	$\frac{3.8(f/f_{\mathsf{p}})^{2.8}}{1+2.8(f/f_{\mathsf{p}})^{3.8}}$	$(f/f_{\mathsf{p}})^3\left(\frac{7}{4+3(f/f_{\mathsf{p}})^2}\right)^{7/2}$	$\frac{(f/f_{\mathsf{p}})^3}{(1+f/f_{\mathsf{p}})^{11/3}[1+8\pi(f/H)]}$

Experimental sensitivities



Effective degrees of freedom

$$g^x_{\mathrm{eff},\rho}(T_x) \equiv \frac{\rho_x(T_x)}{\rho^{\mathrm{rel}}_{\mathrm{bos}}(T_x)\big|_{g=1}} = g_x \frac{15}{\pi^4}\int_{z_x}^{\infty} \mathrm{d}u_x \, \frac{u_x^2\sqrt{u_x^2 - z_x^2}}{e^{u_x} \pm 1} \,,$$

$$g^x_{\mathrm{eff},P}(T_x) \equiv \frac{P_x(T_x)}{P^{\mathrm{rel}}_{\mathrm{bos}}(T_x)\big|_{g=1}} = g_x \frac{15}{\pi^4}\int_{z_x}^{\infty} \mathrm{d}u_x \, \frac{\left(u_x^2 - z_x^2\right)^{3/2}}{e^{u_x} \pm 1} \,,$$

$$g^x_{\mathrm{eff},s}(T_x) = \frac{3\, g^x_{\mathrm{eff},\rho}(T_x) + g^x_{\mathrm{eff},P}(T_x)}{4} \,,$$

where $u_x = \sqrt{m_x^2 + p^2}/T_x$ and $z_x = m_x/T_x$. Sum over all SM and DS species:

$$g^{\mathrm{tot}}_{\mathrm{eff},\rho} = g^{\mathrm{SM}}_{\mathrm{eff},\rho}(T_{\mathrm{SM}}) + g^{\mathrm{DS}}_{\mathrm{eff},\rho}(T_{\mathrm{SM}})\, \xi^4(T_{\mathrm{SM}})$$

$$g^{\mathrm{tot}}_{\mathrm{eff},s} = g^{\mathrm{SM}}_{\mathrm{eff},s}(T_{\mathrm{SM}}) + g^{\mathrm{DS}}_{\mathrm{eff},s}(T_{\mathrm{SM}})\, \xi^3(T_{\mathrm{SM}})$$

Mediator cannibalism

Conserved comoving mediator entropy $s_\text{med}\, a^3 = \text{const}$ gives

$$\frac{\mathrm{d}\ln s_\text{med}}{\mathrm{d}t} = \frac{\mathrm{d}\ln s_\text{med}}{\mathrm{d}\ln \rho_\text{med}} \frac{\dot{\rho}_\text{med}}{\rho_\text{med}} = -3\, H(t)\ ,$$

from which follows that

$$\dot{\rho}_\text{med} = -3\, \frac{\mathrm{d}\ln \rho_\text{med}}{\mathrm{d}\ln s_\text{med}}\, H(t)\, \rho_\text{med}(t)\ .$$

For $\mu_\text{med} = 0$, one can find function $\rho_\text{med}(s_\text{med})$, independent of particle species:

$$\frac{\mathrm{d}\ln \rho_\text{med}}{\mathrm{d}\ln s_\text{med}} = \frac{\mathrm{d}\rho_\text{med}}{\mathrm{d}s_\text{med}} \frac{s_\text{med}}{\rho_\text{med}} = \frac{\mathrm{d}\bar{\rho}_\text{med}}{\mathrm{d}\bar{s}_\text{med}} \frac{\bar{s}_\text{med}}{\bar{\rho}_\text{med}} = \frac{\mathrm{d}\ln \bar{\rho}_\text{med}}{\mathrm{d}\ln \bar{s}_\text{med}} = \frac{\mathrm{d}\ln \bar{\rho}}{\mathrm{d}\ln \bar{s}}$$

with $\bar{s}_\text{med} \equiv 2\,\pi^2\, s_\text{med}/(g_\text{med}\; T_\text{DS}^3)$ and $\bar{\rho}_\text{med} \equiv 2\,\pi^2\, \rho_\text{med}/(g_\text{med}\; T_\text{DS}^4)$.

Mediator cannibalism

That yields

$$\dot{\rho}_{\text{med}} \simeq -3\,\zeta\,H\,\rho_{\text{med}} - \Gamma\,\rho_{\text{med}}$$

with

$$\zeta(t) = \begin{cases} \dfrac{\mathrm{d}\ln\bar{\rho}}{\mathrm{d}\ln\bar{s}}\,(\rho_{\text{med}}) & \text{for} \quad \Gamma_{\text{nc}}(t) \geq H(t) \\ 4/3 & \text{for} \quad \Gamma_{\text{nc}}(t) < H(t), \quad t < \tilde{t} \\ 1 & \text{for} \quad \Gamma_{\text{nc}}(t) < H(t), \quad t \geq \tilde{t} \end{cases},$$

where $\tilde{t} = 7\,t_{\text{cd}}\,(T_{\text{DS}}^{\text{cd}}/m_{\text{med}})^2$ denotes the time when the mediator gets non-relativistic. Number changing process rate is approximated by

$$\Gamma_{\text{nc}} \simeq \Gamma_{32} \simeq \langle\sigma_{32}\,v^2\rangle\,n_{\text{med}}^2$$

Mediator cannibalism

The averaged cross section reads

$$\langle\sigma_{32}\, v^2\rangle = \frac{25\,\sqrt{5}\,\alpha_{32}^3}{3072\,\pi\, m_{\mathsf{med}}^5} + \mathcal{O}\left(\frac{T_{\mathsf{DS}}}{m_{\mathsf{med}}}\right).$$

where

$$(4\,\pi\,\alpha_{32})^3 \equiv \left(\frac{\kappa_3}{m}\right)^2 \left[\left(\frac{\kappa_3}{m}\right)^2 + 3\,\kappa_4\right]^2$$

for a potential $V(\phi) = \frac{m^2}{2}\,\phi^2 + \frac{\kappa_3}{3!}\,\phi^3 + \frac{\kappa_4}{4!}\,\phi^4$. In our model: $\alpha_{32} = 2.3\,\lambda$.

Coupled set of ODEs underlying the entropy injection

$$\bar{a}' = \frac{\bar{a}}{\theta_{\mathrm{H}}} \sqrt{r + \frac{f_{\mathrm{mat}}}{\bar{a}^3} + \frac{f_{\mathrm{rad}}}{\bar{a}^4} \, \frac{\gamma}{\gamma_{\mathrm{cd}}} \, \frac{\mathcal{S}}{\mathcal{G}^{1/3}}} \,,$$

$$\mathcal{S}' = \frac{r\,\bar{a}^4}{f_{\mathrm{rad}}} \, \mathcal{G}^{1/3} \, \gamma_{\mathrm{cd}} \,,$$

$$r' = -r - 3\,\frac{\bar{a}'}{\bar{a}}\,\zeta\, r \,,$$

$$\mathcal{G}' = -\frac{3}{4}\,\frac{T_{\mathrm{SM}}^{\mathrm{cd}}\,\mathcal{G}\,\hat{\mathcal{G}}}{\mathcal{S}^{3/4}\,\bar{a}}\,\frac{4\,\mathcal{S}\,\bar{a}' - \mathcal{S}'\,\bar{a}}{T_{\mathrm{SM}}^{\mathrm{cd}}\,\hat{\mathcal{G}}\,\mathcal{S}^{1/4} + 3\,\mathcal{G}^{4/3}\bar{a}} \,,$$

$$\gamma' = \hat{\gamma}\;T_{\mathrm{SM}}^{\mathrm{cd}}\,\frac{3\,\mathcal{G}\,\bar{a}\,\mathcal{S}' - 12\,\mathcal{G}\,\bar{a}'\,\mathcal{S} - 4\,\mathcal{G}'\,\bar{a}\,\mathcal{S}}{12\,\mathcal{G}^{4/3}\,\mathcal{S}^{3/4}\,\bar{a}^2} \,.$$

with initial condition $\bar{a}_{\mathrm{cd}} = \mathcal{S}_{\mathrm{cd}} = r_{\mathrm{cd}} = \mathcal{G}_{\mathrm{cd}} = 1$ and γ_{cd}.

- Normalized scale factor $\bar{a} = a/a_{\mathrm{cd}}$
- Characteristic time scale $\theta_{\mathrm{H}} = \sqrt{3\,m_{\mathrm{Pl}}^2\,\Gamma^2\rho_{\mathrm{med}}^{\mathrm{cd}}}$
- Normalized mediator energy density $r = \rho_{\mathrm{med}}/\rho_{\mathrm{med}}^{\mathrm{cd}}$
- Normalized initial DM density $f_{\mathrm{mat}} = \rho_{\mathrm{DM}}^{\mathrm{cd}}/\rho_{\mathrm{med}}^{\mathrm{cd}}$
- Normalized initial radiation energy density $f_{\mathrm{rad}} = \rho_{\mathrm{rad}}^{\mathrm{cd}}/\rho_{\mathrm{med}}^{\mathrm{cd}}$
- Normalized DOFs $\gamma = g_{\mathrm{eff},\rho}^{\mathrm{SM}}/g_{\mathrm{eff},s}^{\mathrm{SM}}$
- Normalized DOFs $\mathcal{G} = g_{\mathrm{eff},s}^{\mathrm{SM}}/g_{\mathrm{eff},s}^{\mathrm{SM,cd}}$
- Normalized SM entropy $\mathcal{S} = \left(S_{\mathrm{SM}}/S_{\mathrm{SM}}^{\mathrm{cd}}\right)^{4/3}$

Redshift and dilution of the GW background

After its emission, the GW signal gets red-shifted:

$$h^2\,\Omega_{\mathsf{GW}}(f) = \mathcal{R}h^2\;\Omega^{\mathsf{n}}_{\mathsf{GW}}\left(\frac{a_0}{a_{\mathsf{n}}}\,f\right)$$

Energy density:

$$\mathcal{R}h^2 \simeq \frac{2.4\cdot 10^{-5}}{D_{\mathsf{SM}}^{4/3}}\left(\frac{g^{\mathsf{SM,0}}_{\mathsf{eff},s}}{g^{\mathsf{SM,n}}_{\mathsf{eff},s}}\right)^{4/3}\frac{g^{\mathsf{tot,n}}_{\mathsf{eff},\rho}}{2}$$

Frequency:

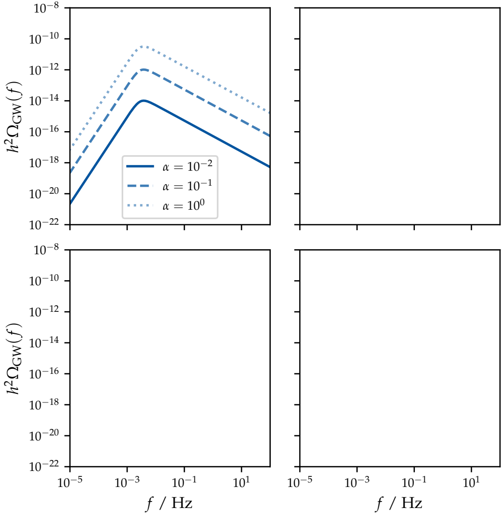
$$\frac{a_0}{a_{\mathsf{n}}} = D_{\mathsf{SM}}^{1/3}\left(\frac{g^{\mathsf{SM,n}}_{\mathsf{eff},s}}{g^{\mathsf{SM,0}}_{\mathsf{eff},s}}\right)^{1/3}\frac{T^{\mathsf{n}}_{\mathsf{SM}}}{T^{0}_{\mathsf{SM}}}$$

Parametrization of the GW signal

Transition strength:

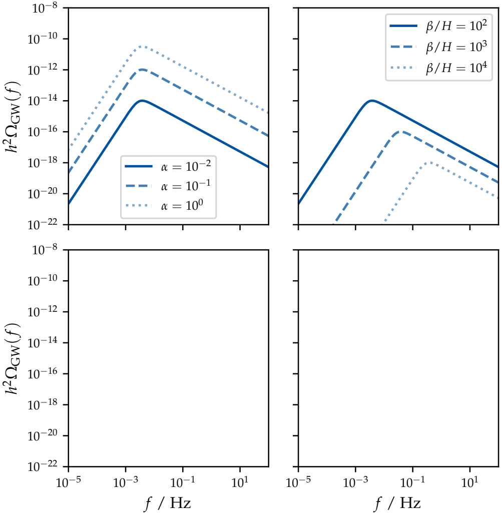
$$\alpha = \frac{\epsilon}{\rho^{\mathrm{n}}_{\mathrm{rad}}}$$

relates the latent heat ϵ of the transition with the energy density $\rho^{\mathrm{n}}_{\mathrm{rad}}$ of the surrounding heat bath. For fixed $T^{\mathrm{n}}_{\mathrm{DS}}$: $\rho^{\mathrm{n}}_{\mathrm{rad}} \propto \xi_{\mathrm{n}}^{-4}$. The transition strength thus grows $\propto \xi_{\mathrm{n}}^{4}$!



Parametrization of the GW signal

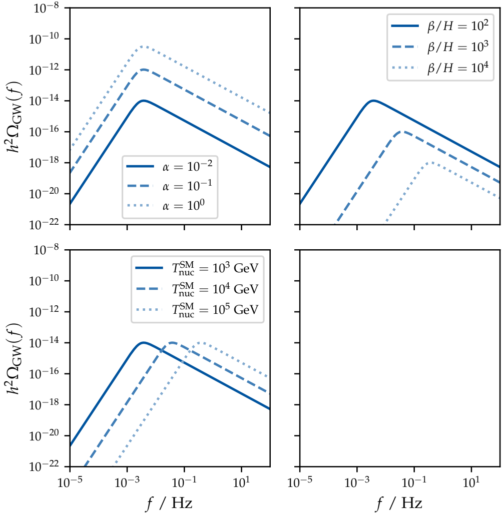
Inverse time scale:
The computation of β/H is complicated, but shows no relevant dependence of the temperature ratio between the sectors. Larger β/H indicate fast transitions. In that case, many small bubbles collide, resulting in weak signals at high frequencies.



Parametrization of the GW signal

Nucleation temperature:

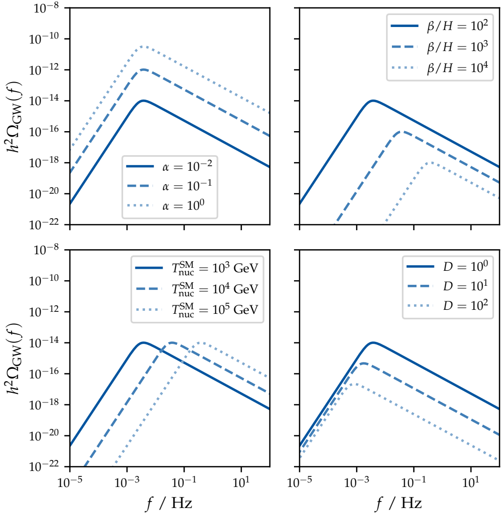
Keeping $T_{\mathrm{DS}}^{\mathrm{n}}$ fixed, a larger temperature ratio ξ_{n} at nucleation leads to a lower $T_{\mathrm{SM}}^{\mathrm{n}}$. This corresponds to lower peak frequencies.



Parametrization of the GW signal

Dilution:

The redshift to lower frequencies and signals strengths increases with the dilution factor. D grows with the temperature ratio ξ_{n}, as more energy is injected into the SM bath from the dark sector. Unlike D_{SM}, D saturates for high temperature ratios.



The $\mathrm{U}(1)_\mathrm{D}$ model in detail

Lagrangian:

$$\mathcal{L} \supset |D_\mu \, \Phi|^2 + |D_\mu \, H|^2 - \frac{1}{4} \, B'_{\mu\nu} \, B'^{\mu\nu} - \frac{\epsilon}{2} \, B'_{\mu\nu} \, B^{\mu\nu} - V(\Phi, H) \ ,$$

$$D_\mu \, \Phi = \left(\partial_\mu + i \, g \, B'_\mu\right) \Phi \ ,$$

$$V_{\text{tree}}(\Phi, H) = -\mu^2 \, \Phi^* \, \Phi + \lambda \, (\Phi^* \, \Phi)^2 - \mu_H^2 \, H^\dagger \, H + \lambda_H \, (H^\dagger \, H)^2 + \lambda_p \, (\Phi^* \, \Phi) \, (H^\dagger \, H) \ .$$

Mass spectrum:

$$m^2_{(h,\,\phi)}(h, \phi) = \begin{pmatrix} -\mu_H^2 + 3 \, \lambda_H \, h^2 + \frac{\lambda_p}{2} \phi^2 & \lambda_p \, h \, \phi \\ \lambda_p \, h \, \phi & -\mu^2 + 3 \, \lambda \, \phi^2 + \frac{\lambda_p}{2} h^2 \end{pmatrix} ,$$

$$m^2_{G^0,G^+}(h, \phi) = -\mu_H^2 + \lambda_H \, h^2 + \frac{\lambda_p}{2} \phi^2 \ ,$$

$$m^2_\varphi(h, \phi) = -\mu^2 + \lambda \, \phi^2 + \frac{\lambda_p}{2} h^2 \ .$$

The $\mathrm{U}(1)_{\mathsf{D}}$ model in detail

For $\lambda_p, \epsilon \to 0$ and $\mu^2 = \lambda\, v^2$, the field-dependent dark Higgs and dark photon masses are given by

$$m_{\mathsf{DP}} = g\,\phi \overset{T=0}{=} g\,v\,, \qquad\qquad m_{\mathsf{DH}} = \sqrt{2\,\lambda}\,\phi \overset{T=0}{=} \sqrt{2\,\lambda}\,v\,.$$

The corresponding Debye masses are

$$\Pi_\Phi(T_{\mathsf{DS}}) = \left(\frac{\lambda}{3} + \frac{g^2}{4}\right)\, T_{\mathsf{DS}}^2\,, \qquad\qquad \Pi^{\mathsf{L}}_{A'}(T_{\mathsf{DS}}) = \frac{g^2}{3}\, T_{\mathsf{DS}}^2\,.$$

- Quartic dark Higgs coupling: λ
- $U(1)_{\mathsf{D}}$ gauge coupling: g
- Dark Higgs lifetime: τ

- Dark Higgs VEV: $v = \frac{\mu}{\sqrt{\lambda}}$
- Temperature ratio: $\xi_{\mathsf{n}} = \left.\frac{T_{\mathsf{DS}}}{T_{\mathsf{SM}}}\right|_{\mathsf{n}}$

Signal-to-noise ratios for LISA and the ET

Compute the overlap of the signals $h^2\,\Omega_{\mathsf{GW}}(f)$ and expected sensitivities $h^2\,\Omega_{\mathsf{obs}}(f)$ and weight it with the duration of the observation t_{obs} to obtain a signal-to-noise measure:

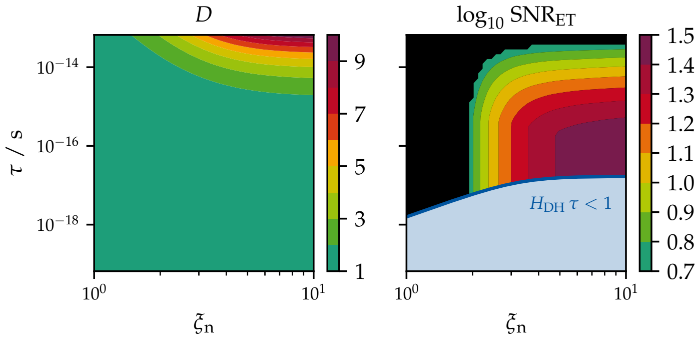
$$\rho^2 = t_{\mathsf{obs}} \int_{f_{\mathsf{min}}}^{f_{\mathsf{max}}} \mathrm{d}f \, \left[\frac{h^2\,\Omega_{\mathsf{GW}}(f)}{h^2\,\Omega_{\mathsf{obs}}(f)} \right]^2$$

If ρ exceeds a certain threshold value for a given signal, the signal is observable.

To analyze the impact of ξ_{n} and τ on the observability of the signals produced by our model, consider the benchmark points

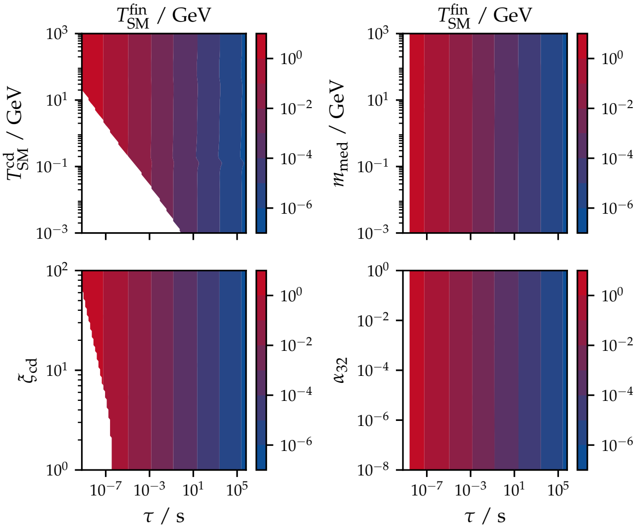
Benchmark point	λ	g	v
LISA	$1.5 \cdot 10^{-3}$	0.5	2 TeV
ET	$1.5 \cdot 10^{-3}$	0.5	10 PeV

ET benchmark point, $v = 10\,\mathrm{PeV}$



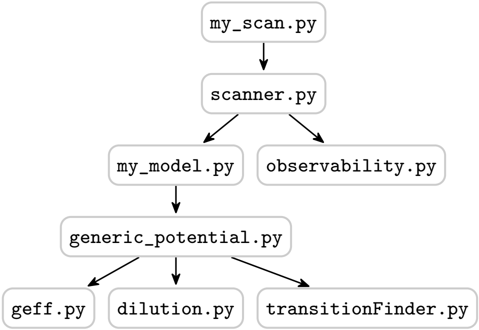
Same picture as for LISA, but weaker signal-to-noise ratios due to steeper ET sensitivity curves. Highest observability for hot and short-lived dark sectors. Signals observable for $\xi_{\mathrm{n}} > 2$ and $10^{-17}\,\mathrm{s} \lesssim \tau \lesssim 10^{-14}\,\mathrm{s}$.

Final temperature independent of all input parameters except lifetime



Our extensions to CosmoTransitions

Structure:



my_scan.py

scanner.py

my_model.py

observability.py

generic_potential.py

geff.py

dilution.py

transitionFinder.py

Example output: