A natural SUSY Higgs near 125 GeV

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Complements of particle physics,
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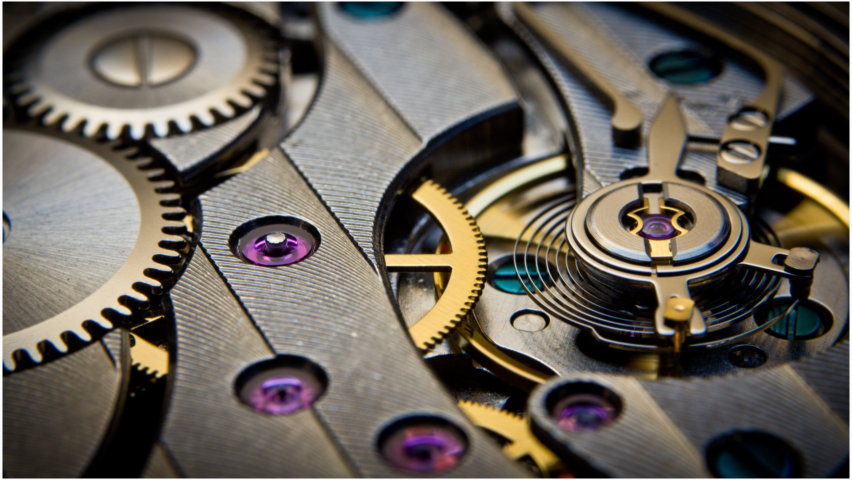

Figure 1: Fine tuning in a nutshell.

Introduction: Fine tuning, hierarchy and naturalness

Fine tuning: Process in which the parameters of a theory are adjusted precisely to fit with observations.

Hierarchy: Existence of dimensionless ratios between free parameters of a theory which are not $\mathcal{O}(1)$.

Naturalness: Property of a theory, whose parameters are neither *hierarchical* nor *fine-tuned*.

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Naturalness: Property of a theory, whose parameters are neither *hierarchical* nor *fine-tuned*.

Example: An albeit perfectly predictive theory with the following parameters

- 1.26
- 0.8
- $3.21831287362871 \cdot 10^{31}$

is both fine-tuned and hierarchical and would thus be regarded as *unnatural*.

Solution: ~~Acceptance and suffering, Anthropic Principle, God or~~ lack of deep understanding.

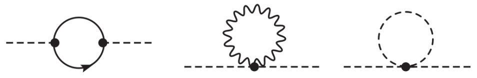

Figure 2: Radiative corrections due to top quarks, gauge bosons and virtual Higgs bosons.

SM: scalar particle masses not protected against radiative corrections

Higgs mass: $m_h^2 = 2\mu^2 + \delta m_h^2$ with

$$\delta m_h^2 \simeq \frac{3}{4\pi^2}\left(-\lambda_t^2 + \frac{g^2}{4} + \frac{g^2}{8\cos^2\theta_{\mathrm{W}}}\right)\Lambda^2$$

due to loop diagrams with top quarks, gauge bosons and virtual Higgs bosons.

- Little Hierarchy problem
    - $\Lambda \sim 10\,\text{TeV}$
    - $\delta m_h^2 \sim (100 - 10 - 5)\,(200\,\text{GeV})^2$
  - $\rightsquigarrow$ Fine tuning $\sim 1\,\%$

- Big Hierarchy problem
    - GUT scale 10^{15} GeV
    - New particles X, Y and Φ
    - $\delta m_h^2 \sim \mathcal{O}\left(\Lambda^2\right) \sim m_\Phi^2 \sim 10^{30}\,\text{GeV}^2$
  - $\rightsquigarrow$ Fine tuning $\sim 10^{-26}$

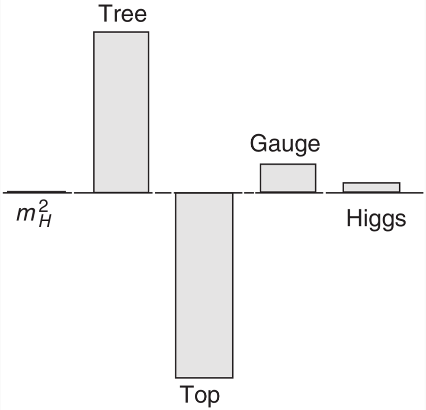

Figure 3: The little Hierarchy problem.

Possible Solutions:

- $\Lambda < 1\,\mathrm{TeV}$
- Composite Higgs
- Extra dimensions
- New symmetry: SUSY

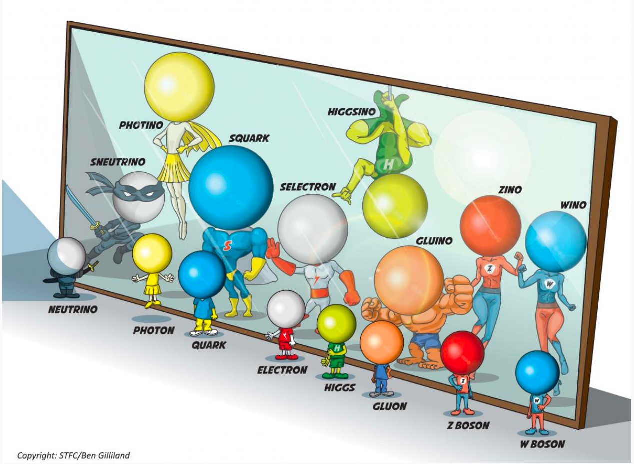

Figure 4: A supersketch.

Three SUSYs

MSSM

SUSY with minimal number of new particles and interactions w.r.t. SM

NMSSM

MSSM with solved μ problem, more Higgses and parameter $\lambda \lesssim 0.7$.

λSUSY

NMSSM with augmented validity $\lambda \lesssim 2$ and interesting features.

$$m_h^2 \leq m_Z^2 \cos^2 2\beta + \delta_t^2 + \lambda^2 v^2 \sin^2 2\beta$$

where $\delta_t^2 = \frac{3}{2\pi} \frac{m_t^4}{v^2} \left[\ln \left(\frac{m_{\tilde{t}_1} m_{\tilde{t}_2}}{m_t^2} \right) + \frac{X_t^2}{m_{\tilde{t}_1} m_{\tilde{t}_2}} \left(1 - \frac{X_t^2}{12 m_{\tilde{t}_1} m_{\tilde{t}_2}} \right) \right]$

Stop mass mixing

The squark and slepton masses are determined by the soft SUSY breaking parameters $m_{Q_i}, m_{U_i}, m_{D_i}, m_{L_i}, m_{E_i}$ where $i = 1 - 3$ is the family index.

Stop sector

$\tilde{t}_L$ and $\tilde{t}_R$ have same quantum numbers and mix to $\tilde{t}_1$ and $\tilde{t}_2$ where $m_{\tilde{t}_1} < m_{\tilde{t}_2}$

$$M_{\tilde{t}}^2 \simeq \begin{pmatrix} m_{Q_3}^2 + m_t^2 & m_t \left(A_t - \mu^* \cot\beta\right) \\ m_t \left(A_t^* - \mu \cot\beta\right) & m_{U_3}^2 + m_t^2 \end{pmatrix}$$

Assume no $\not{C}\!P$ in SUSY: $A_t, \mu \in \mathbb{R}$.

Define $X_t \equiv A_t - \mu \cot\beta$ and $m_{\tilde{t}}^2 \equiv m_{\tilde{t}_1} m_{\tilde{t}_2}$

No mixing: $X_t = 0$

Maximal mixing: $X_t = \sqrt{6}\, m_{\tilde{t}}$

MSSM

Phenomenology of the MSSM Higgs bosons

H_u, $H_d \rightsquigarrow$ 8 real scalar fields:
3 absorbed into $W^\pm$ and Z,
5 physical Higgs bosons:
h^0, H^0, A^0 and $H^\pm$

For $m_A \gg m_Z$:

$$m_h^2\big|_{\text{tree}} \to m_Z^2 \cos^2 2\beta$$

Including radiative contribution:

$$m_h^2 = m_Z^2 \cos^2 2\beta + \delta_t^2$$

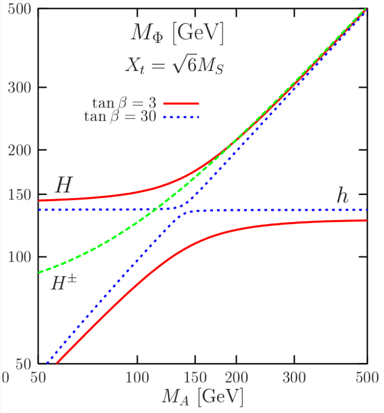

Figure 5: MSSM Higgs mass spectrum in dependence of m_A for $m_{\tilde{t}} = m_{\tilde{t}_1} = m_{\tilde{t}_2} = 2\,\text{TeV}$.

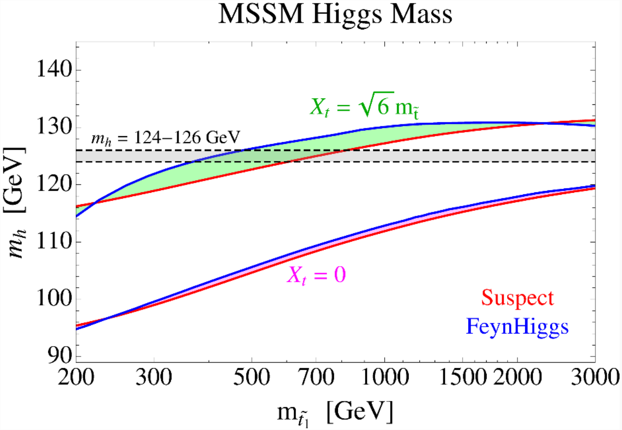

Figure 6: m_h in dependence of $m_{\tilde{t}_1}$ and X_t in MSSM scenario where $\tan\beta = 20$.

Border of parameter space:

- Maximal mixing $X_t = \sqrt{6}\, m_{\tilde{t}}$
- High $\tan\beta = 20$
- Else: Multi-TeV stops
- Highly *unnatural*

$\rightsquigarrow$ How to quantize unnaturalness?

Fine tuning, quantized

Many different, reasonable measures. In our case:

$$\Delta_{m_h} = \max_{p_i} \left| \frac{\partial \ln m_h^2}{\partial \ln p_i} \right|$$

p_i:

- Messenger scale Λ
- Higgsino mass μ, $B\mu$
- Soft breaking m_{Q_3}, m_{U_3}
- Higgs doublet $m_{H_u}^2, m_{H_d}^2$
- Trilinear coupling A_t

Contributions due to radiative corrections, e.g.

$$\delta m_{H_u}^2 = -\frac{3y_t^2}{8\pi^2} \left(m_{Q_3}^2 + m_{U_3}^2 + |A_t|^2 \right) \ln \left(\frac{\Lambda}{m_{\tilde{t}}} \right)$$

$\Delta \sim$	$\mathcal{O}(1)$	natural
$\Delta \gtrsim$	10	rather unnatural
$\Delta \gtrsim$	1000	fine-tuned

Or equivalently: *Degree of fine tuning* $\equiv \frac{1}{\Delta}$

Fine tuning in the MSSM

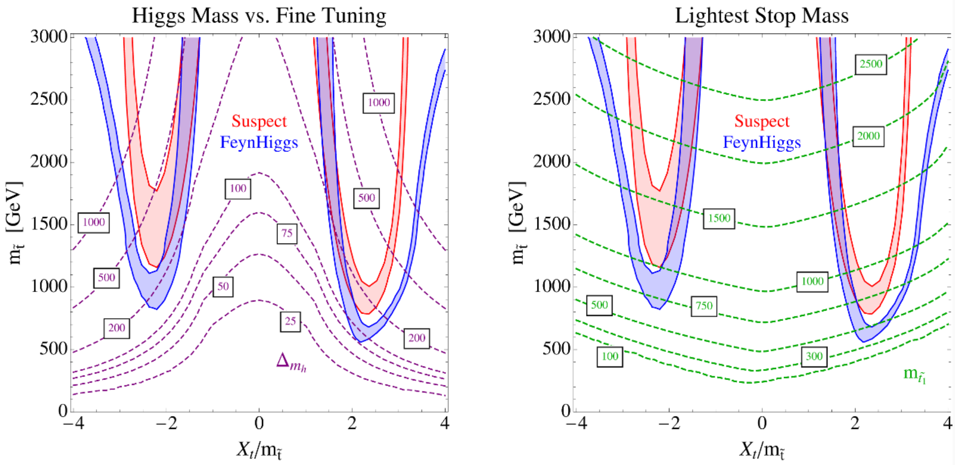

Figure 7: Necessary $m_{\tilde{t}}$ to accommodate a 125 GeV Higgs with iso-value curves for fine tuning and $m_{\tilde{t}_1}$ in dependence of X_t in MSSM scenario with $\Lambda = 10$ GeV.

Observable effects

$$R_{\gamma\gamma} \equiv \frac{\sigma(gg \to h) \times \mathrm{Br}(h \to \gamma\gamma)|_{\mathrm{MSSM}}}{\sigma(gg \to h) \times \mathrm{Br}(h \to \gamma\gamma)|_{\mathrm{SM}}}$$

Heavy stop loops suppress the Higgs-to-gluon coupling

$\Rightarrow$ A natural MSSM would lead to a depleted event rate $R_{\gamma\gamma}$

$\Rightarrow$ Expect effect of $R_{\gamma\gamma} = 80 - 90\%$ in natural theory.

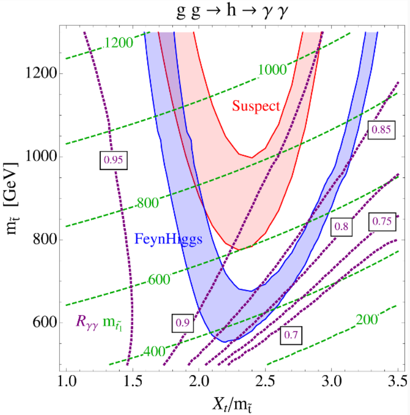

Figure 8: Zoom-in of figure 7. Violet: Ratio of $gg \to h \to \gamma\gamma$ event rate w.r.t. SM prediction.

MSSM Result

Most *natural* parameter choice:

- Maximal mixing $X_t = \sqrt{6}\, m_{\tilde{t}}$
- Large vev ratio $\tan\beta \gtrsim 20$
- Large stop mass $m_{\tilde{t}} > 600\,\mathrm{GeV}$
- Large CP-uneven Higgs mass $m_A \gg m_Z$

No fine tuning better than $\mathcal{O}(1\,\%)$ can be achieved, even with an ultra-low messenger scale of $\Lambda = 10\,\mathrm{TeV}$. Small deviations, e.g. from maximal mixing, increase the necessary stop masses quickly in a multi-TeV regime which would render the theory highly unnatural. ↯

Is a more natural setting possible?

NMSSM

The μ problem of the MSSM

MSSM: superpotential on GUT scale, soft breaking Lagrangian on EW scale.

However, the higgsino mass parameter μ enters the superpotential, even though it should be on the EW scale, ironically giving rise to another hierarchy problem.

Solution:

$$\mu H_u H_d \to (\lambda S + \hat{\mu})\, H_u H_d \to (\lambda \langle S \rangle + \hat{\mu})\, H_u H_d = \mu_{\text{eff}} H_u H_d$$

Introduce singlet Higgs superfield S: MSSM $\to$ NMSSM. For perturbativity up to the GUT scale: $\lambda \lesssim 0.7$

The extended superpotential and soft breaking terms

Superpotential:

$$W \supset (\lambda S + \mu)\, H_u H_d + \frac{M_S}{2} S^2$$

Soft breaking term:

$$V_{\text{soft}} \supset m_{H_u}^2 \left|H_u\right|^2 + m_{H_d}^2 \left|H_d\right|^2 + m_S^2 \left|S\right|^2 + \left(B\mu H_u H_d + \lambda A_\lambda S H_u H_d + \text{h.c.}\right)$$

Note:

M_S	singlet mass
m_S	SUSY breaking mass

Phenomenology:

Added singlino $\tilde{S}$, new A, new H. Completely new mass hierarchy is possible, due to singlet-doublet mixing.

$$m_h^2 \leq m_Z^2 \cos^2 2\beta + \lambda^2 v^2 \sin^2 2\beta + \delta_t^2$$

The new term is rather big for large values of λ, so that no super heavy stops are required

New preferred parameter region

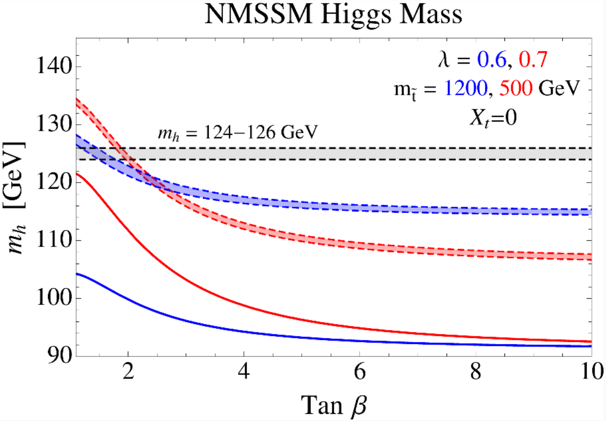

Figure 9: Higgs mass in dependence of $\tan\beta$, λ and $m_{\tilde{t}}$ in NMSSM scenario. The dashed lines include the new $\lambda^2 v^2$ term.

Other region of parameter space naturally preferred

- Small $\tan\beta \lesssim 2$
- Large $\lambda \sim 0.7$
- Reduced necessary δ_t^2
- Mixing effects X_t reduced

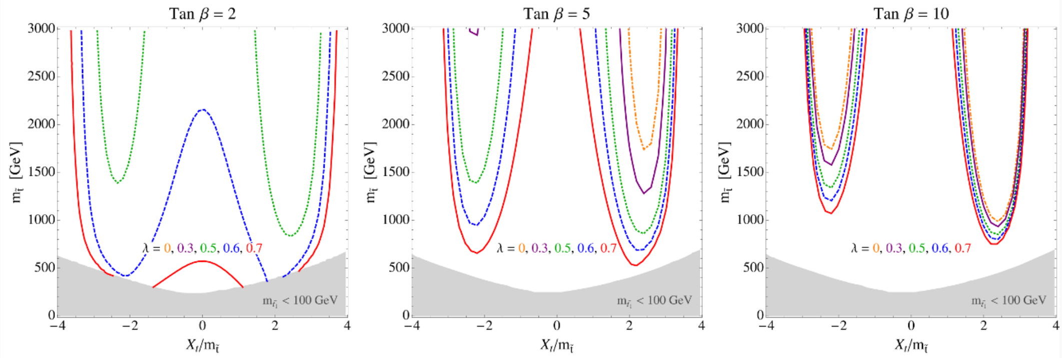

Figure 10: Necessary $m_{\tilde{t}}$ to accommodate a 125 GeV Higgs in dependence of X_t, λ and $\tan\beta$ in NMSSM scenario. For a good choice of parameters only modestly large stop masses of $m_{\tilde{t}_1} > 300$ GeV are necessary, making the theory less fine-tuned eventually.

Amount of necessary fine tuning in NMSSM

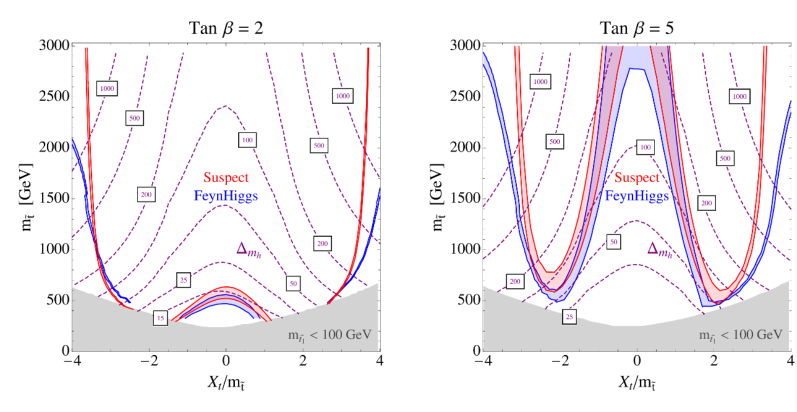

Figure 11: Necessary $m_{\tilde{t}}$ for $m_h = 125\,\text{GeV}$ with iso-value curves for Δ_{m_h} in dependence of X_t and $\tan\beta$ in NMSSM. For $\tan\beta = 2$, $\Delta_{m_h} \lesssim 15$ can be achieved, while $\tan\beta = 5$ requires large stop masses and thus a worse fine tuning.

Optimal choice of parameters

$\Delta_{m_h} \sim 15$ can be achieved in NMSSM with a particular choice of parameters, even though this requires rather heavy stops with $m_{\tilde{t}} > 400$ GeV. For less massive stops with $m_{\tilde{t}} \sim 200$ GeV, a higher mixing and thus a worse $\Delta_{m_h} > 50$ is necessary.

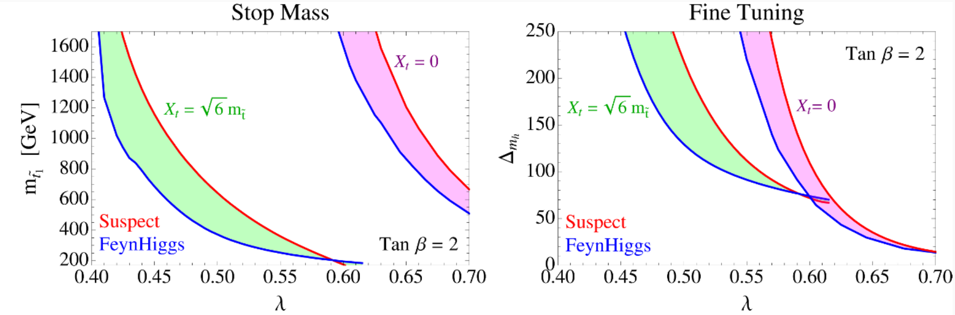

Figure 12: Necessary $m_{\tilde{t}_1}$ and fine tuning Δ_{m_h} to accommodate a 125 GeV Higgs in dependence of λ and X_t in NMSSM scenario.

Summary NMSSM and comparison with MSSM

We see that a substantially better degree of fine tuning of “only” $5 - 10\,\%$ can be achieved, however, also only at a border of the respective parameter space:

NMSSM:

- $\tan\beta < 2$
- $X_t = 0$
- $m_A = 0.5\,\text{TeV}$
- $\Lambda = 10\,\text{TeV}$
- $\mu_{\text{eff}} = 200\,\text{GeV}$
- $\lambda \simeq 0.7$, as large as possible

MSSM:

- $\tan\beta > 20$
- $X_t = \sqrt{6}\, m_{\tilde{t}}$
- $m_A = 1\,\text{TeV}$
- $\Lambda = 10\,\text{TeV}$
- $\mu = 200\,\text{GeV}$

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- $\Lambda = 10\,\text{TeV}$
- $\mu = 200\,\text{GeV}$

What happens when $\lambda > 0.7$?

λ-SUSY

What happens when $\lambda > 0.7$?

$\rightsquigarrow$ **Landau pole** below the GUT scale, **non-perturbative** and **unpredictive** theory, tempers the gauge coupling unification. ↯ However, it can be shown, that these problems become negligible up to 10 TeV as long as $\lambda \lesssim 2$: λ-SUSY.

$$m_h^2 \leq m_Z^2 \cos^2 2\beta + \underbrace{\lambda^2 v^2}_{(200\text{–}300\,\mathrm{GeV})^2} \sin^2 2\beta + \delta_t^2$$

Singlet-doublet mixing becomes necessary!

Singlet-doublett mixing

The two scalars s and h mix when the off-diagonal elements become too large

$$\mathcal{M}^2\big|_{\text{tree}} = \begin{pmatrix} m_Z^2\cos^2 2\beta + \lambda^2 v^2 \sin^2 2\beta & \lambda v(\mu, M_S, A_\lambda) \\ \lambda v(\mu, M_S, A_\lambda) & m_S^2 \end{pmatrix}$$

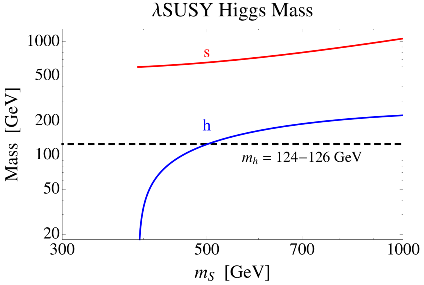


Figure 13: The two scalar masses in dependence of m_S.

For a singlet soft breaking mass near $m_S = 500$ GeV ($\lambda = 2$, $\tan\beta = 2$), mixing reduces m_h from 280 GeV to the measured of value 125 GeV. m_S needs no fine tuning, as can be seen by the low slope near the reference point.

Approach:

Fix a benchmark point with $\Delta_{m_h} = 5.2$ in the parameter space and change two parameters while keeping $m_h = 125\,\text{GeV}$ constant

$\lambda = 2$	$\tan\beta = 2$
$\mu = 200\,\text{GeV}$	$M_S = 0\,\text{GeV}$
$m_S = 510\,\text{GeV}$	$m_{H^+} = 470\,\text{GeV}$
$m_{Q_3} = m_{U_3} = 500\,\text{GeV}$	
$A_t = A_\lambda = 0$	

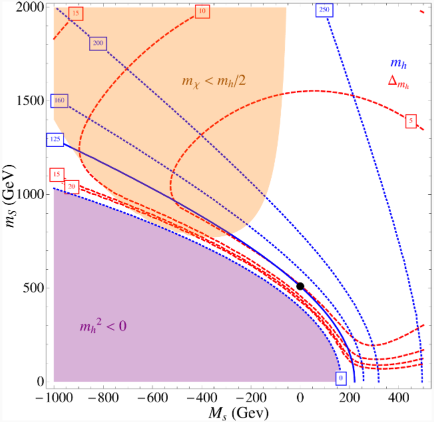


Figure 14: m_h and Δ_{m_h} as a function of singlet mass M_S and soft mass m_S.

Results:

- Δ_{m_h} increases quickly when m_h drops
- $\Delta_{m_h} \sim \mathcal{O}(1)$ in a large region of parameter space
- $\Leftrightarrow$ m_h largely independent of exact choice of parameters
- Excluded: tachyonic Higgses, invisible Higgs decay into neutralinos

Analog for the following graphs...

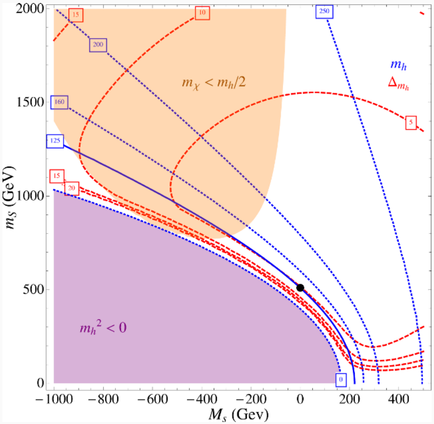


Figure 15: m_h and Δ_{m_h} as a function of singlet mass M_S and soft mass m_S.

Exploration of parameter space

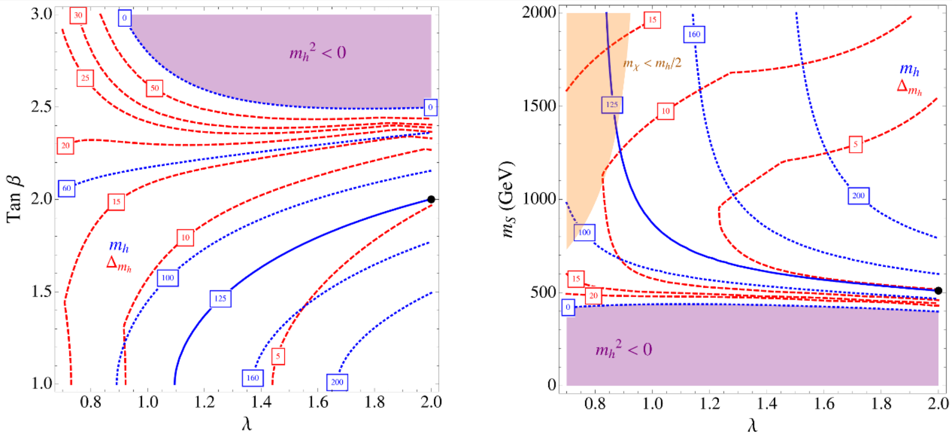


Figure 16: m_h and Δ_{m_h} as a function of λ and $\tan\beta$ (left) as well as λ and the soft mass m_S (right). **Along the 125 GeV lines, Δ_{m_h} does not change strongly.**

Properties of benchmark point and sparticle masses

Benchmark point:
Natural ($\Delta_{m_h} = 5$) Higgs boson with $125\,$GeV due to $\theta_{hs} = 0.12$ doublet-singlet mixing is favored.

$m_{h_2,h_3} = 522\,$GeV, $m_{A_1,A_2} = 580\,$GeV

The graph shows in dependence of λ, that even Terascale sparticles are allowed without contributing dominantly to the fine tuning.

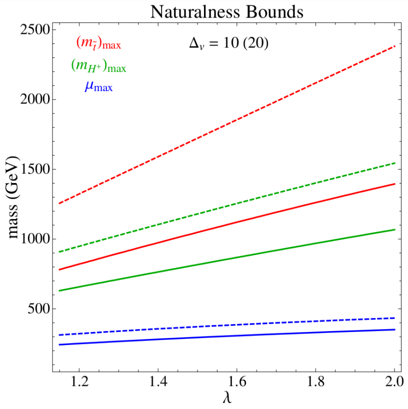
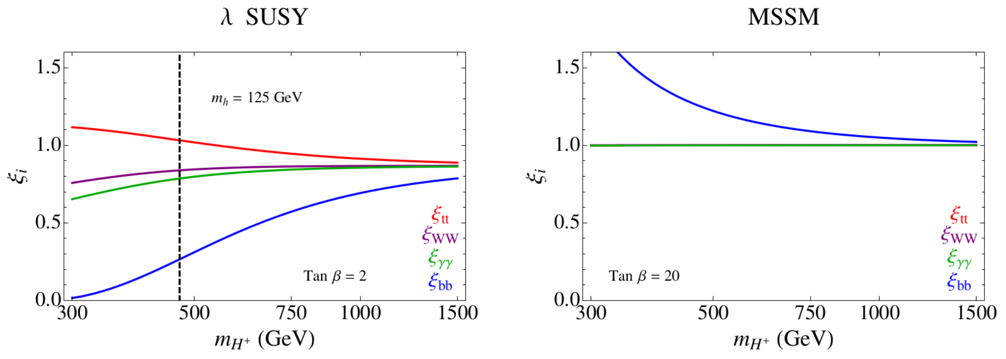


Figure 17: Bounds on the *natural* sparticle masses in dependence of λ. Here $\Delta_v = 10$ is used to obtain more conservative limits.

Observational consequences

A particular Higgs phenomenology is expected. Especially, the important Higgs-bottom coupling $\xi_{bb} = y_b^2/\left. y_b^2\right|_{\text{SM}}$ changes. In SM: $\text{BR}(h \to bb) \simeq 58\,\%$.



$$\left.\xi_{bb}\right|_{\text{tree}} \to 1 - |\sin 4\beta| \tan\beta \left(\frac{\lambda v}{m_H^+}\right)^2 \qquad \left.\xi_{bb}\right|_{\text{tree}} \to 1 + |\sin 4\beta| \tan\beta \left(\frac{m_Z}{m_A}\right)^2$$

Observational consequences

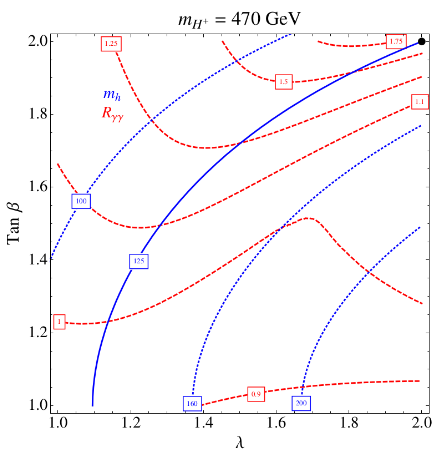


Figure 18: Iso-value curves for m_h and $R_{\gamma\gamma}$ in dependence of λ and $\tan\beta$.

Naturally favored parameter region: Expect depleted Higgs-bottom coupling $\xi_{bb} \simeq 0.3 \rightsquigarrow$ Enhancement of the other final states, e.g. enhanced $\gamma\gamma$ event rate $R_{\gamma\gamma} = 130 - 150\,\%$.

Summary and review

Conclusion

Good news: $m_h = 125.10 \pm 0.14\,\text{GeV} \rightsquigarrow$ The presented analysis is relevant.

- **MSSM:** $1/\Delta_{m_h} = 1\,\%$ at boundary of parameter space
- **NMSSM:** $1/\Delta_{m_h} = 5 - 10\,\%$ at boundary of parameter space
- **λ-SUSY:** $1/\Delta_{m_h} = 10 - 20\,\%$ in large region of parameter space
    - Stop masses don't have to be heavy
    - However, stops can be as heavy as $1.5\,\text{TeV}$ before contributing to Δ_{m_h}
    - Theory is falsifiable with current methods: $R_{\gamma\gamma} \simeq 150\,\%$

Critics and open questions

- Domain wall problem of the added singlet Higgs S
- CP phases are not included
- Impact of $\lambda \simeq 2$ on other parts of physics (Very early universe, cosmic rays, astrophysics, ...)

Thank you for your attention!

Literature

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