Reply to the Referee: JHEP_073P_0426

We thank the referee for the careful reading of our manuscript and for the constructive comments. We are glad that the referee considers a revised version publishable in JHEP, and we have revised the manuscript accordingly. In the course of this revision we have also improved the readability of the draft, in particular by moving more technical material to the appendices (see points 2 and 5). Below we reproduce the referee's comments in italics and give our replies; the corresponding changes are highlighted in red in the revised manuscript.

General remark (review character and novelty).

We agree that two of the three models (the Abelian Higgs and the conformal $U(1)'$ dark sector) have been discussed before. They have not, however, been studied at the level adopted here: we update the analysis to the NANOGrav 15 yr data, apply a consistent data analysis and a uniform treatment of the GW spectra across all three models, study the complementary laboratory and cosmological constraints on each model's parameter space, and, for the first time, confront the different model classes within a single comparative study. This comparison is a standalone contribution that does not exist in the literature, and it rests on the dedicated numerical framework presented in our companion code paper [53]. We have added a corresponding statement to the introduction to make this scope explicit.

1) *Check the SMBH uncertainty band in the right panel of Fig. 1. The upper range of the band looks lower than it should be.*

We thank the referee for prompting us to re-examine this. The grey band shows the 1σ variation of the GW amplitude of the SMBHB contribution resulting from a global fit including both the SMBHBs as well as a phase transition signal. We point out that it is *not* the maximal envelope of allowed spectra, and that it is in addition parameterisation-dependent. Moreover, the parameterisation of the SMBHB signal is an an active topic in the literature. We therefore specified our use of the (A, γ) parameterisation recommended by the NANOGrav new-physics search [20] and added the following remark, together with references to recent discussions of SMBHB spectral uncertainties, clarifying both the σ-level nature of the band and its dependence on the chosen parameterisation.

We note that the grey band in this figure represents the 1σ band of the SMBHB contribution to the total GW signal steming from a global fit including both the SMBHB and the phase transition signal to the NANOGrav 15 yr data. The grey line and the corresponding width of the grey band are dominated by the adopted prior distribution of the amplitude A and slope γ, following ref. [20]. We further point out that the grey band does not show the maximal envelope of allowed SMBHB spectra, which moreover is parameterisation-dependent [15, 20, 70, 71, 72].

2) *Section 3.1 applies for $T_p \sim T_c$ and thin walls, but PTA data require $T_p \ll T_c$ and thick walls. Either revise this section or remove it.*

We thank the referee for this clarification and agree. Section 3.1 was intended only to build intuition for the generic correlation between α and β/H, and it explicitly states that the thin-wall estimates break down in the thick-wall regime relevant for the PTA data, where the full numerical machinery is required. To make the status of this section clearer and to improve the readability of the main text (see also point 5), we have moved it to an appendix.

3) *In footnote 4, the authors mention that because of computational limitations, to make the confidence regions in the right panel of Fig. 8, they had to terminate their sampling before* *convergence was reached, that the regions for the Abelian Higgs and flip-flop models should be regarded as indicative, and that this does not affect their results qualitatively. However, the regions in Figs. 10–12 also appear to be the result of incomplete sampling. The authors should enlarge their samples and redo these figures. It is worth the trouble for a journal publication.*

We partly agree. The conformal and Abelian dark Higgs posteriors appear well converged. The genuinely critical case is the flip-flop model, for which we agree that a longer chain would be warranted for a fully rigorous statistical interpretation; this is exactly why we flagged this caveat in footnote 4. We appreciate the referee's view that this would be worth the effort, and we do see the point. After careful consideration, however, we concluded that the additional computational cost is not justified here, because this limitation is intrinsic to studies of this type: the 2D bounce-action computation is very expensive. Pushing substantially beyond this for a single figure is, in our assessment, computationally unreasonable. Crucially, our main conclusion (the tuning argument) does not depend on the precise contour shapes. It is established already at the qualitative level, and is corroborated both by the strong sensitivity of the signal to small variations of the input parameters and by the bounce-action and nucleation-rate behaviour shown in figure 8. We have revised footnote 4 and the surrounding discussion to make explicit which posteriors are converged and why the residual flip-flop sampling limitation does not affect our conclusions. We further describe the numerical cost to indicate the scale of the problem:

The conformal and dark Abelian Higgs model posteriors appear well converged, whereas the flip-flop model lacks samples to draw meaningful statistical conclusions from this plot: the two-step transition occupies a narrow ridge in its six-dimensional parameter space, directly related to the tuning discussed below. We refrained from drawing more samples, as the 2D bounce-action computation is a numerical bottleneck: each of the over 500,000 evaluated model realisations required up to 1 h on a single CPU, corresponding to 10 nodes with each 56 cores running over more than a month. Extending it substantially up to the point of convergence is hence computationally unreasonable and would require dedicated computational resources. Crucially, however, the tuning conclusion does not depend on the precise contour shapes: it is established qualitatively and is confirmed by the strong dependence of the signal to small input-parameter variations (cf. table 1) and by the bounce-action and nucleation-rate behaviour shown in figure 8.

4) *The text tends to digress from the goal of studying the GW signal at PTAs, e.g., after confirming that the Abelian Higgs model does not reproduce the PTA signal, they go on to discuss dark photons/dark Higgs phenomenology. Most of this could be removed or moved to an Appendix.*

We would prefer to keep this material in the main text. The discussion of complementary constraints (collider, beam-dump, supernova, BBN/CMB) alongside the GW interpretation is, in our view, a standalone strength of the work and a particularly good match for JHEP: it is precisely this laboratory complementarity that connects the analysis to the JHEP readership, whereas without it a purely cosmological venue might be more natural. Presenting the model interpretation together with its complementary constraints was therefore a deliberate choice; they belong together, in contrast to much of the existing literature, which frequently omits such constraints altogether. We regard this bridging of the GW/cosmology and collider/dark-sector communities as a core and novel feature of the paper. We acknowledge that it comes at the price of some additional length, since these constraints are genuinely more involved than imposing a single bound, and we have tightened the presentation where possible (see points 2 and 5).

We also note that the referee characterises our result as "confirming that the Abelian Higgs model does not reproduce the PTA signal." This does not match our conclusion: the model *can* reproduce the signal, but only at the cost of significant parameter tuning, and this is precisely the quantity we set out to measure and compare.

5) *More generally, the authors should streamline the paper by removing redundant material.*

We have moved section 3.1 to an appendix and shortened the introduction of section 3. This has improved the readability and flow of the main text and we thank the referee for the helpful comment.

6) *A minor point: the definition of α should appear earlier. It is first defined in Section 3, but is already used in Fig. 1 in Section 2.*

We thank the referee for catching this oversight. We have moved the definition of α to appear in Section 2, preceding its first use in Fig. 1.

We hope that, with these revisions, the referee finds the manuscript suitable for publication in JHEP.