

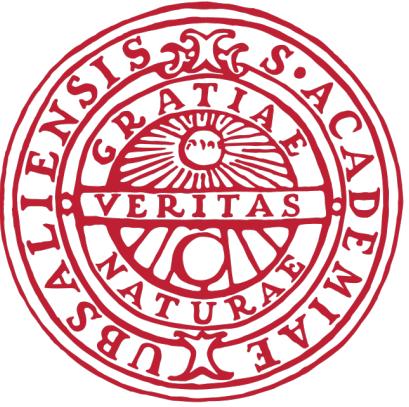
# Exploring the dark universe with gravitational wave backgrounds

Seminar talk at IPNP Prague, 04.12.2024

**Carlo Tasillo,  
Uppsala University**

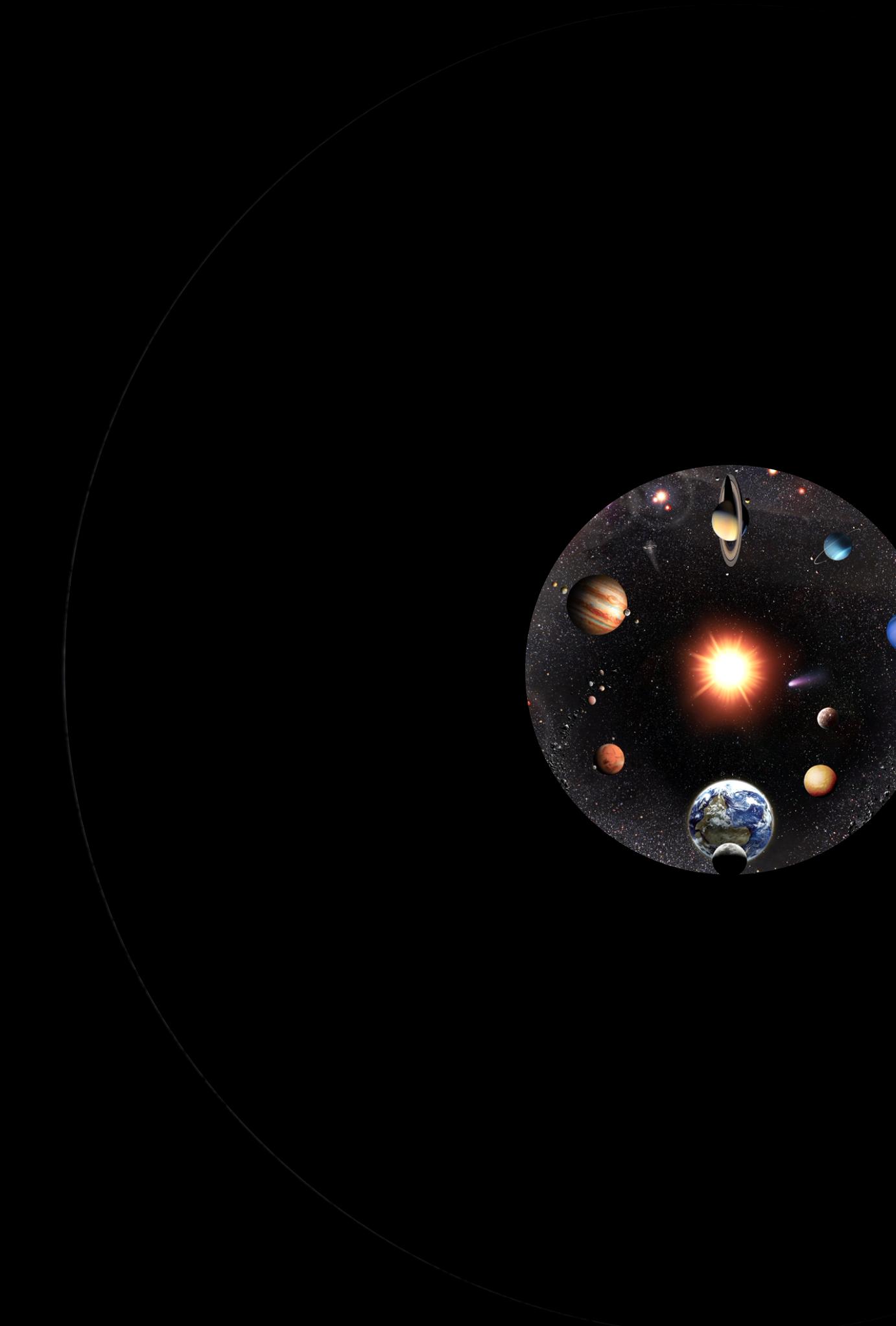
Based on work with Sowmiya Balan, Torsten Bringmann, Paul Frederik Depta, Tomás Gonzalo, Felix Kahlhöfer, Thomas Konstandin, Jonas Matuszak, Kai Schmidt-Hoberg and Pedro Schwaller

JCAP 05 (2024) 065, 2306.17836, and JCAP 11 (2023) 053



UPPSALA  
UNIVERSITET

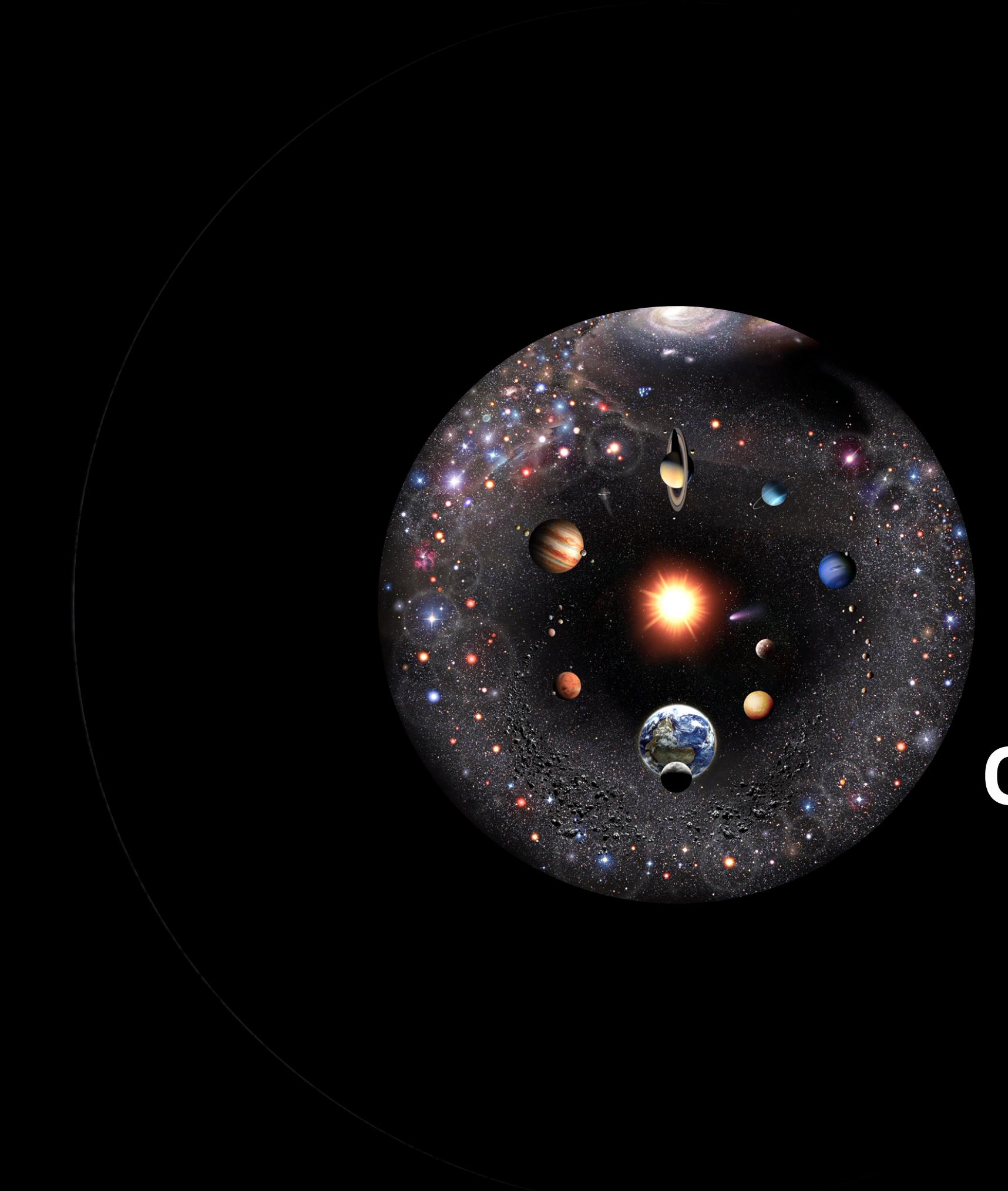
# The observable universe



**Our Solar System**

PABLO  
CARLOS  
BUDASSI

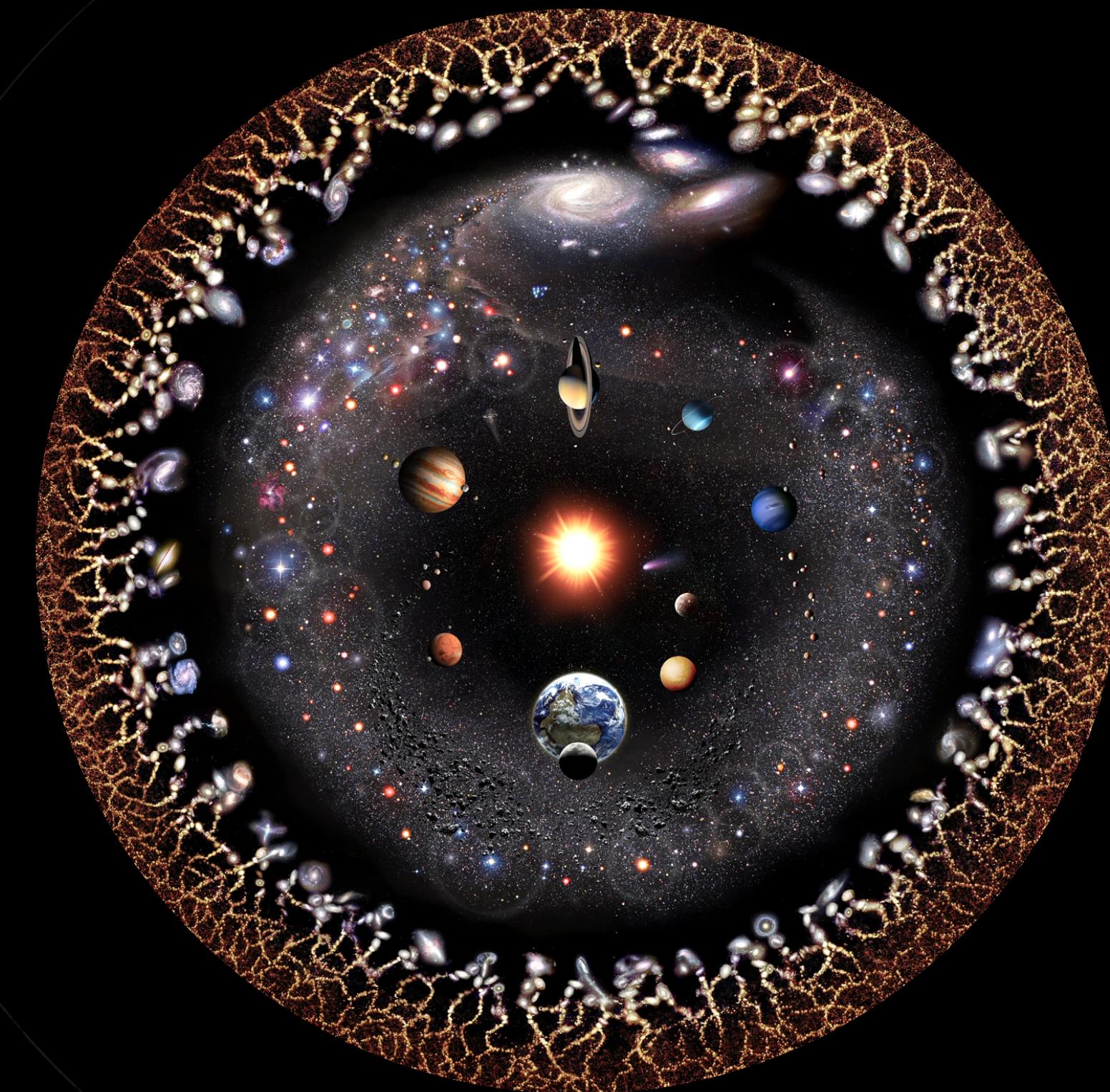
# The observable universe



Our galaxy

PABLO  
CARLOS  
BUDASSI

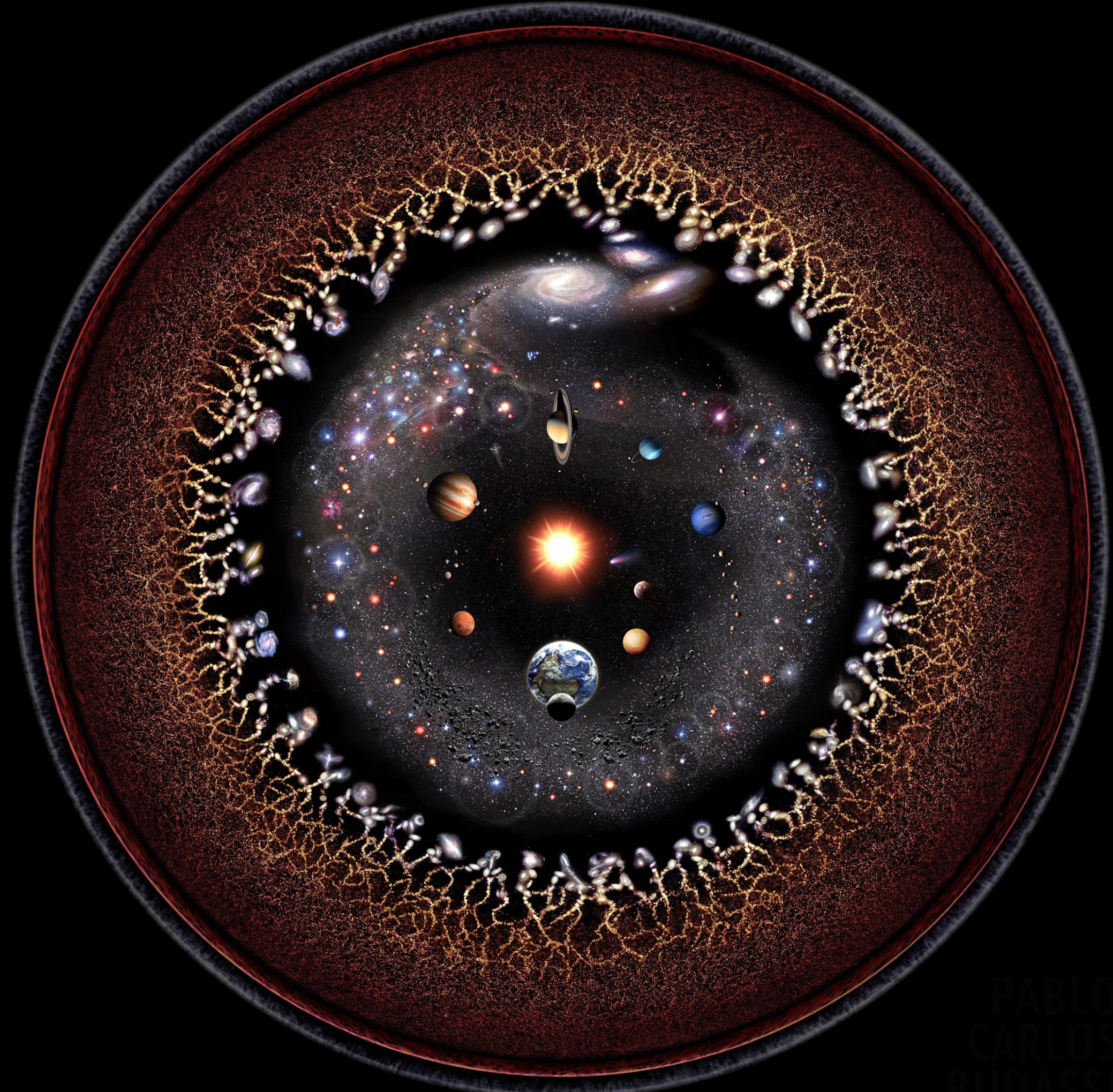
# The observable universe



Other galaxies

PABLO  
CARLOS  
BUDASSI

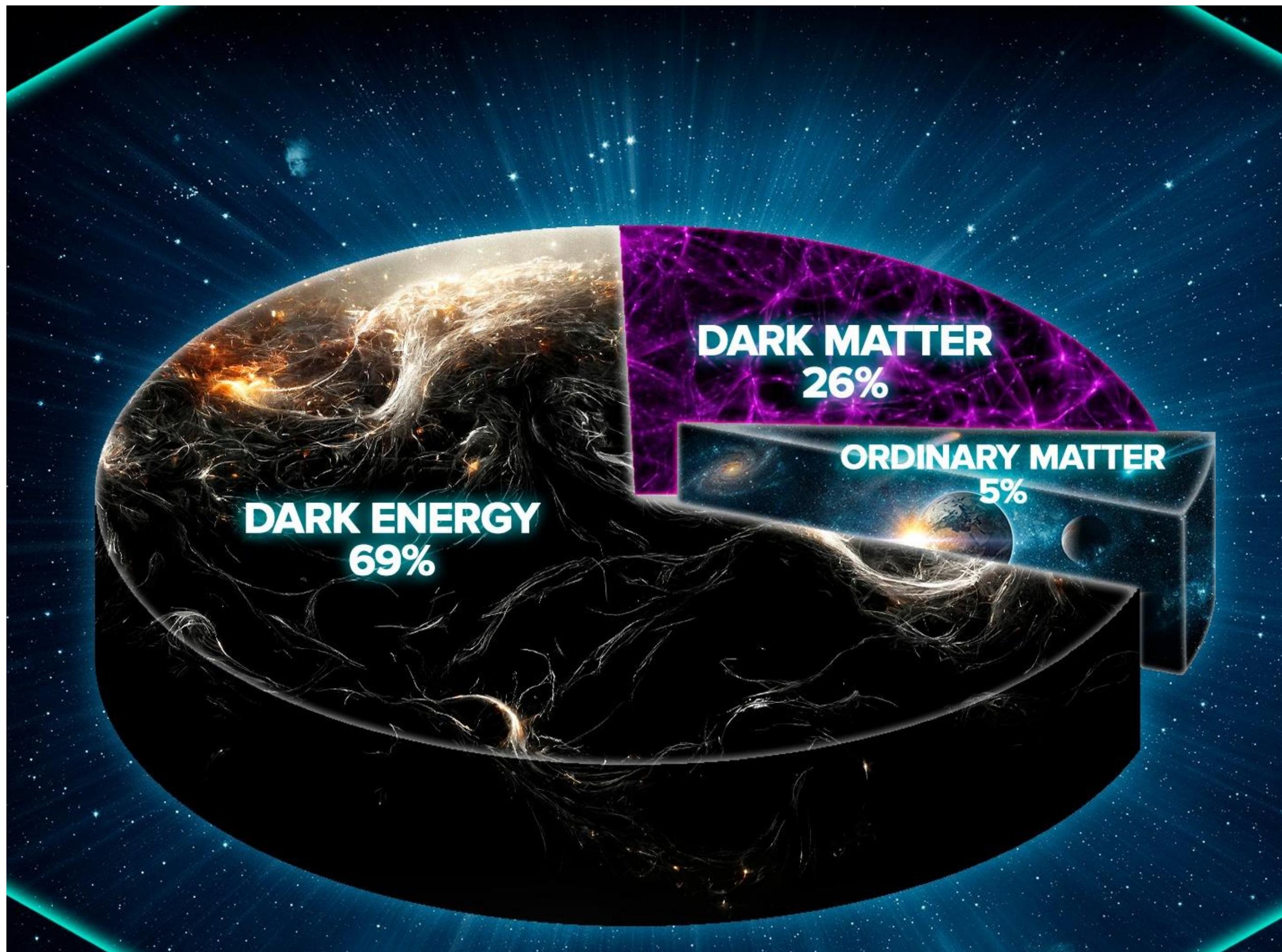
# The observable universe



**The CMB...  
and the CGWB?**

PABLO  
CARLOS  
BUDASSI

# We only understand 5%



[PBS spacetime]

We need

$$\Omega_{\text{DM}} h^2 = 0.12$$

of cold dark matter in order to explain the CMB, galaxy clustering, the bullet cluster, galactic rotation curves, ...

Cirelli+ [2406.01705]



# At Last, There's -

A globe-span-

Astronomers detect 'cosmic bass note' of gravitational waves

Sound comes from the merging of supermassive black holes across the universe, according to scientists

# Gravitational Waves

Scientists have finally 'heard' the chorus of gravitational waves that ripple through the universe

Black Holes in Space

Gravitational wave at the center of the M

# Scientists Re-

come from c-

holes

# The Cosmos Is Thrumming With Gravitational Waves, Astronomers Find

Radio telescopes around the world picked up a telltale hum reverberating across the cosmos, most likely from supermassive black holes merging in the early universe.

It may be

massive black

# of Low-Frequency Gravity

to the waves, w-

and from pa

# Scientists 'hear' cosmic hum from gravitational waves

ing everything in the universe.

# First Evidence of Giant Gravitational Waves Thrills Astronomers

For first time ever, scientists "hear" gravitational waves rippling through the universe

Astronomers are tuning in to a never-before-seen type of gravitational waves that was announced earlier this year

# Monster gravitational waves spotted for first time

**SCIENCE**

## Colossal gravitational waves—trillions of miles long—found for the first time

The Washington Post

In a major discovery, scientists say space-time churns like a choppy sea

The mind-bending finding suggests that everything around us is constantly being rolled by low-frequency gravity waves

**Physics**

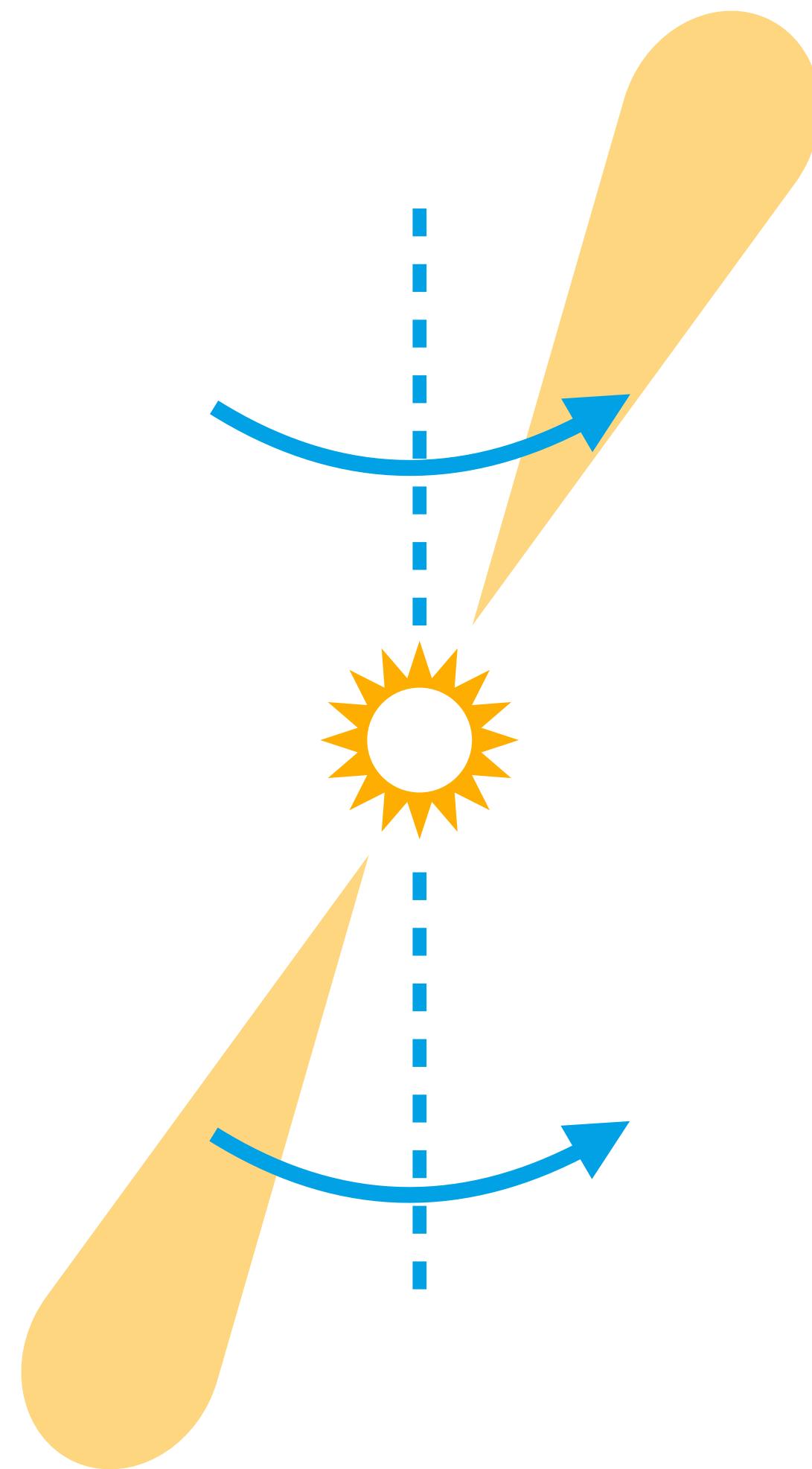
## Gravitational waves produce a background hum across the whole universe

After decades of searching, astronomers have found a distinctive pattern of light, from spinning stars called pulsars, that suggests huge gravitational waves are creating gentle ripples in space-time across the universe

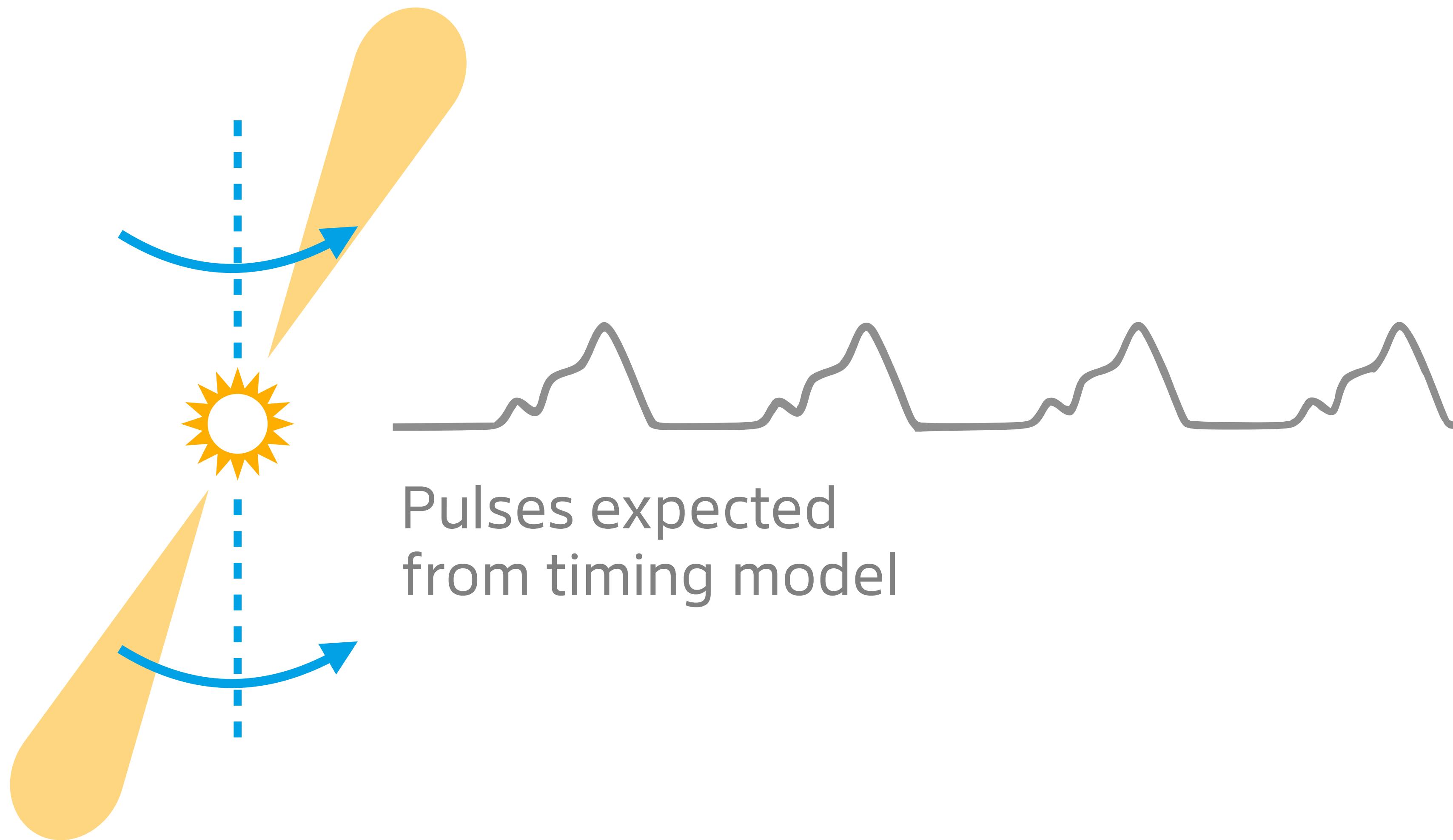
The results are

Luckily, we live in the age of gravitational wave cosmology!

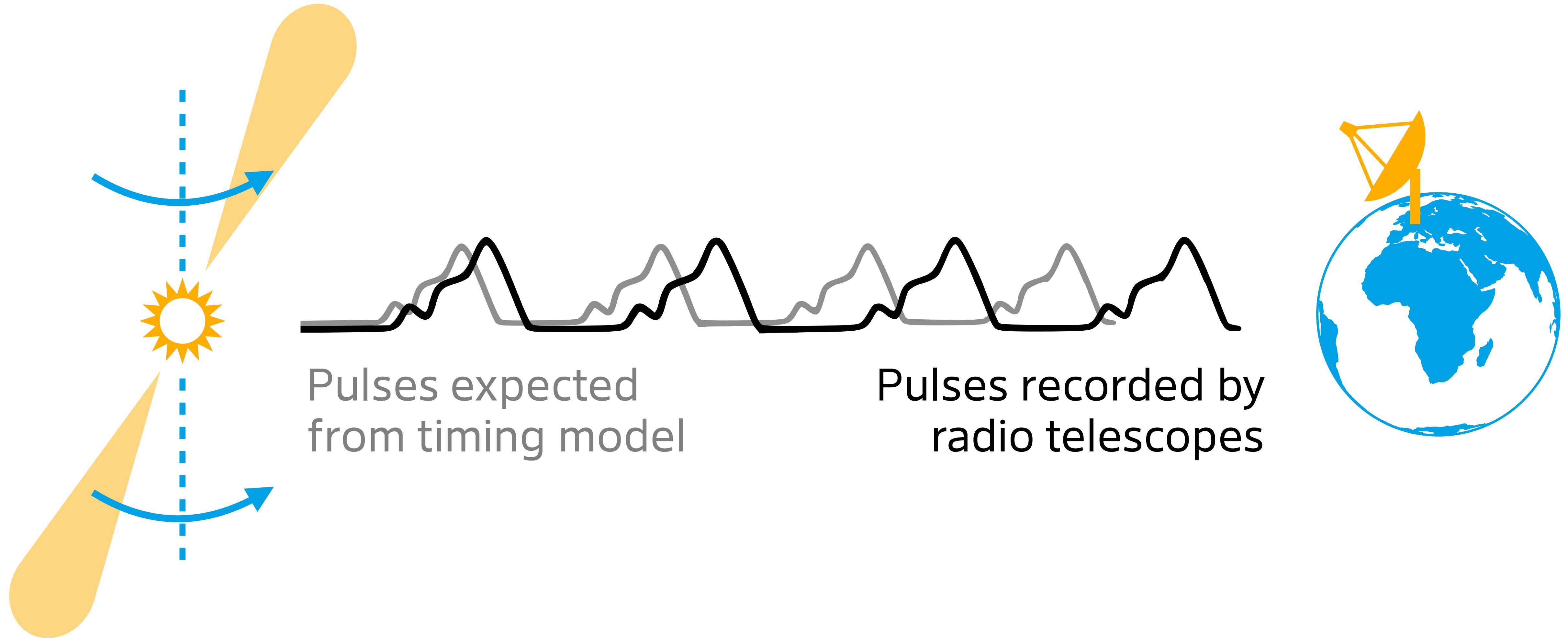
# Pulsar timing arrays



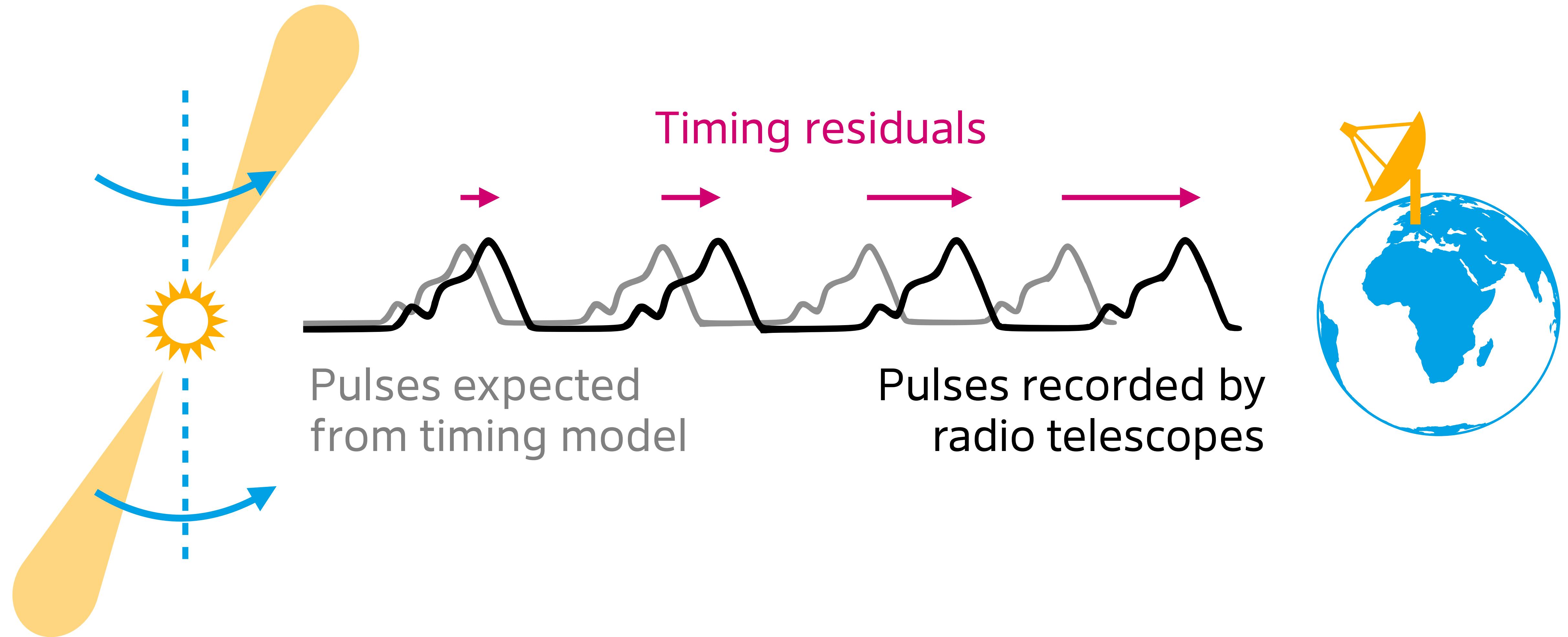
# Pulsar timing arrays



# Pulsar timing arrays

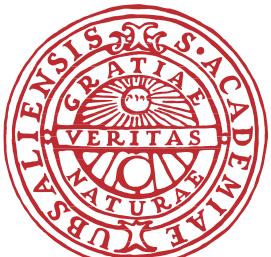
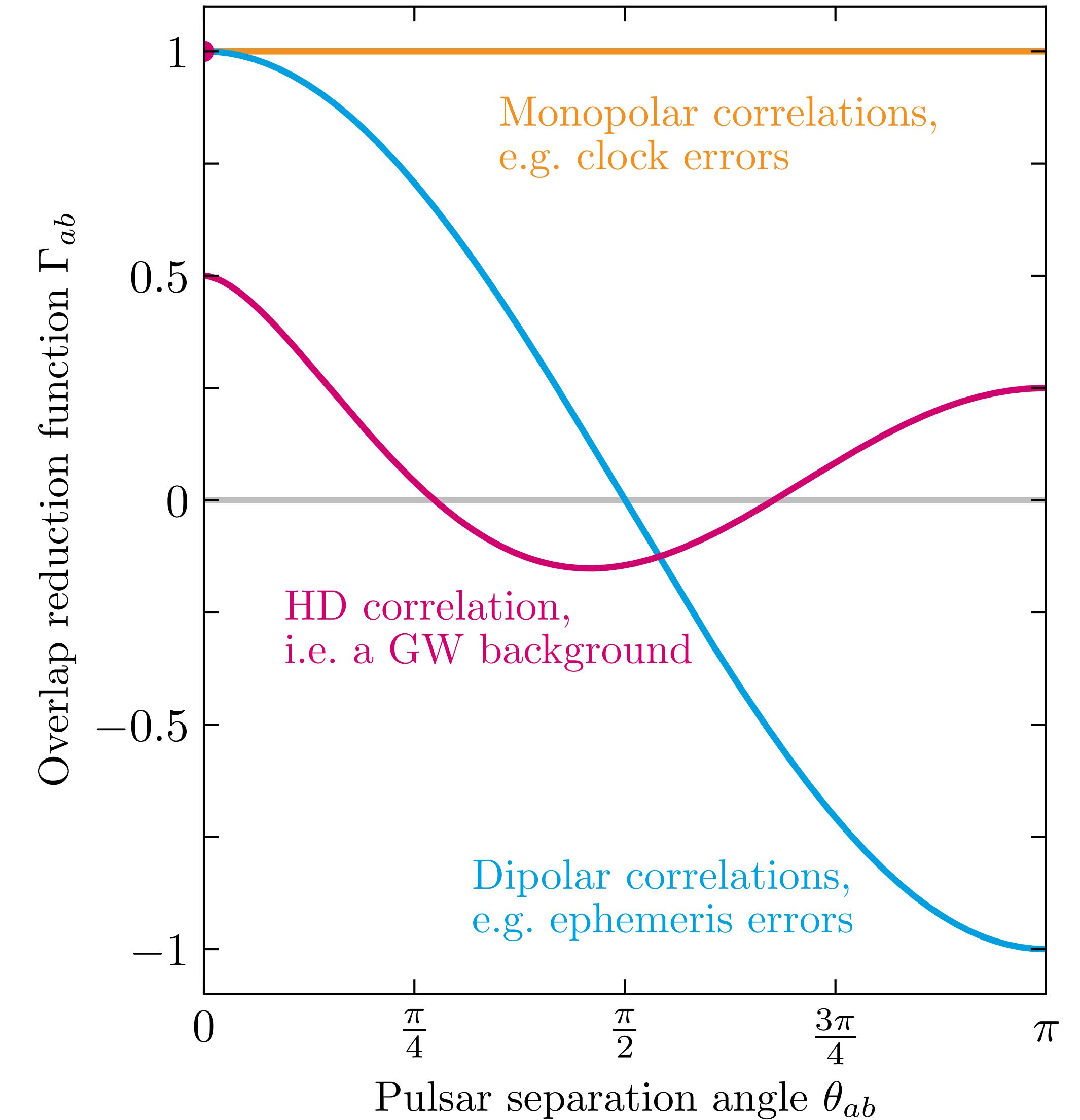


# Pulsar timing arrays

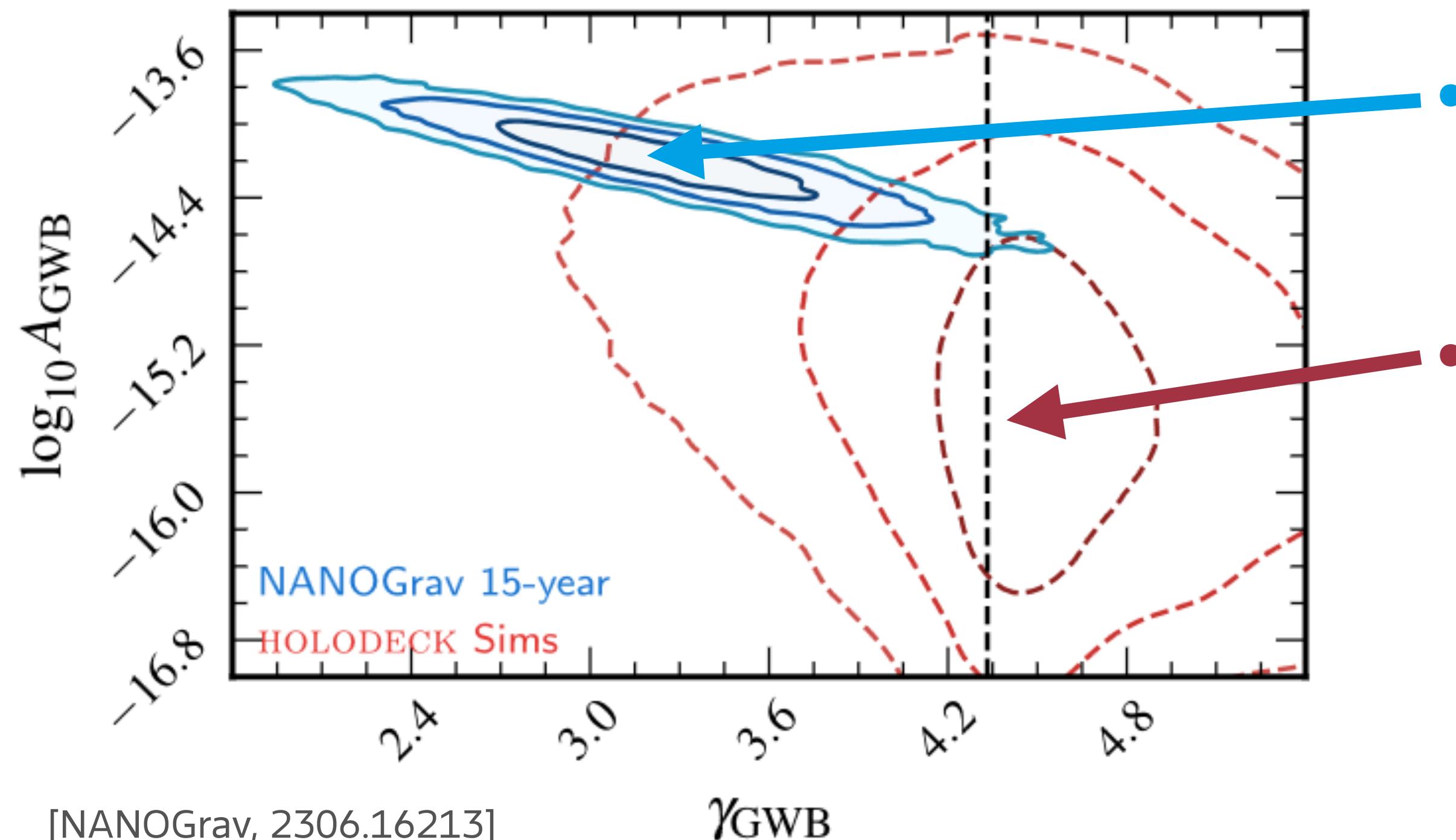


# Searching for the Hellings-Downs correlation

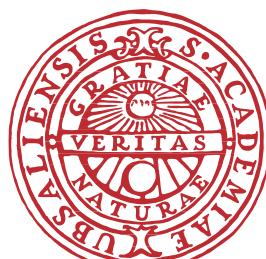
- PTAs found an underlying „common red process“ among  $\mathcal{O}(70)$  pulsars
- Signal could have many sources:
  - Pulsars themselves:  $\mathcal{B} < 10^{-12}$
  - Clock errors:  $\mathcal{B} < 10^{-8}$
  - Ephemeris errors:  $\mathcal{B} < 10^{-7}$
  - GWs:  $\mathcal{B} = 200 - 1000$



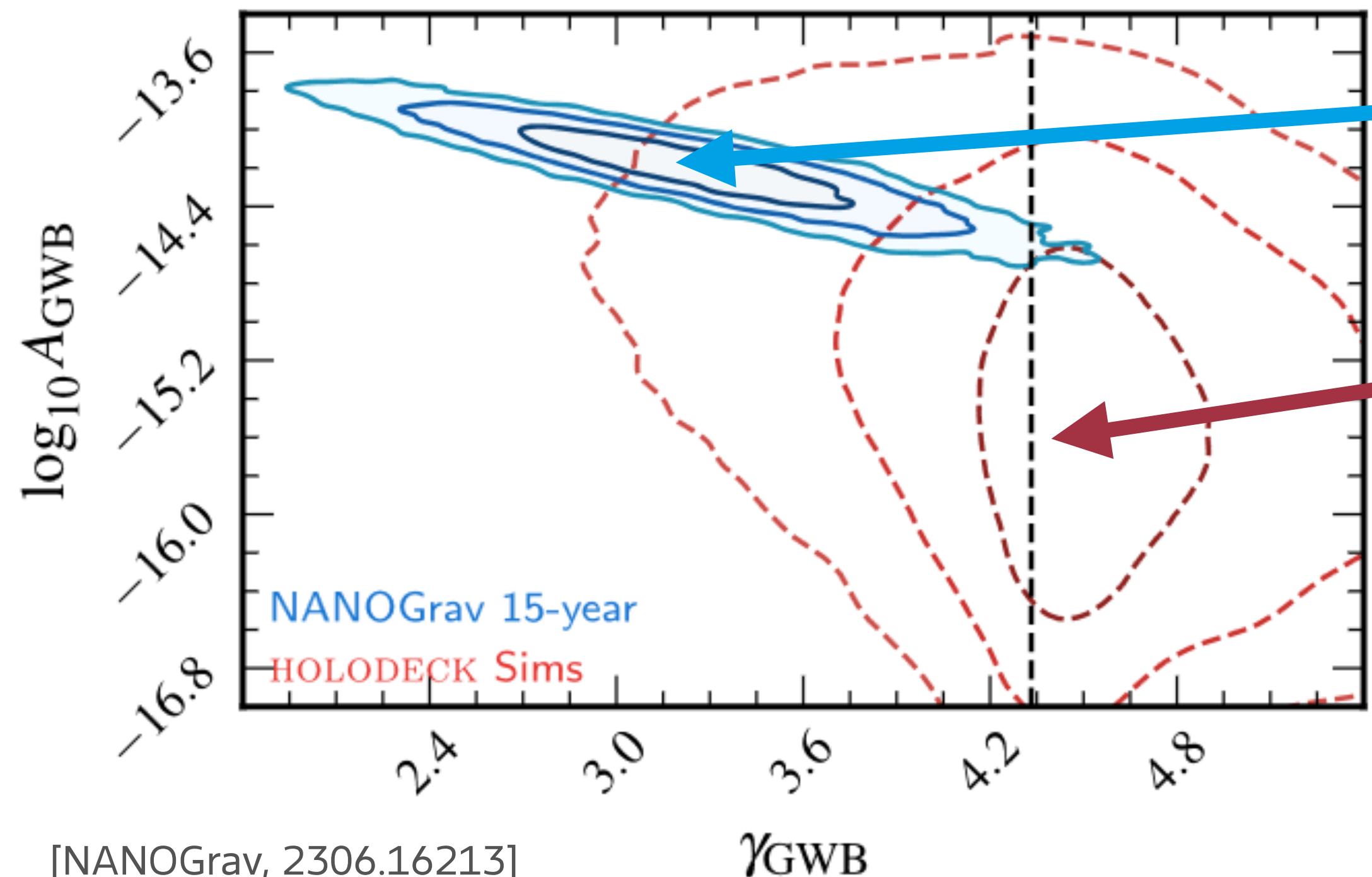
# Merging supermassive black holes



- Observed signal follows a power-law spectrum with amplitude  $A$  and slope  $\gamma$
- Astrophysical simulations based on realistic BH populations predict much weaker signals with higher  $\gamma$

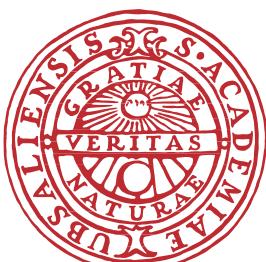


# Merging supermassive black holes

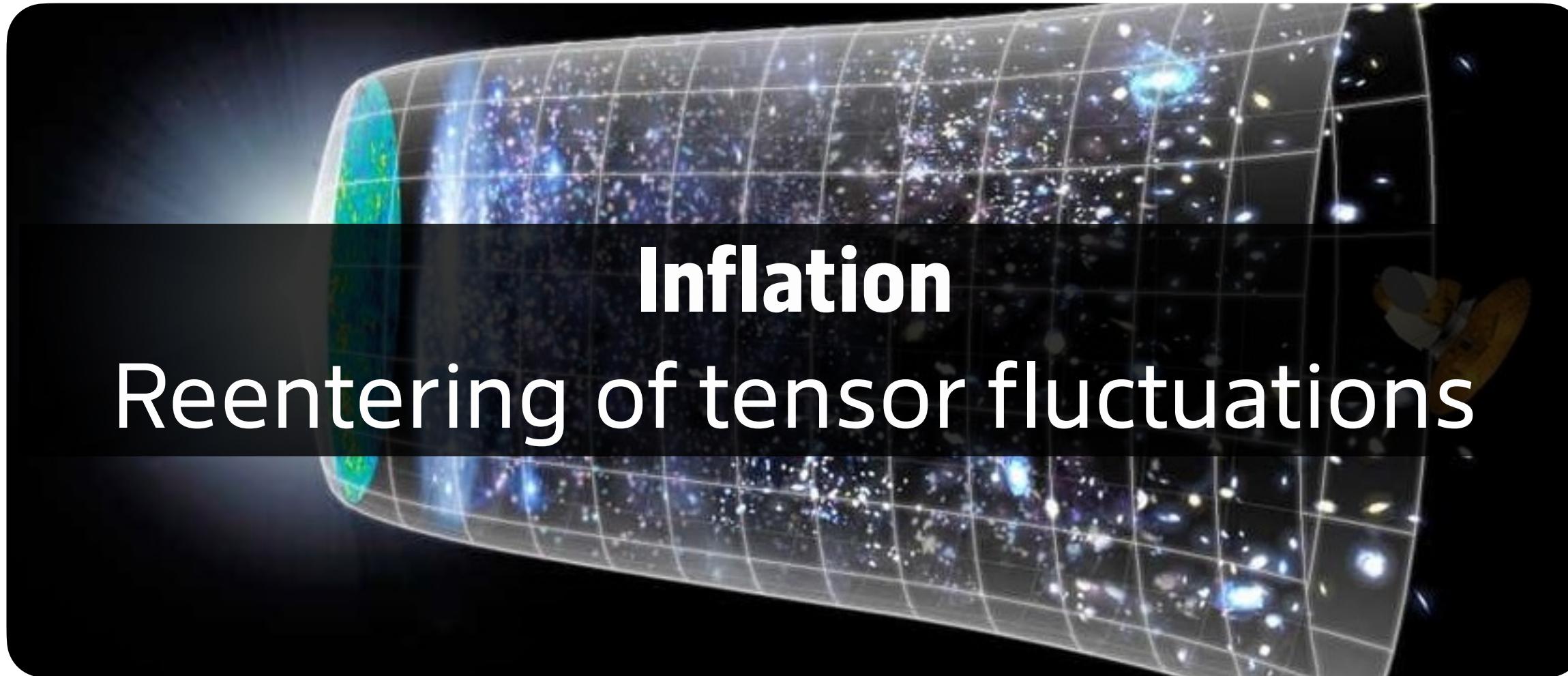


- Observed signal follows a power-law spectrum with amplitude  $A$  and slope  $\gamma$
- Astrophysical simulations based on realistic BH populations predict much weaker signals with higher  $\gamma$

Are there other signal sources?



# Possible cosmological sources of the PTA signal



**Inflation**  
Reentering of tensor fluctuations



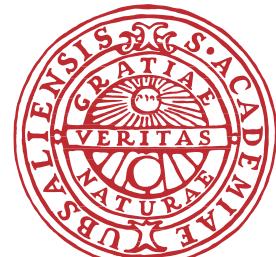
**Phase transitions**  
Connection to dark matter?



**Topological defects**  
Cosmic strings and domain walls



**Primordial black holes**  
But only if they are clustered

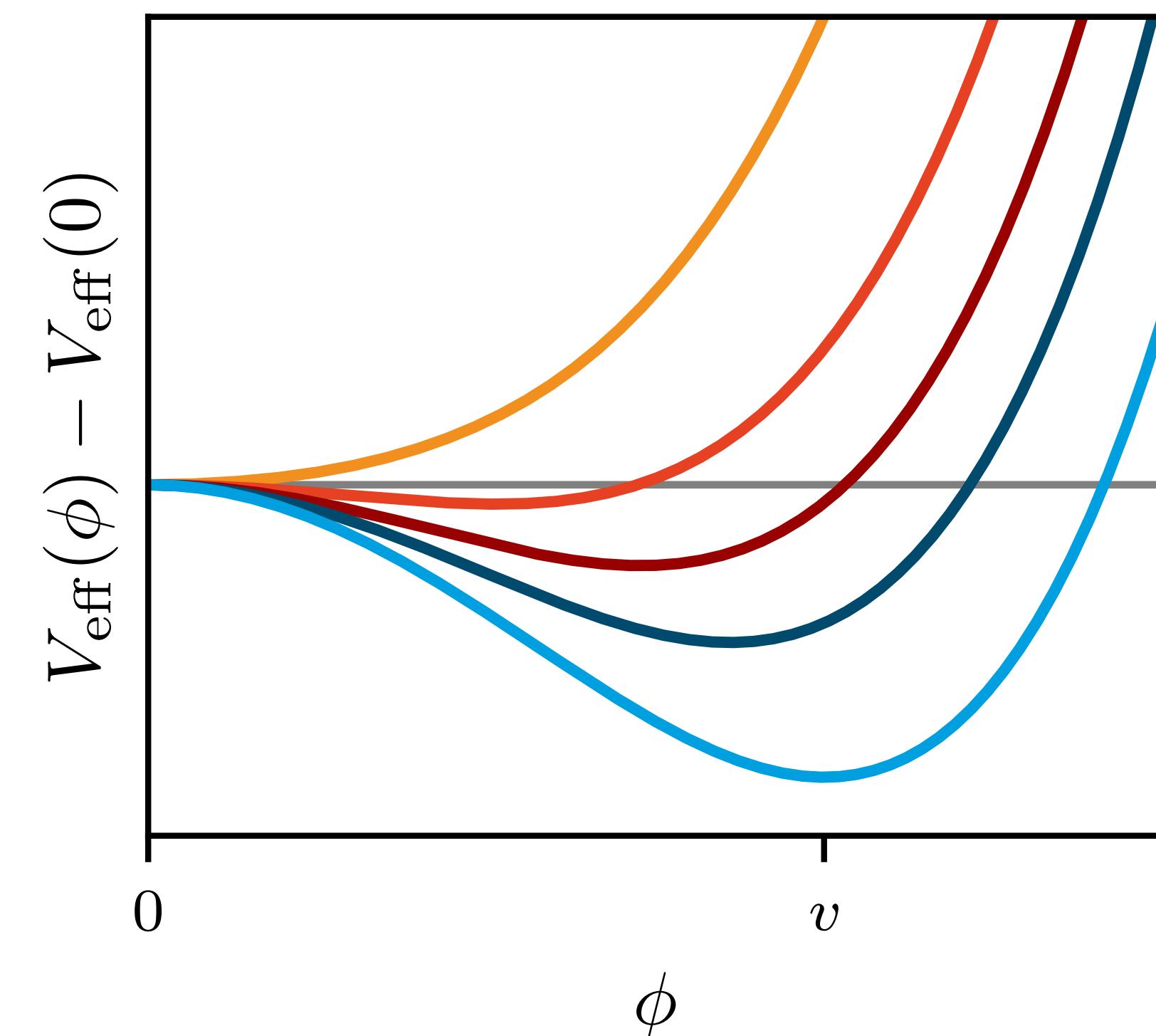




# **Do PTAs observe a dark sector phase transition?**

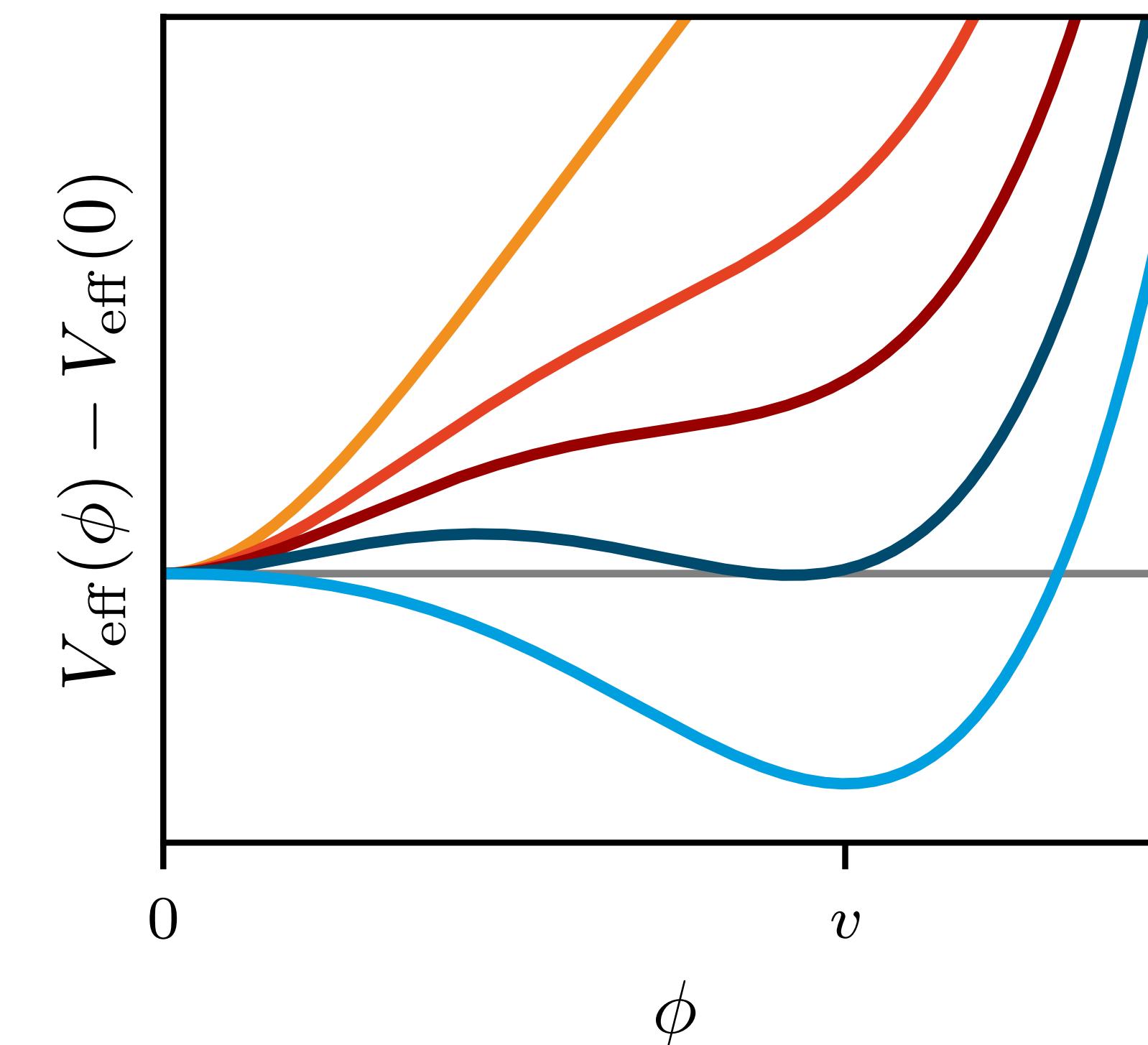
# First-order phase transitions vs. cross-overs

Cross-over phase transition

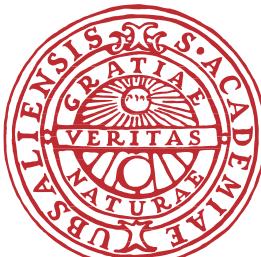


A scalar field “rolls down” from  $\phi = 0$  to  $\phi = v$ , when the plasma cools from **high temperatures** to **low temperatures**.

First-order phase transition

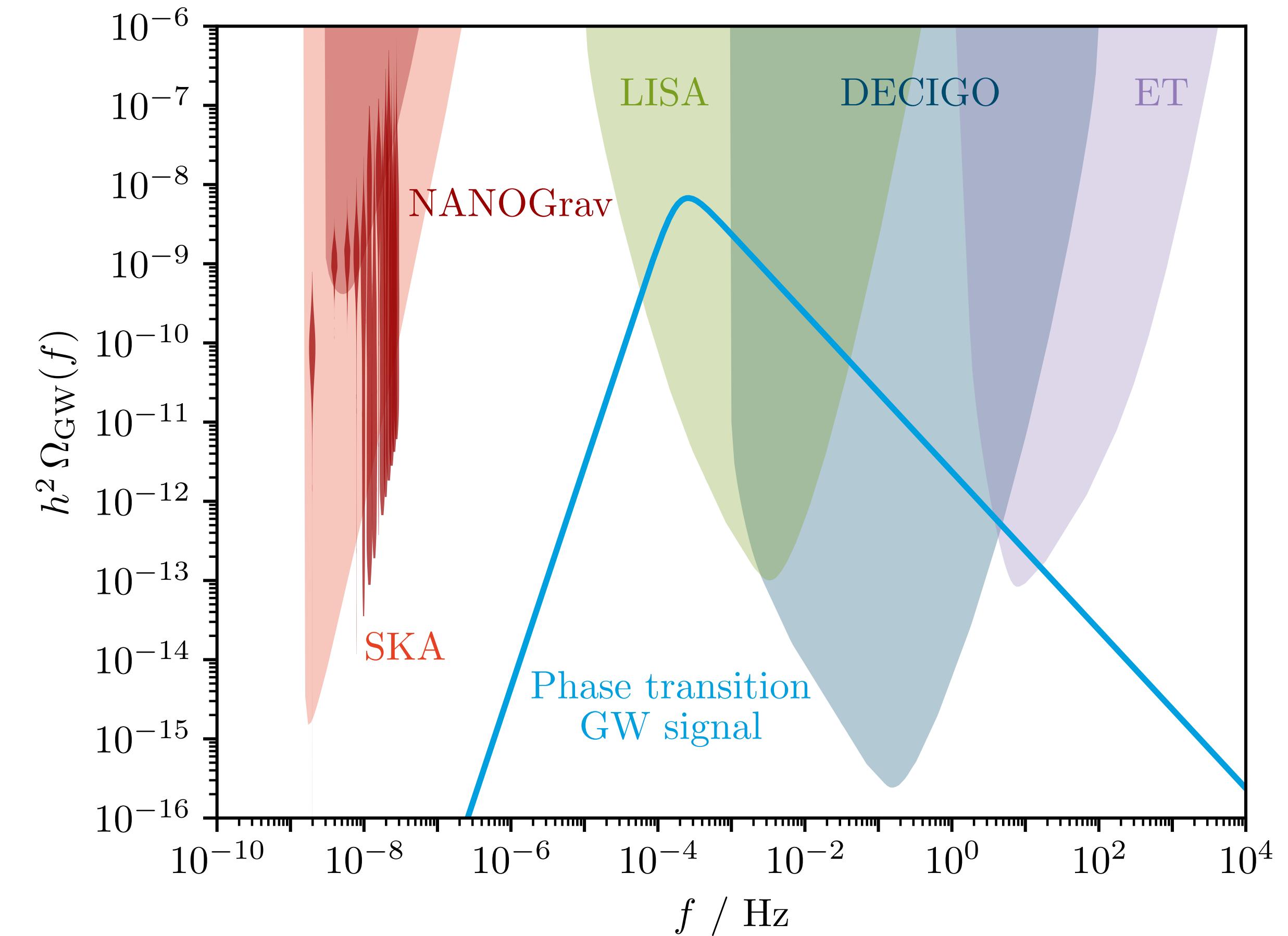
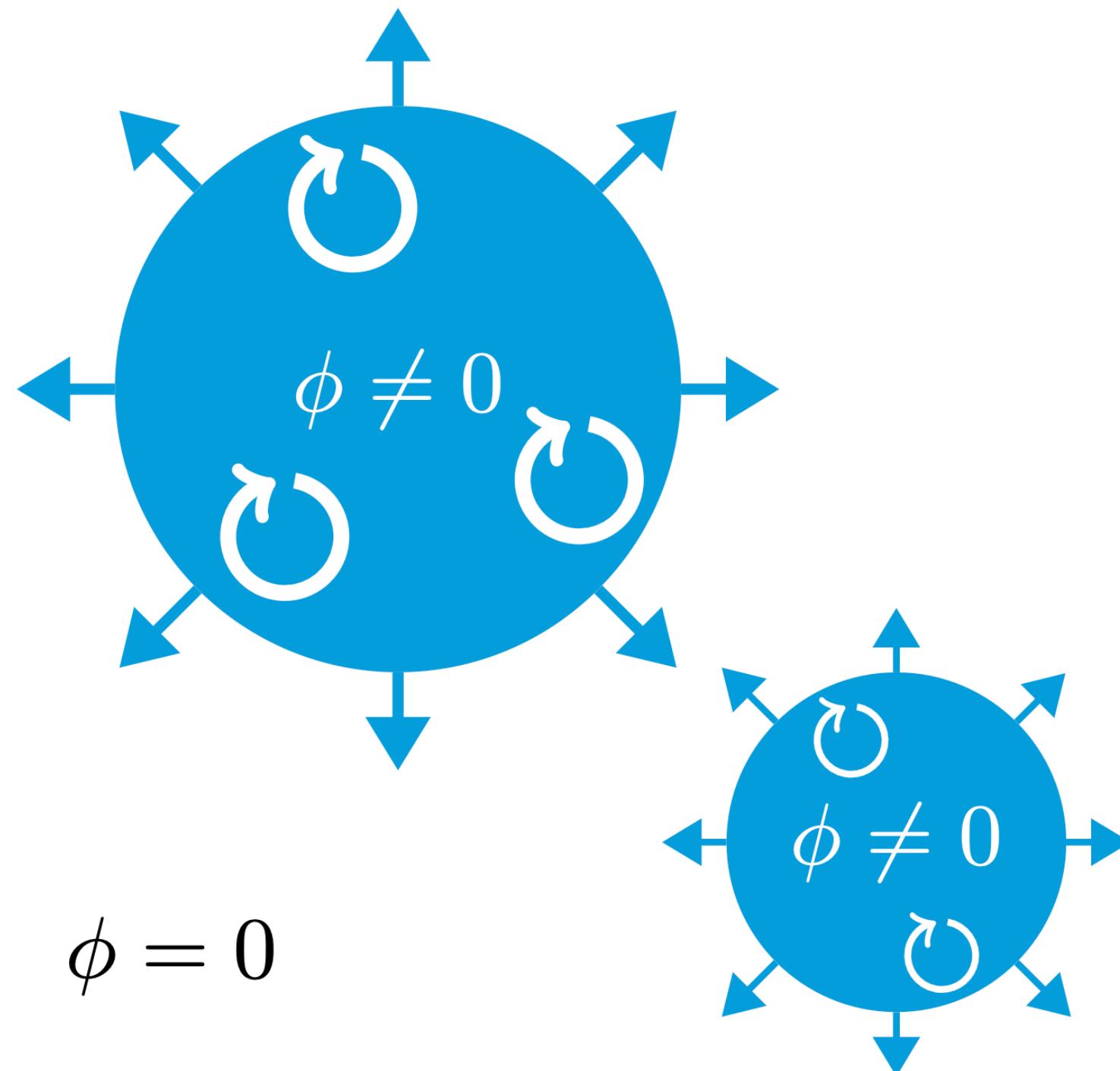


A scalar field tunnels to the true potential minimum  $\phi \neq 0$  to minimize its free energy / its action.

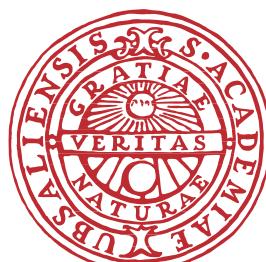


# First-order phase transitions produce GW backgrounds

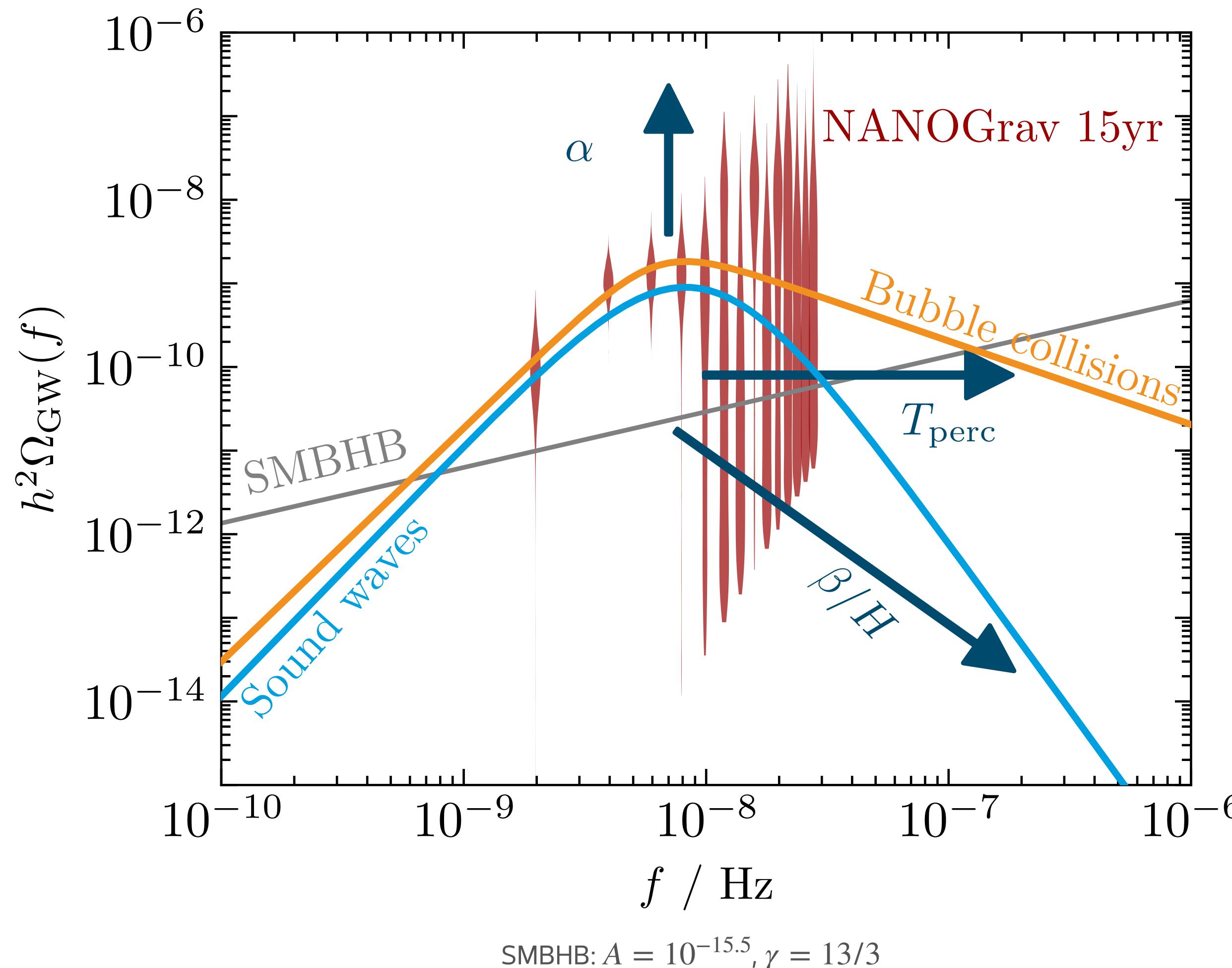
Bubbles of the new phase nucleate,  
collide and perturb the plasma...



... giving rise to an observable stochastic  
gravitational wave background.



# Parametrization of the GW signal

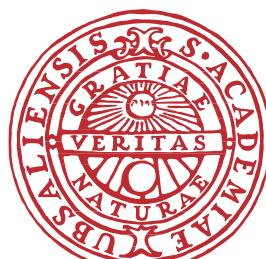


$$h^2 \Omega_{\text{GW}}^{\text{sw,bw}}(f) \simeq 10^{-6} \left( \frac{\alpha}{\alpha + 1} \right)^2 \left( \frac{H}{\beta} \right)^{1,2} \mathcal{S} \left( \frac{f}{f_{\text{peak}}} \right)$$

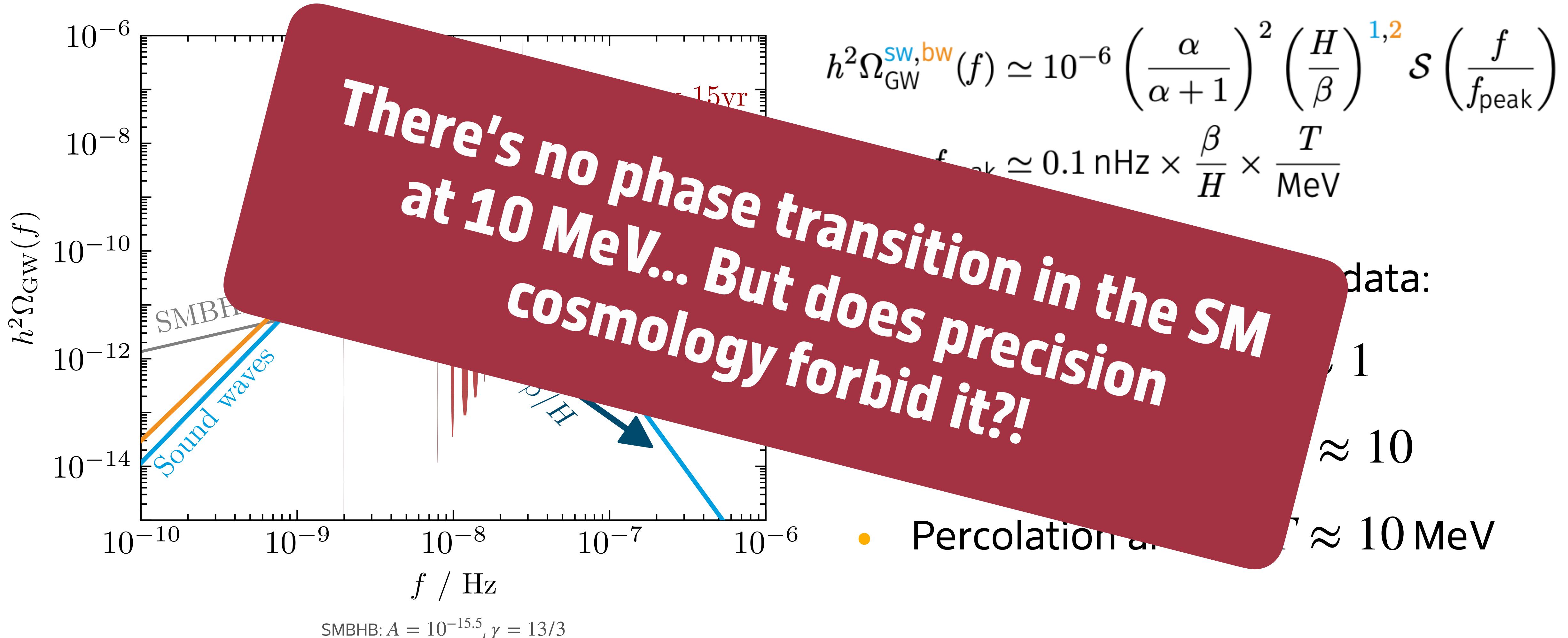
with  $f_{\text{peak}} \simeq 0.1 \text{nHz} \times \frac{\beta}{H} \times \frac{T}{\text{MeV}}$

To fit the new pulsar timing data:

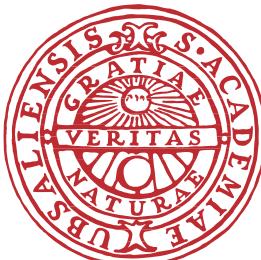
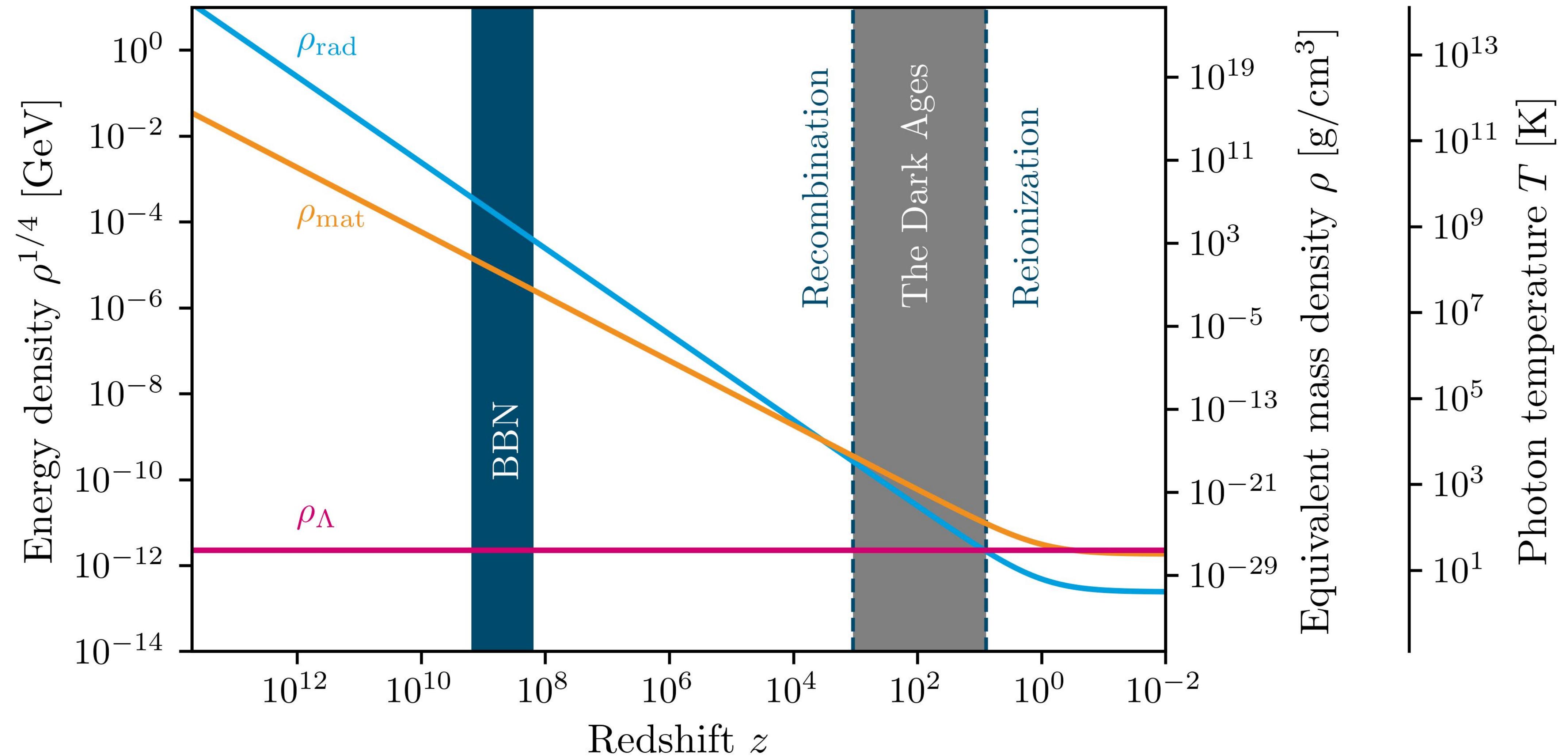
- Strong transitions,  $\alpha \approx 1$
- Slow transitions,  $\beta/H \approx 10$
- Percolation around  $T \approx 10 \text{ MeV}$



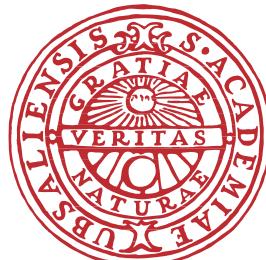
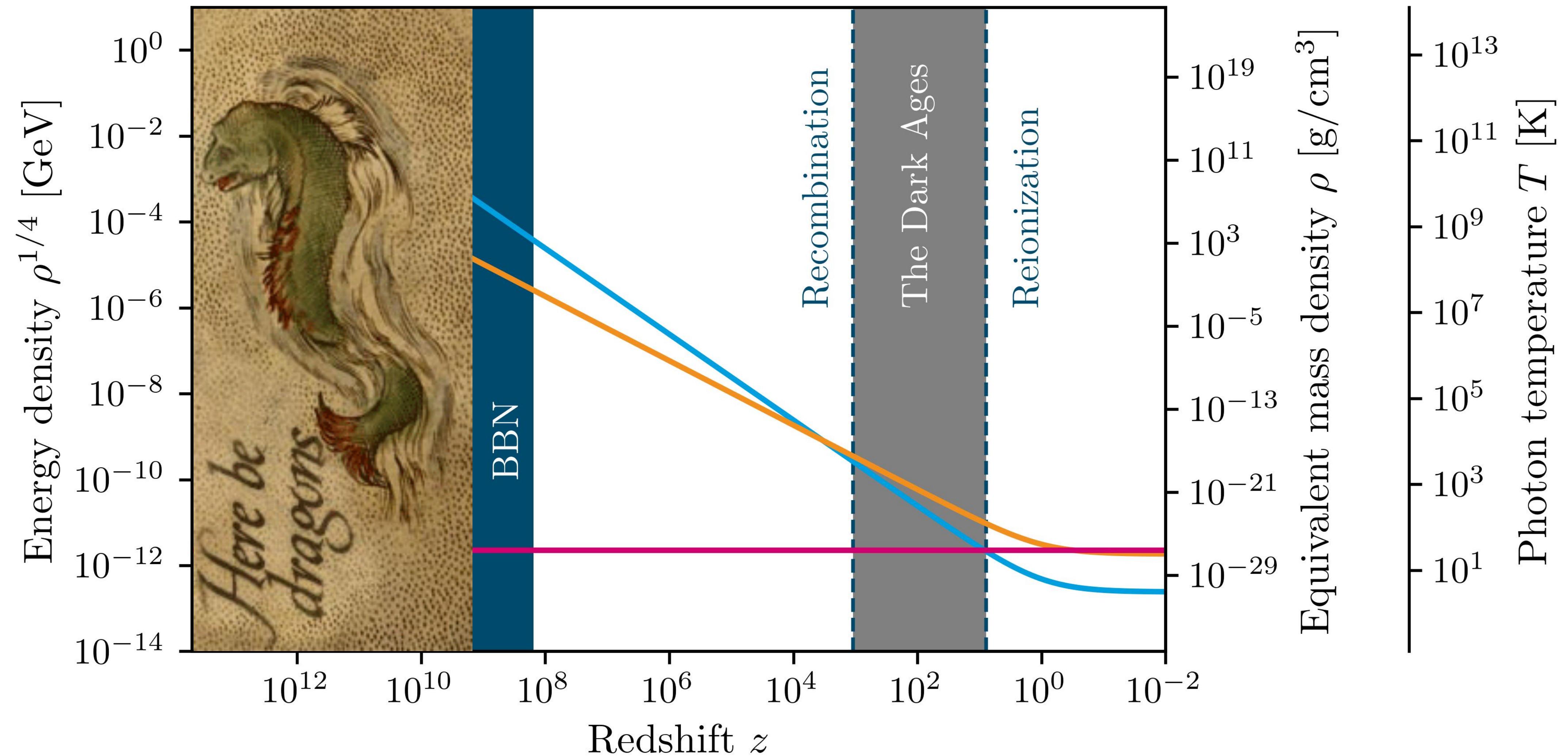
# Parametrization of the GW signal



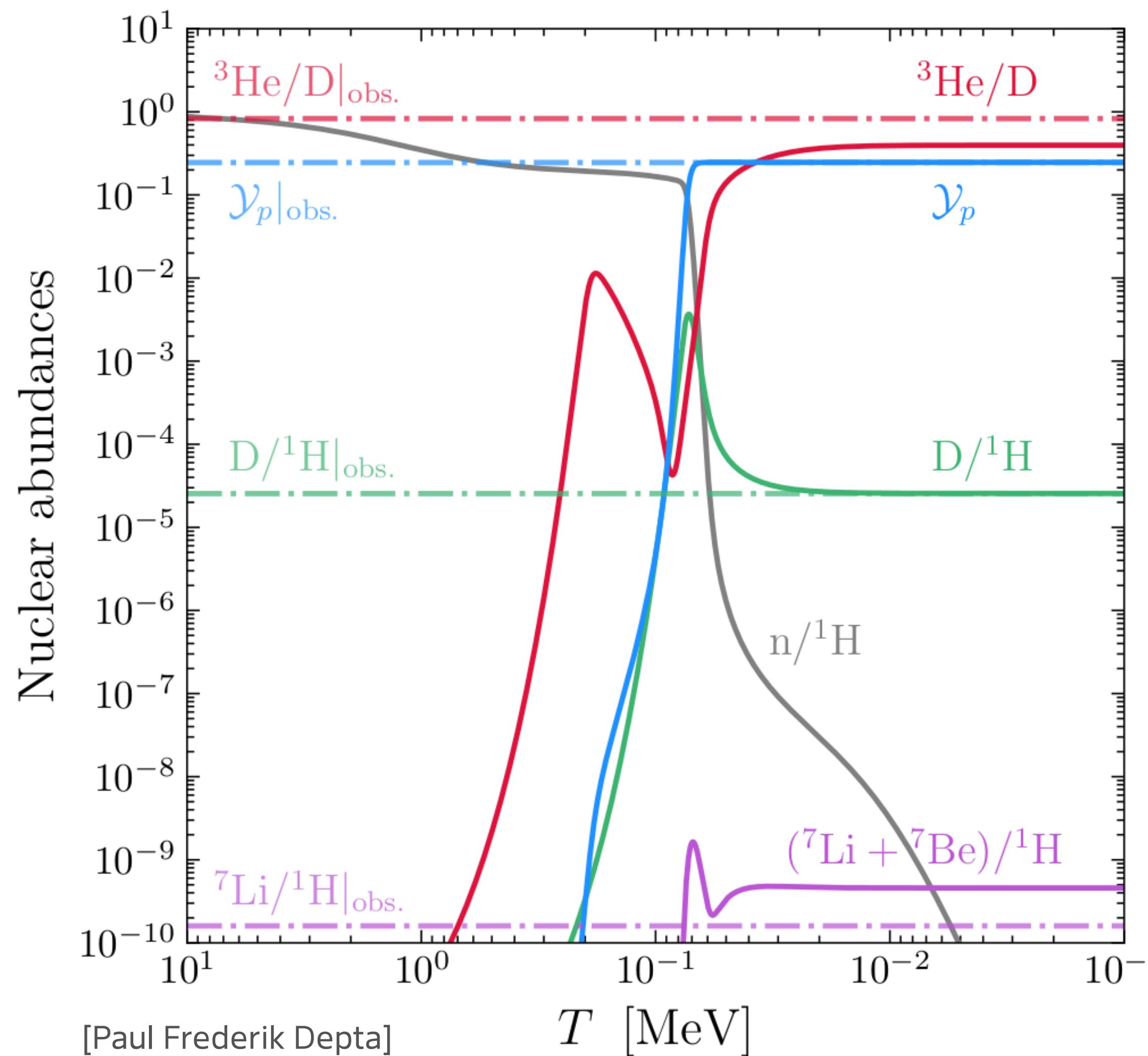
# A brief history of time



# A brief history of time



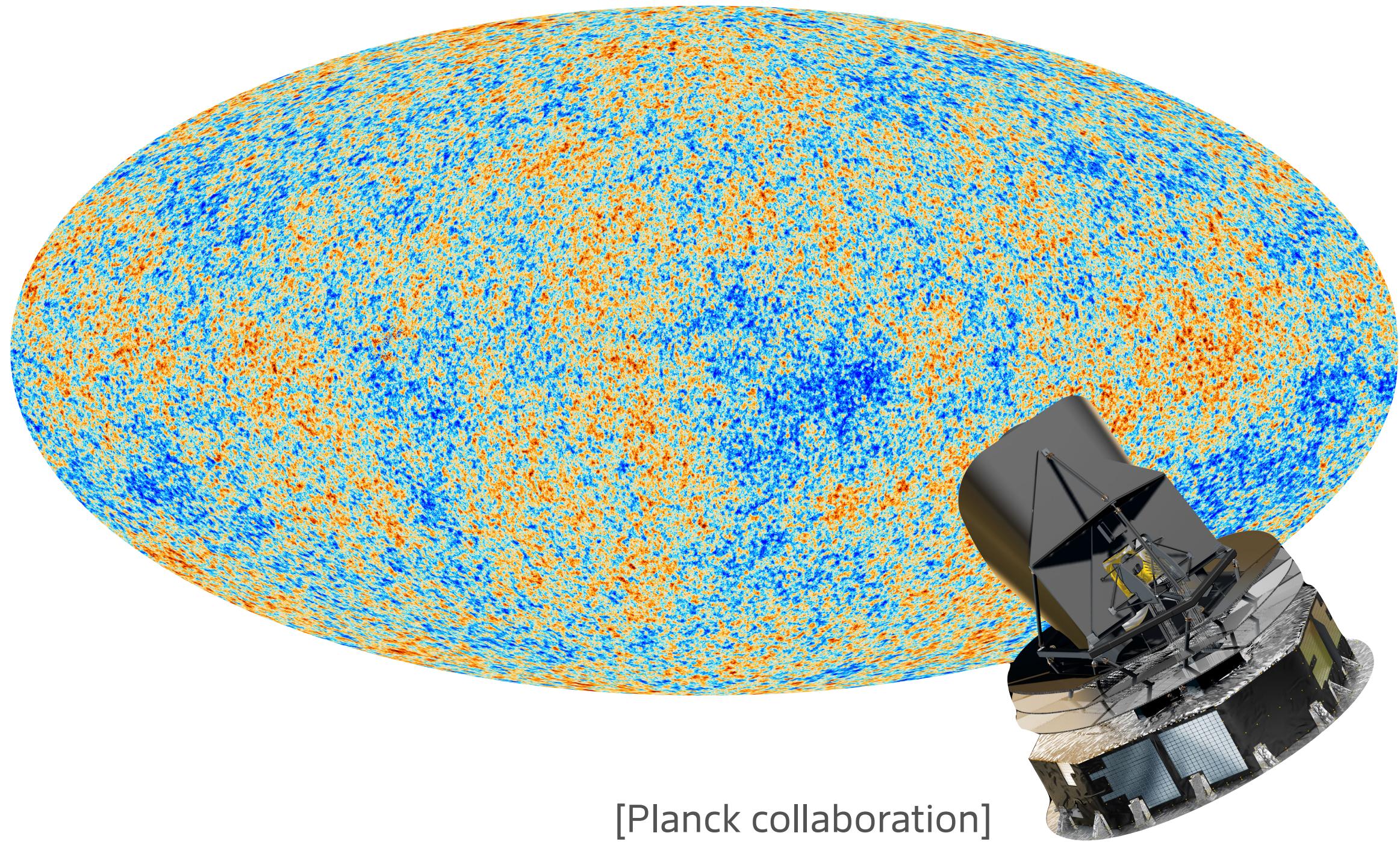
# Big Bang Nucleosynthesis and the Cosmic Microwave Background



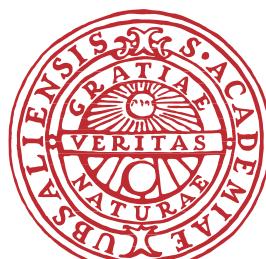
- Observation of primordial light element abundances in good agreement with standard BBN
- $N_{\text{eff}}^{\text{BBN}} = 2.898 \pm 0.141$



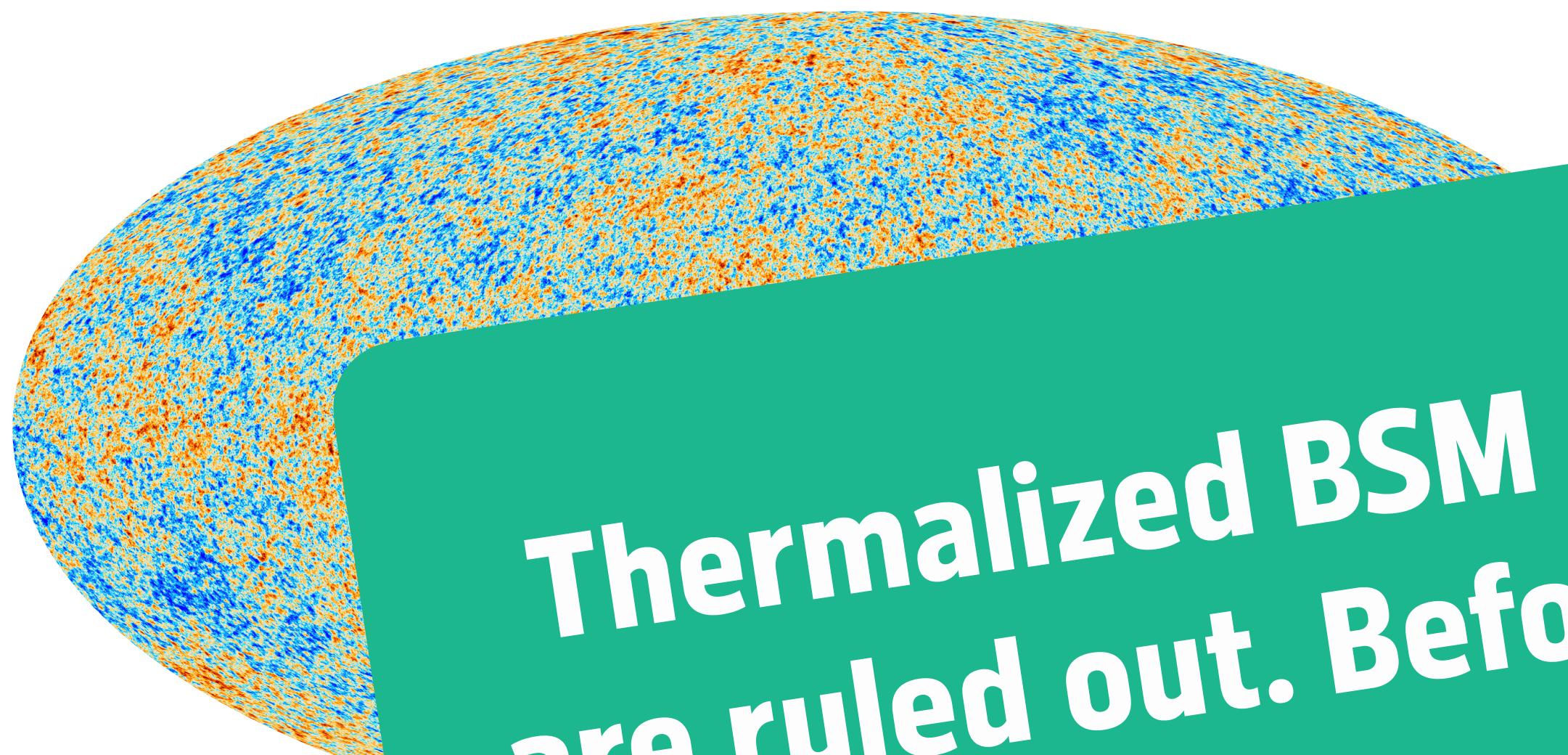
# Big Bang Nucleosynthesis and the Cosmic Microwave Background



- Observation of primordial light element abundances in good agreement with standard BBN
- $N_{\text{eff}}^{\text{BBN}} = 2.898 \pm 0.141$
- $N_{\text{eff}}^{\text{CMB}} = 2.99 \pm 0.17$
- Consistent with  $N_{\text{eff}}^{\text{SM}} = 3.044$  from  $3\nu$  generations



# Big Bang Nucleosynthesis and the Cosmic Microwave Background



Thermalized BSM species at  $T < 1 \text{ MeV}$   
are ruled out. Before: no constraints. 😊

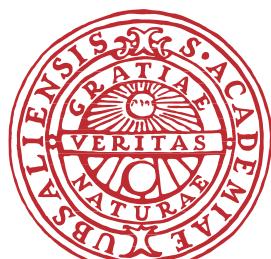
[Planck collaboration]

- Observation of primordial light elements and abundances in the CMB

141

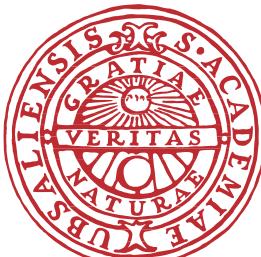
13.1 /

- Consistent with  $N_{\text{eff}}^{\text{SM}} = 3.044$  from  $3\nu$  generations



# Adding more Higgs bosons to the Standard model

There's no strong  
first-order phase transition  
at 10 MeV in the Standard  
Model. :(



# Turning on the light in a dark sector

## Stable dark sector

Additional DS energy density  
accelerates Hubble expansion via

$$\Delta N_{\text{eff}} \gtrsim 6 \times \alpha$$

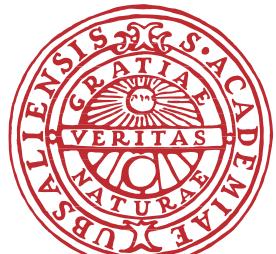
But we need  $\alpha \simeq 1$ ...

$$\Delta N_{\text{eff}} < 0.22 \text{ @ 95 \% C.L.}$$

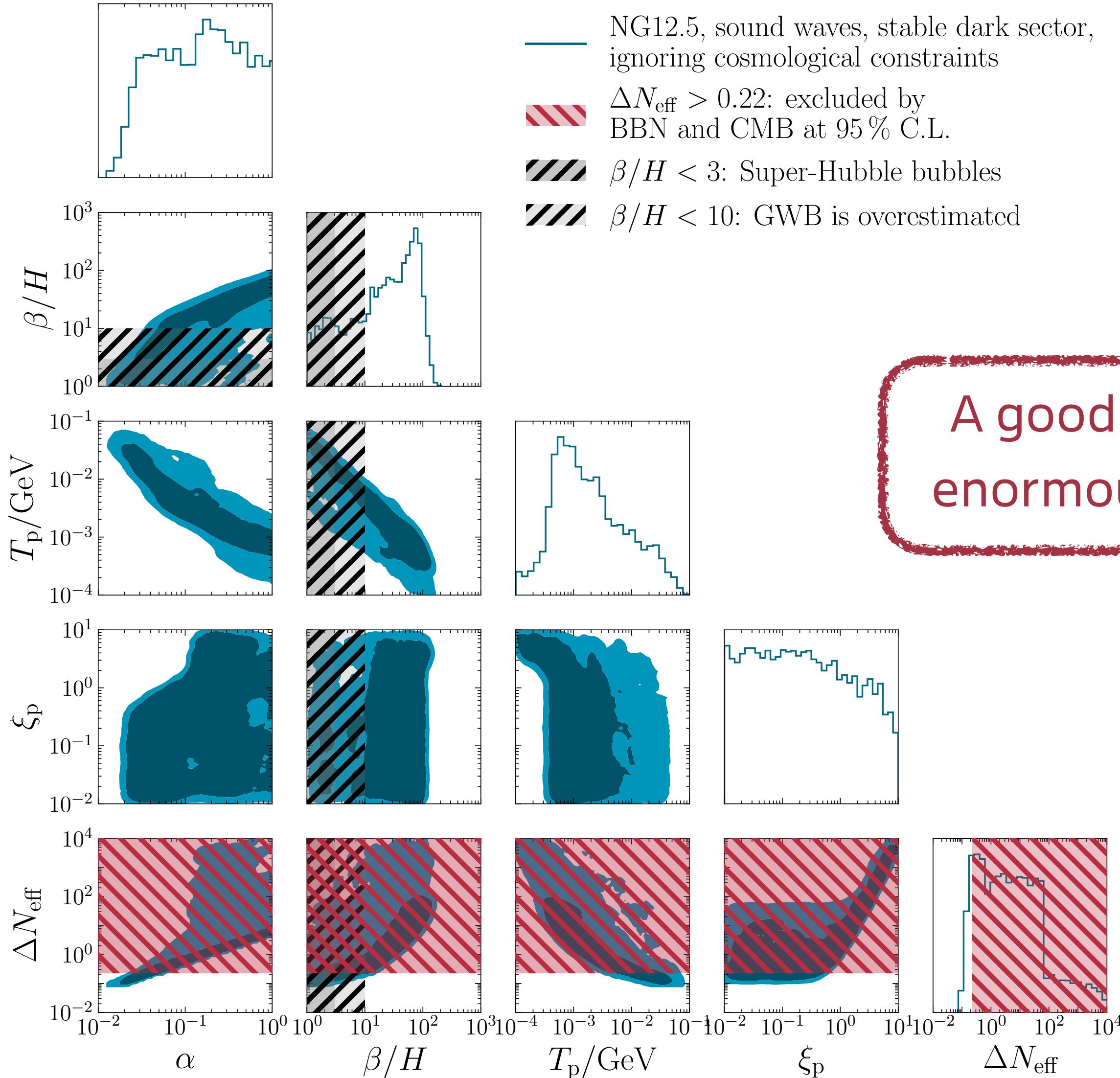


## Decaying dark sector

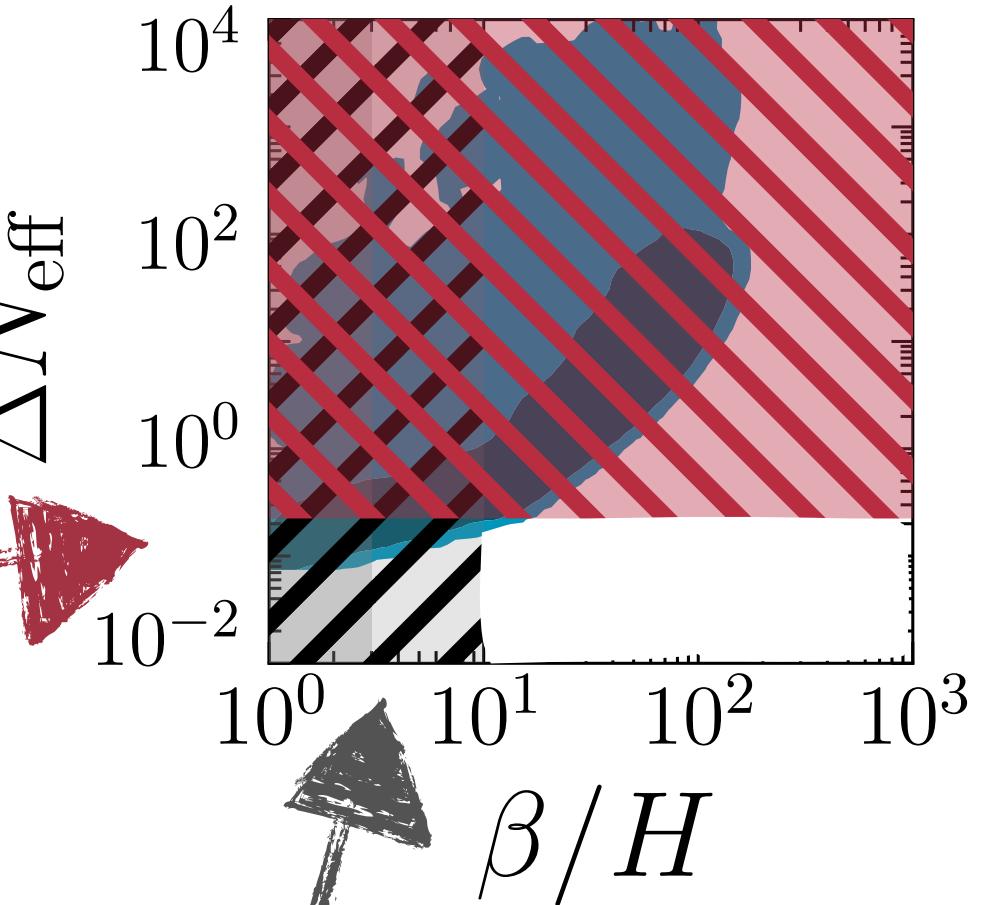
Can the dark sector decay quickly  
enough to the SM to circumvent  
BBN and CMB constraints?



# Stable dark sector phase transition: A naive fit



A good fit would require enormous  $\Delta N_{\text{eff}} \gg 0.22$

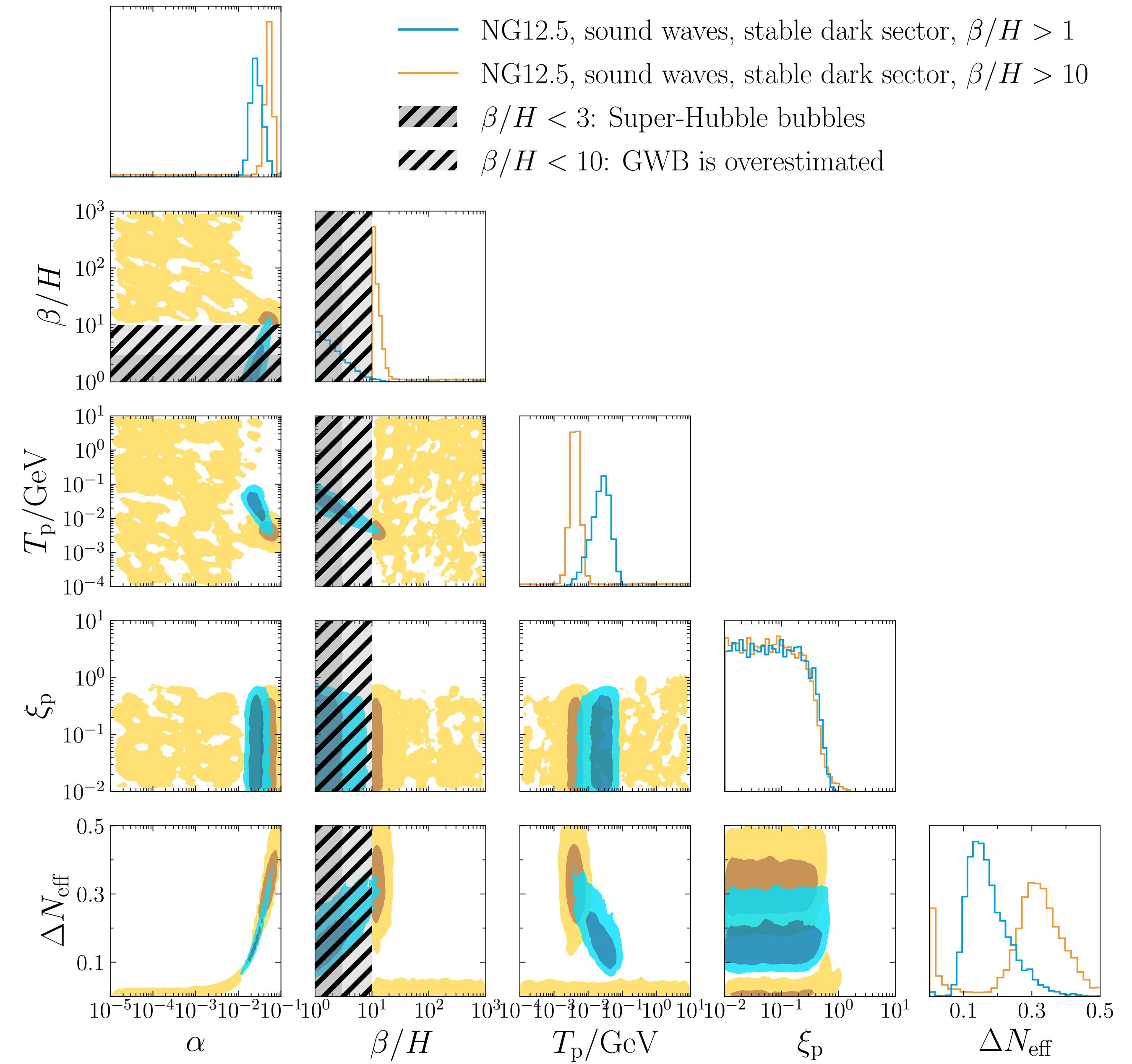


Giant bubble sizes would be needed, violating causality & questioning validity of GW predictions



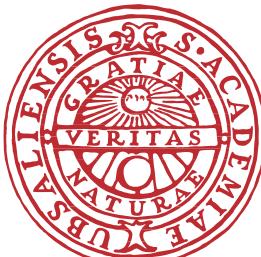
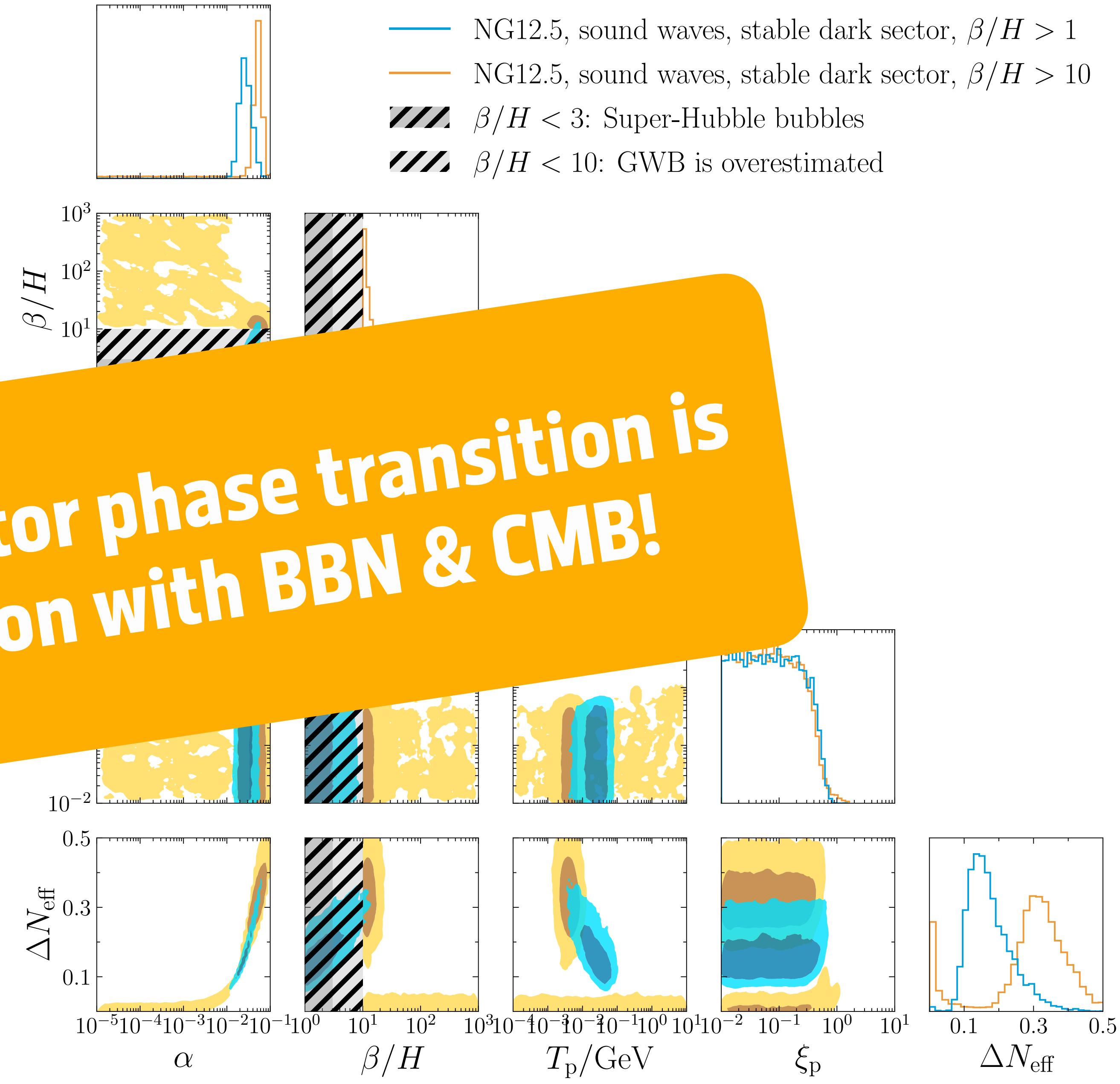
# Global fits

- Combined PTA and BBN/CMB likelihoods in enterprise
- $\beta/H > 1$ : Would fit the data if GW spectrum were reliable
- $\beta/H > 10$ : Shot noise because not explaining PTA data is better than messing up BBN + CMB

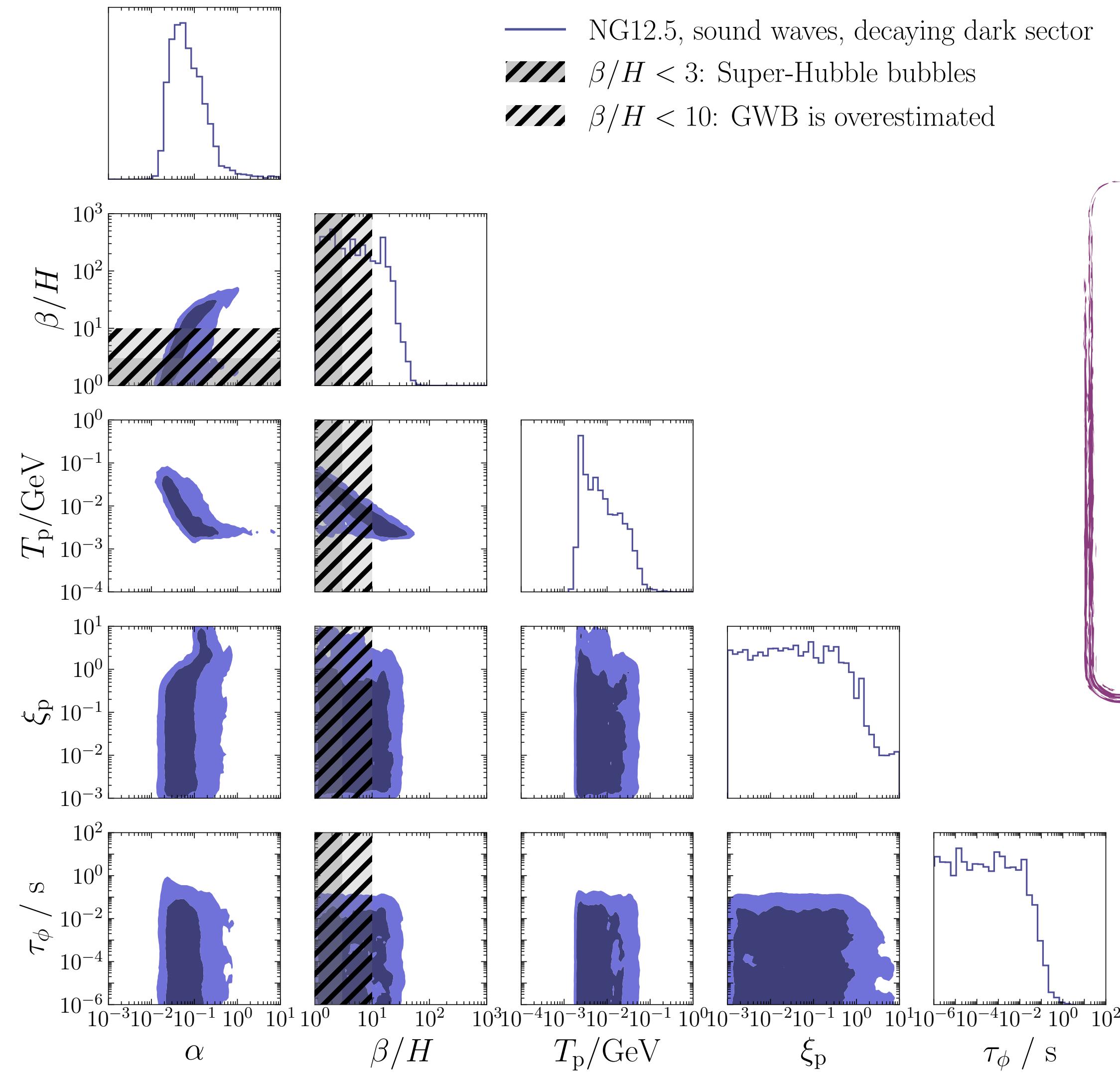


# Global fits

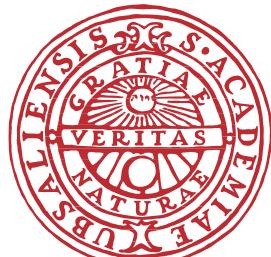
- Combined PTA and BBN/CMB likelihoods in enterprise
- $\beta/H > 1$ : Would do away with GW speed limit
- $\beta/H > 10$ : because data is not up to par with BBN + CMB



# Decays to the rescue

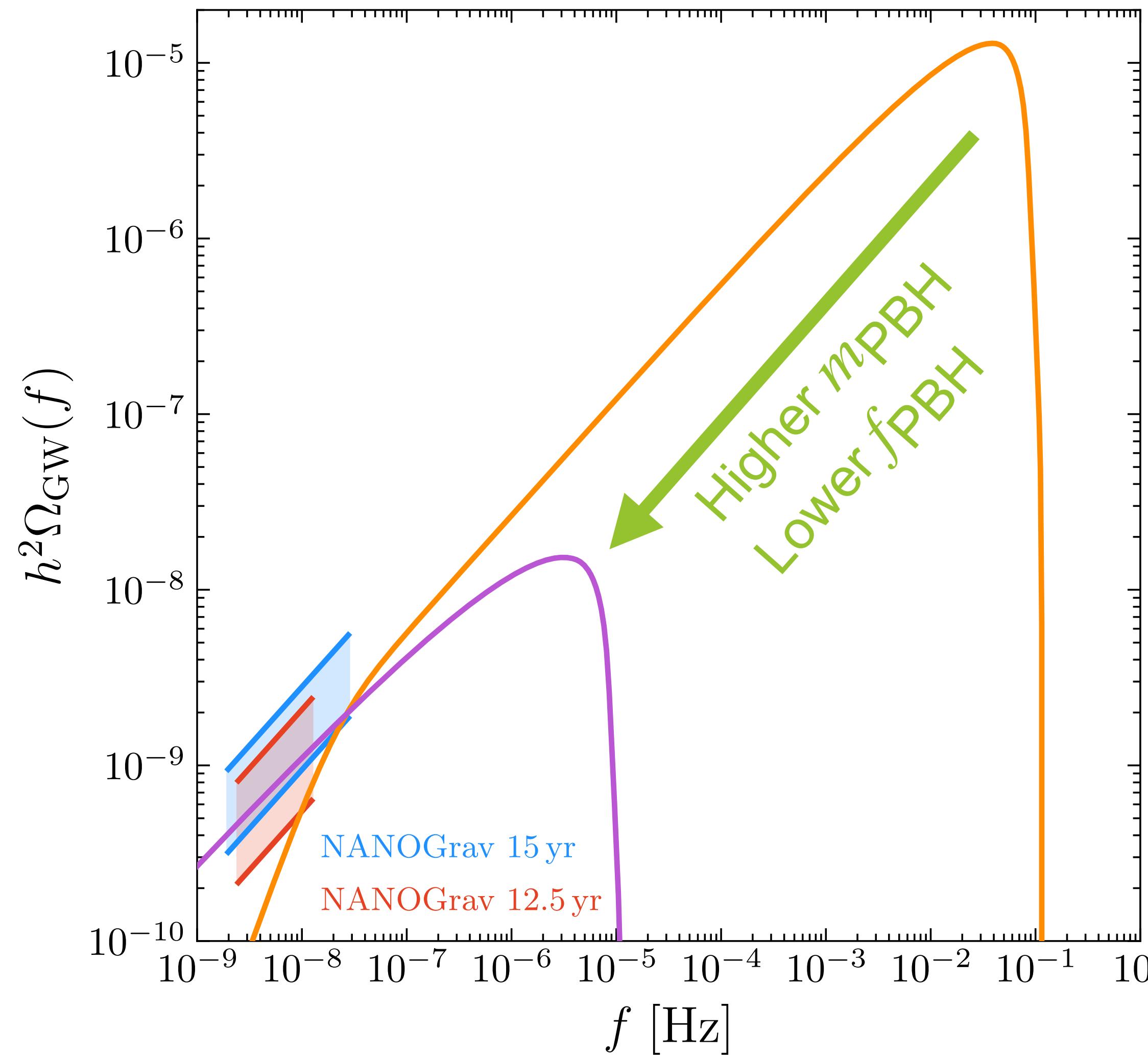


If the dark sector decays quickly  
( $\tau_\phi \lesssim 0.1$  s) before neutrino  
decoupling ( $T \gtrsim 2$  MeV), a great  
fit to PTA data can be achieved!

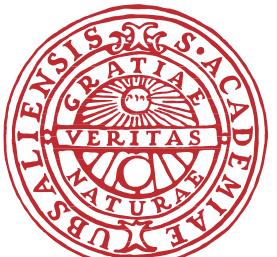


# **Do PTAs observe primordial black hole mergers?**

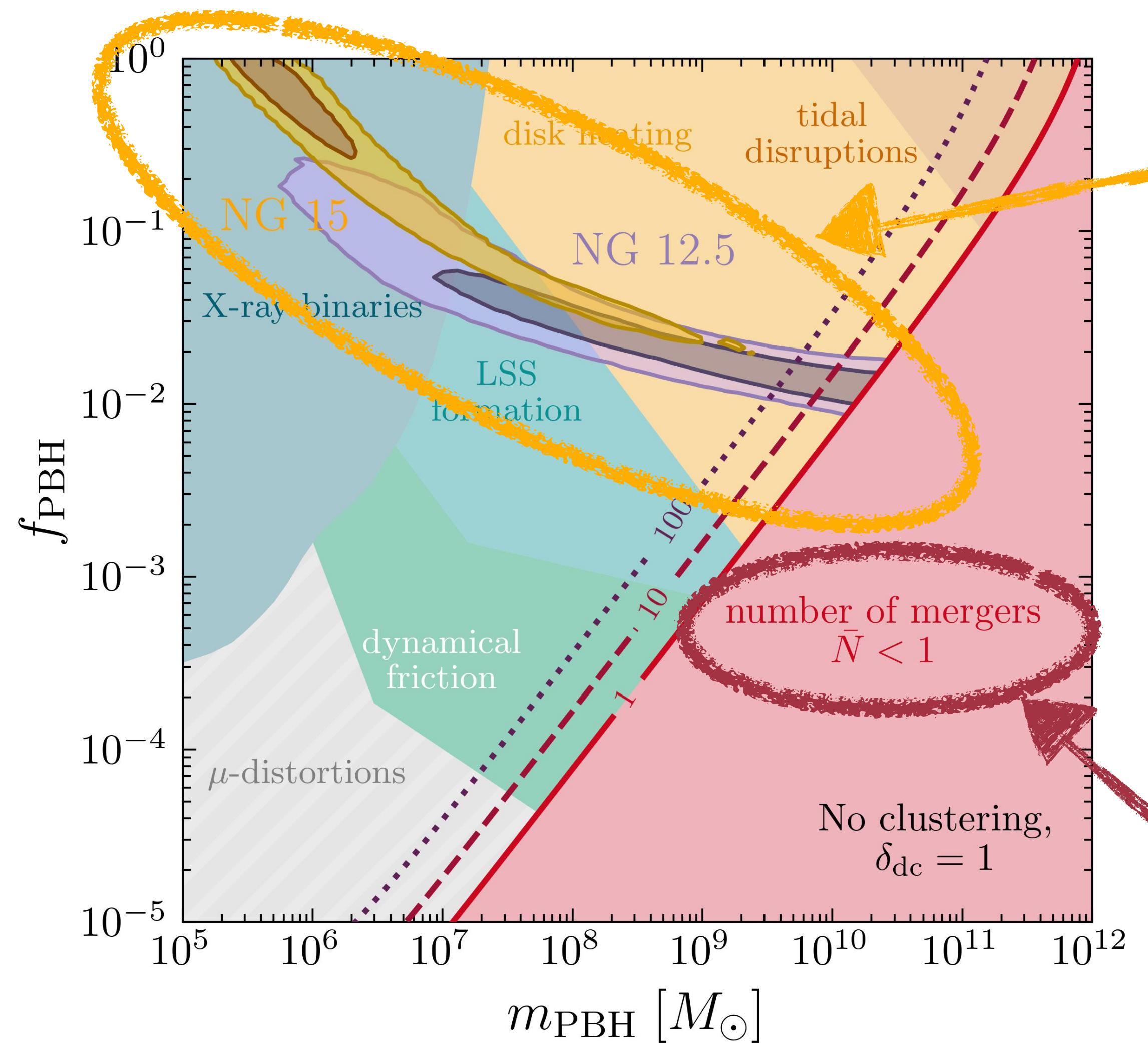
# Supermassive primordial black holes



- Inflation leaves large super-Hubble density perturbations
- BHs form when these come into causal contact again, long before first stars form
- Described by mass  $m_{\text{PBH}}$  and DM fraction  $f_{\text{PBH}}$



# Homogeneously distributed PBHs cannot explain PTA data



Parameter space favored by PTAs is excluded by astrophysical bounds

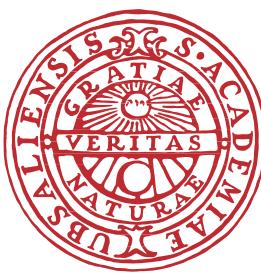
Crucial: excluded regions with small merger numbers. Atal et al. came to the wrong conclusion.



# PBH clustering

$$\delta_{dc} = 1$$

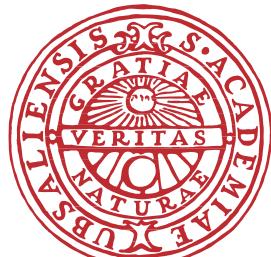
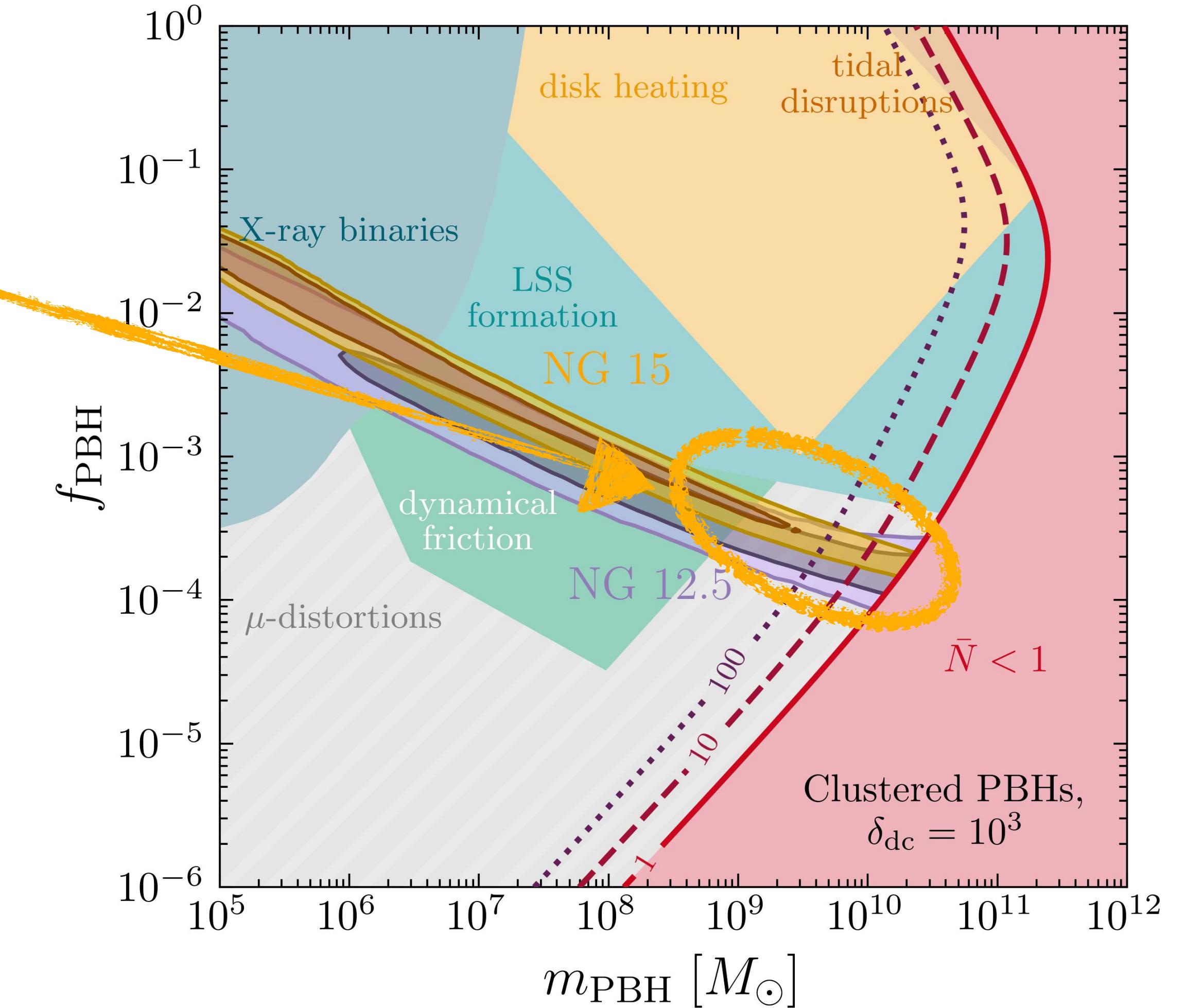
$$\delta_{dc} = 1 + \frac{\delta n}{\bar{n}} \gg 1$$

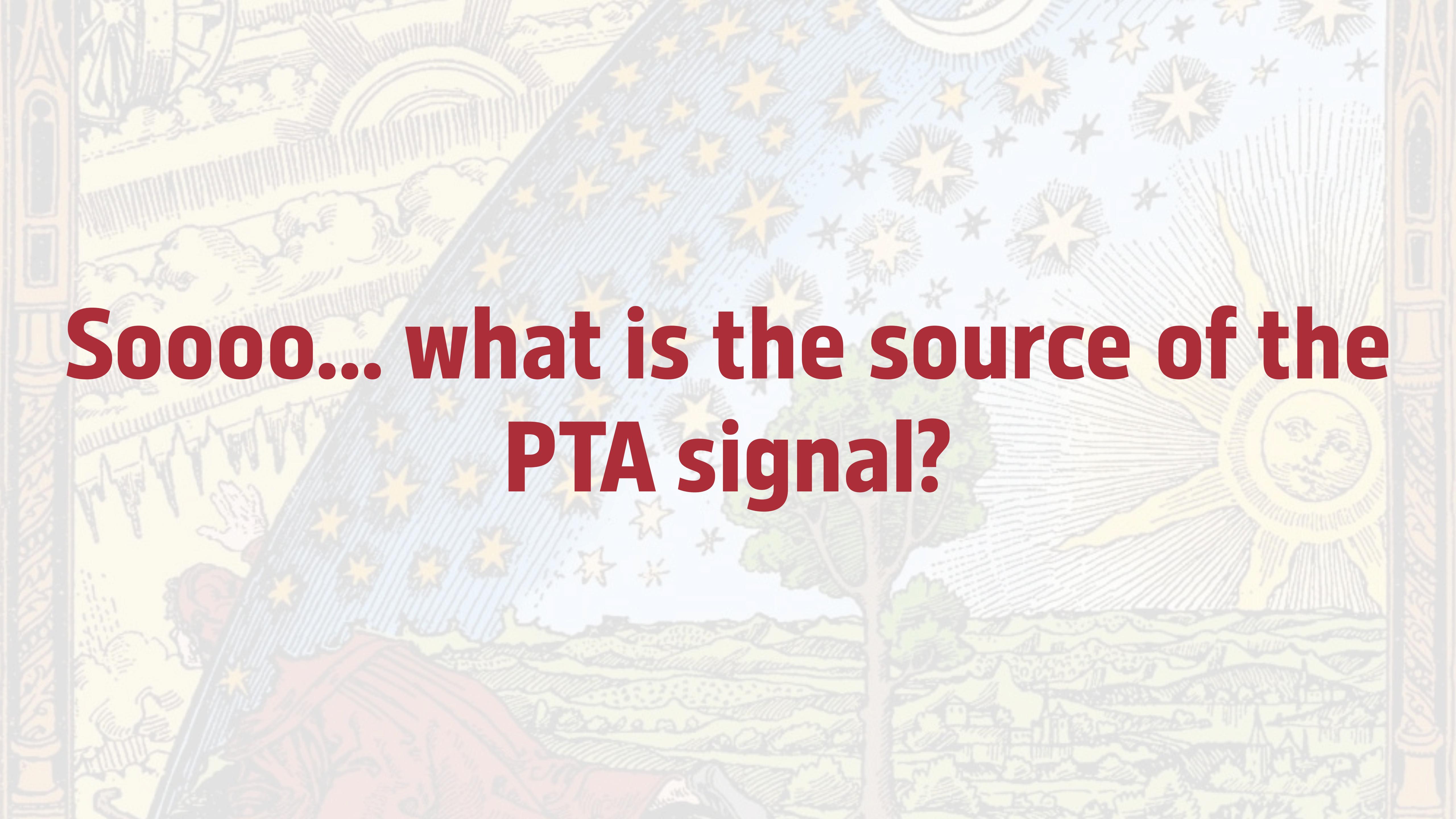


# Clustered PBHs can explain the PTA data

Clustering increases merger rate and shifts the best fit region below constraints:  
Good fit is possible! \*

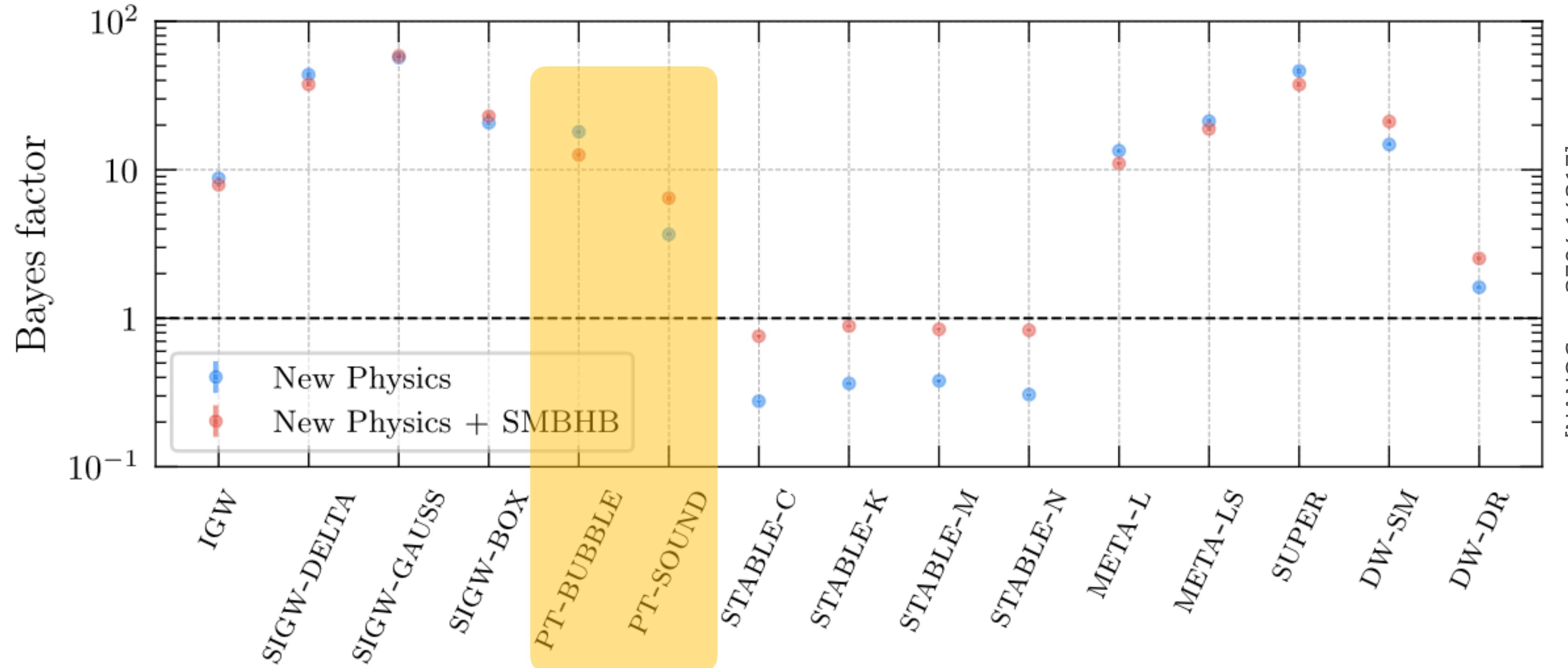
\* Caveats:  $\mu$ -distortion constraints from PBH production need to be circumvented & astrophysical constraints are expected to weaken/shift with clustering





**Soooo... what is the source of the  
PTA signal?**

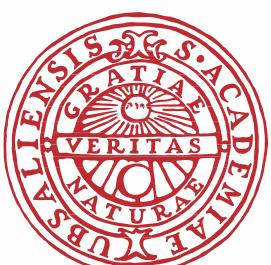
# New physics interpretation of the NANOGrav 15yr data



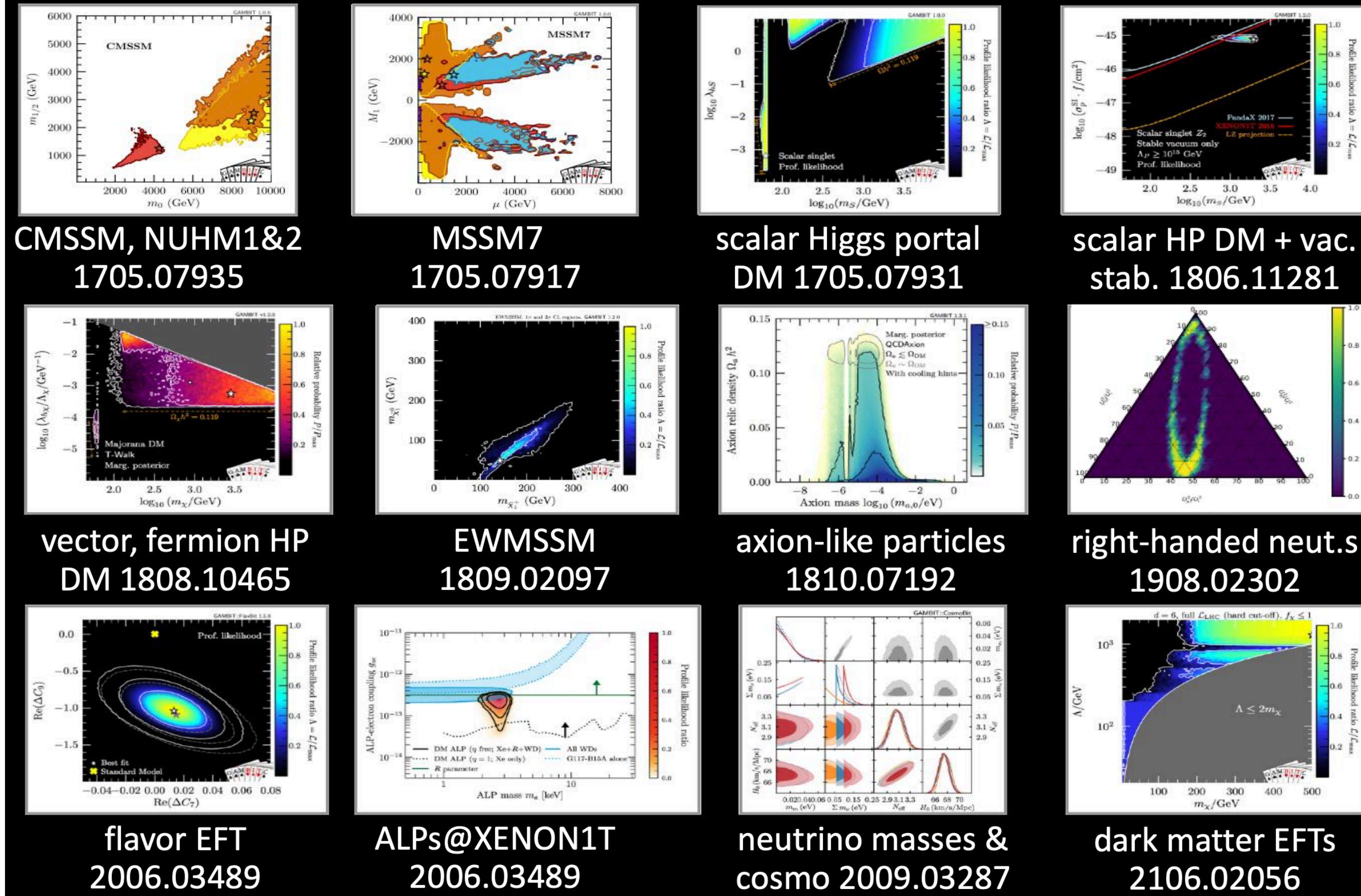
[NANOGrav, 2306.16213]



Cosmological constraints have not been included in the analysis.



# GAMBIT: from Lagrangians to Likelihoods



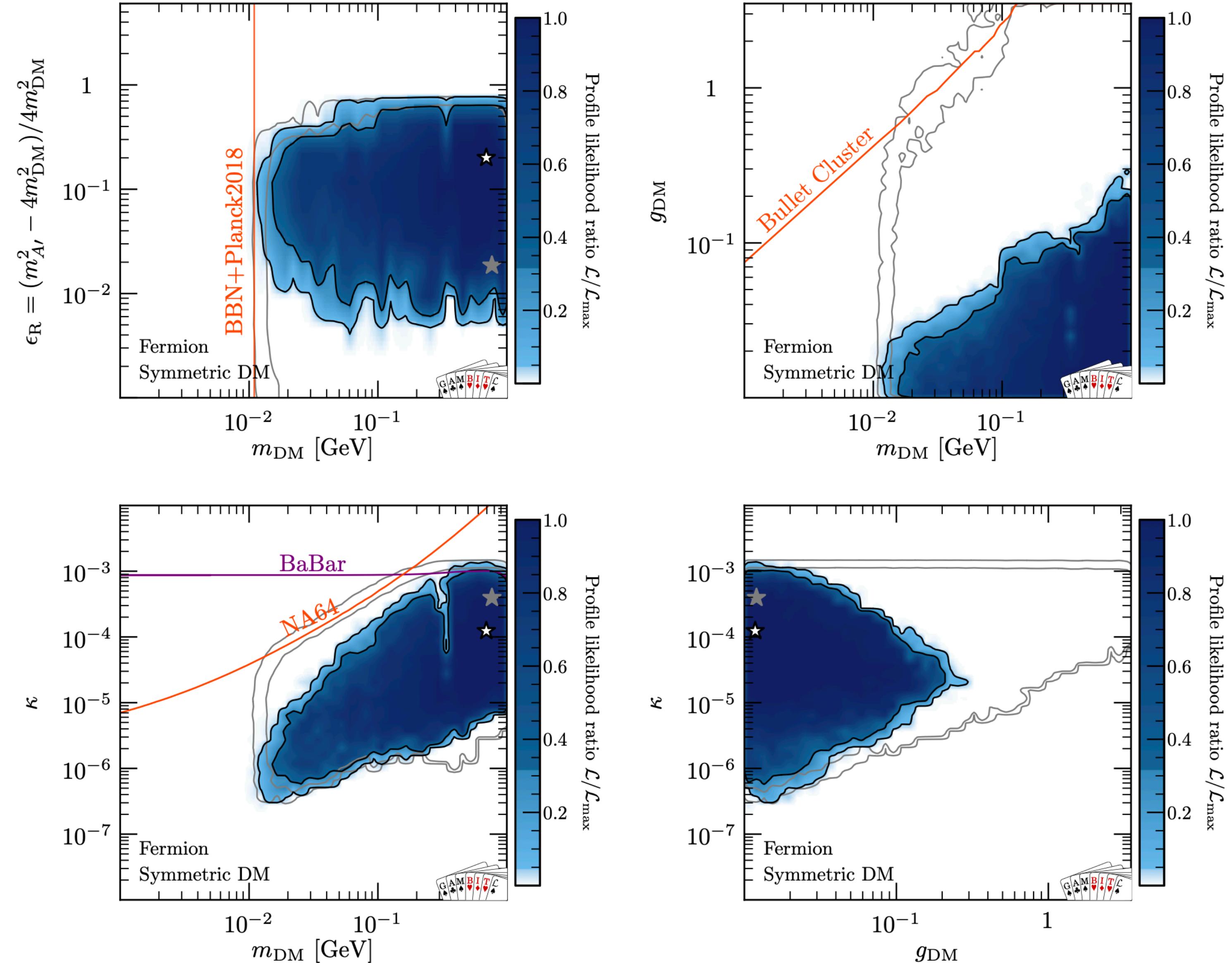
Slide by C. Balázs @ SUSY 2021



Today's models and experiments are too complicated to keep track of all constraints. The solution: GAMBIT

# Global fits: work in progress

- 2405.17548 studied sub-GeV dark matter in a  $U(1)$  setup
- We break that conformal  $U(1)$  and generate very strong GWs
- **Best fit point explains both PTA data and dark matter!**
- Exciting: Best-fit point might be in reach for tests with Belle II, LDMX, SuperCDMS, ...

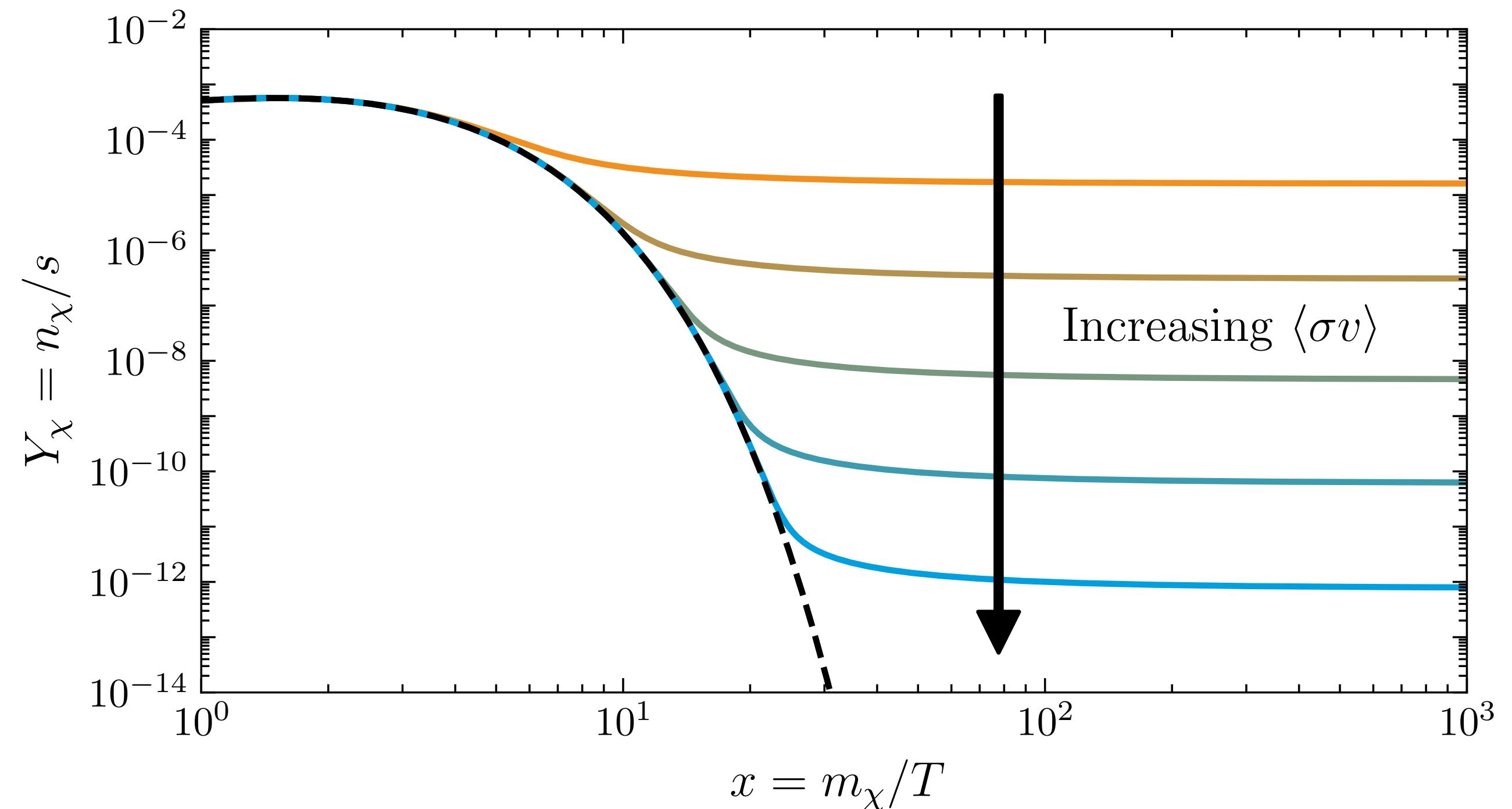
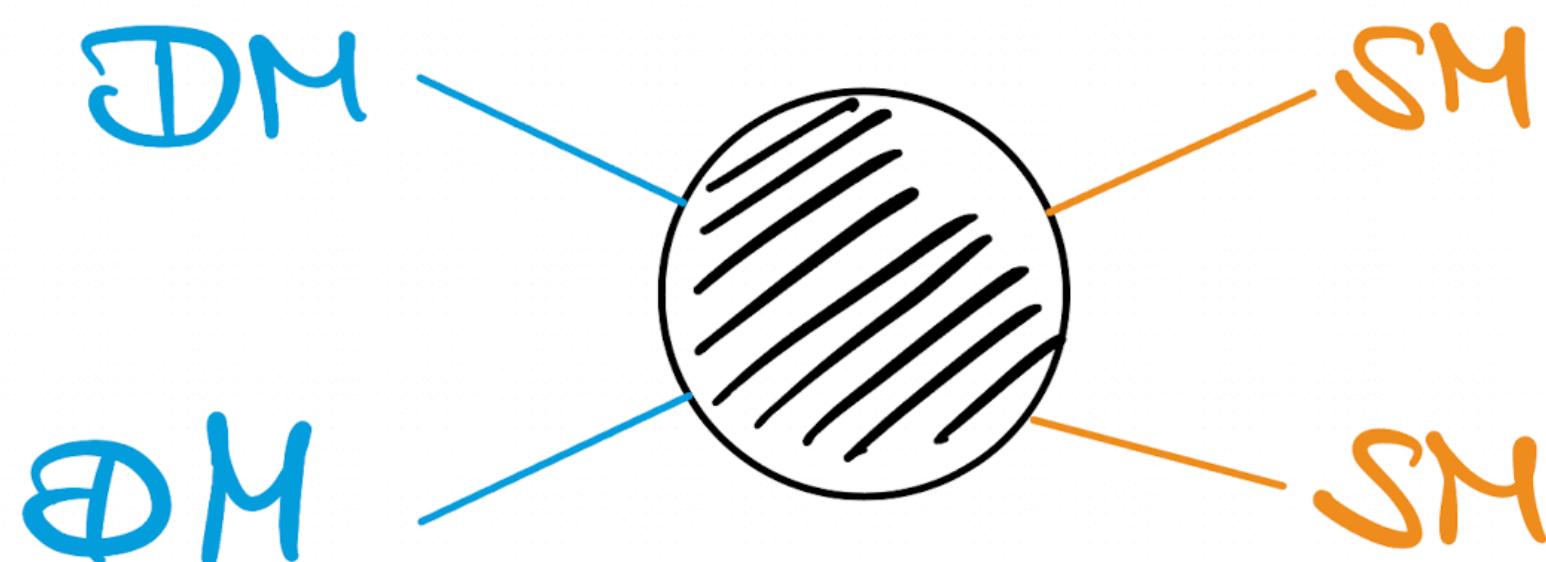




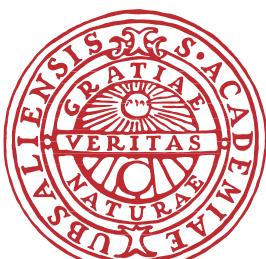
**The (first) LISA miracle.**

# The WIMP miracle

If DM can annihilate into SM particles with a cross section  $\langle \sigma v \rangle$  ...



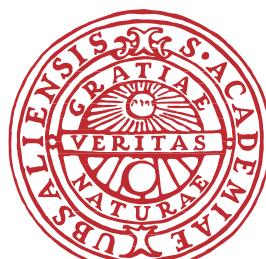
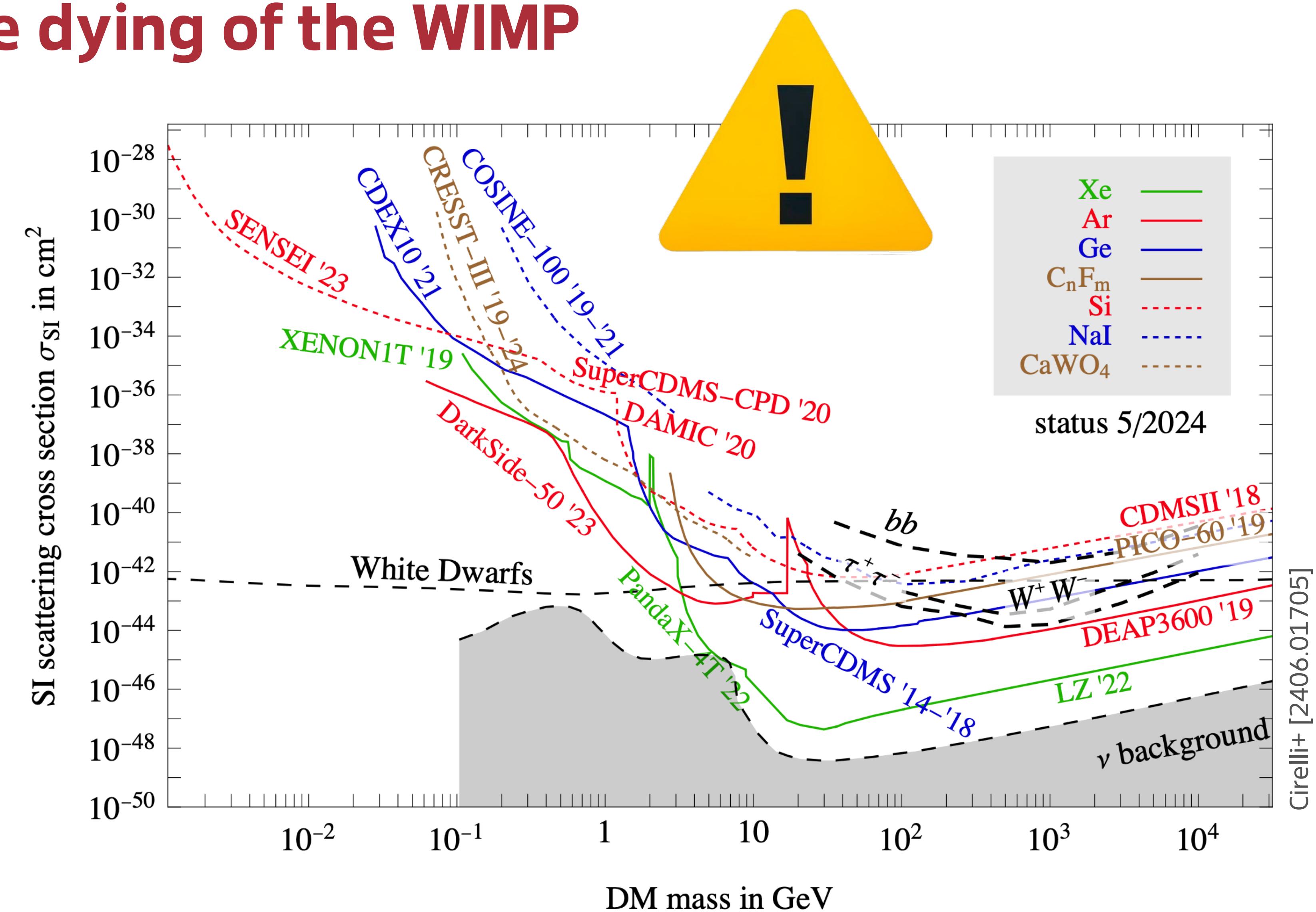
... the DM abundance can freeze out to the observed relic abundance for weak interactions and  $m_{\text{DM}} \simeq \mathcal{O}(\text{TeV})$ .



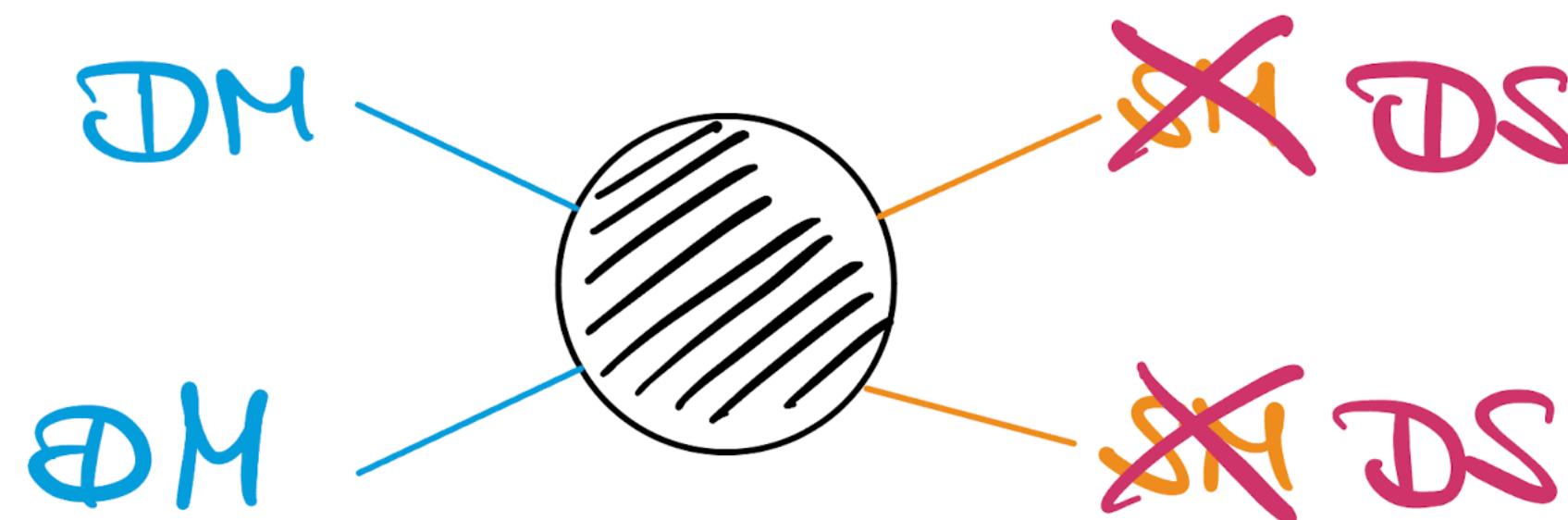
# Rage, rage against the dying of the WIMP

Direct detection experiments put this scenario under pressure, excluding „vanilla” WIMPs.

[Lindner+ 2403.15860]

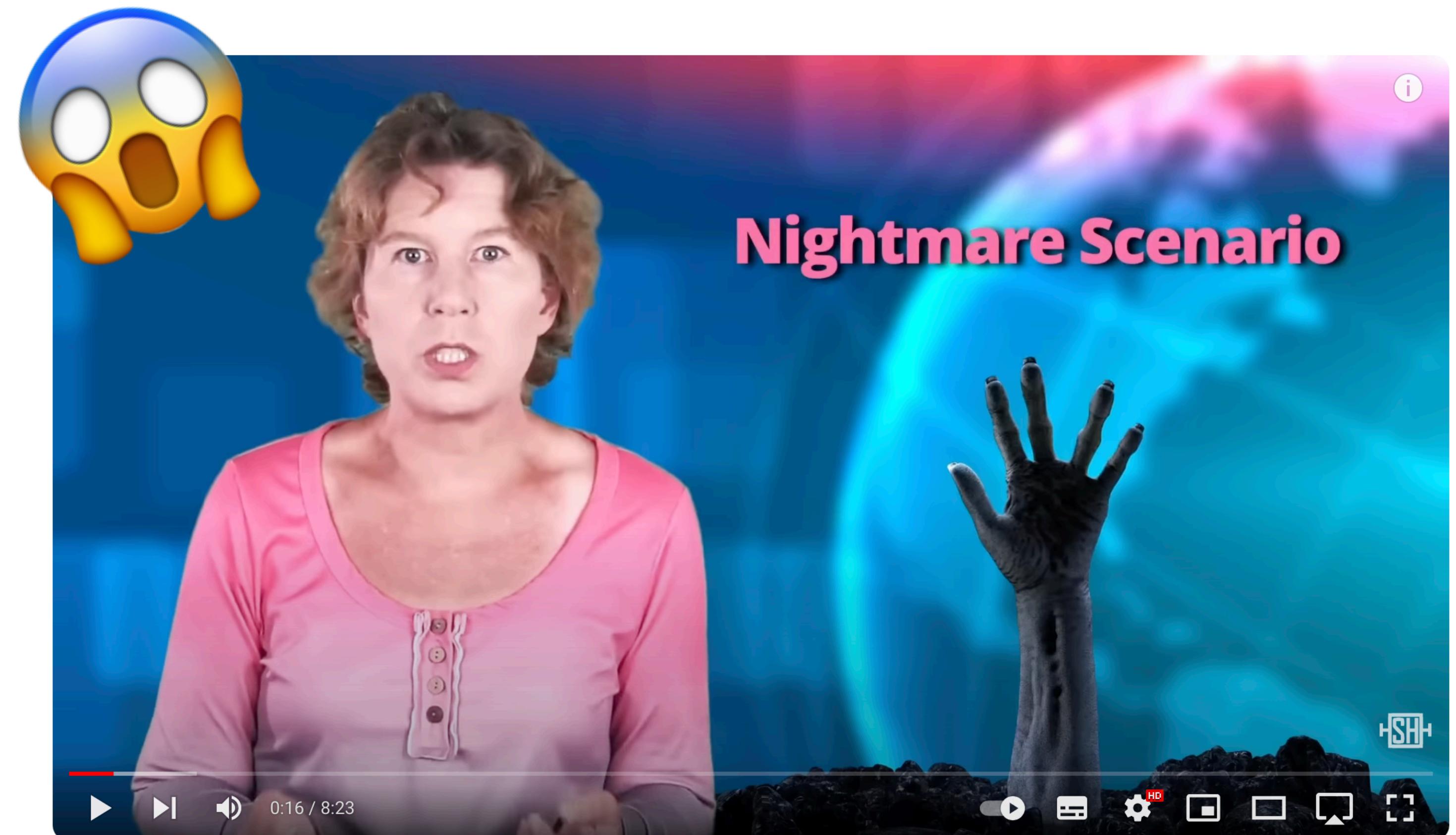


# The nightmare scenario



What if WIMPs evade our detection because they never were in contact with the SM and froze out of a secluded dark sector?

Pospelov+ [0711.4866]



The Nightmare Scenario for Dark Matter is Inching Closer



Sabine Hossenfelder  
1,46 Mio. Abonnenten

Mitglied werden

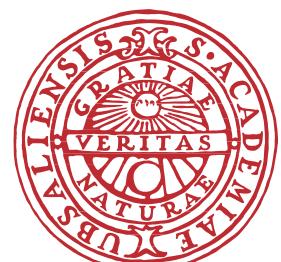
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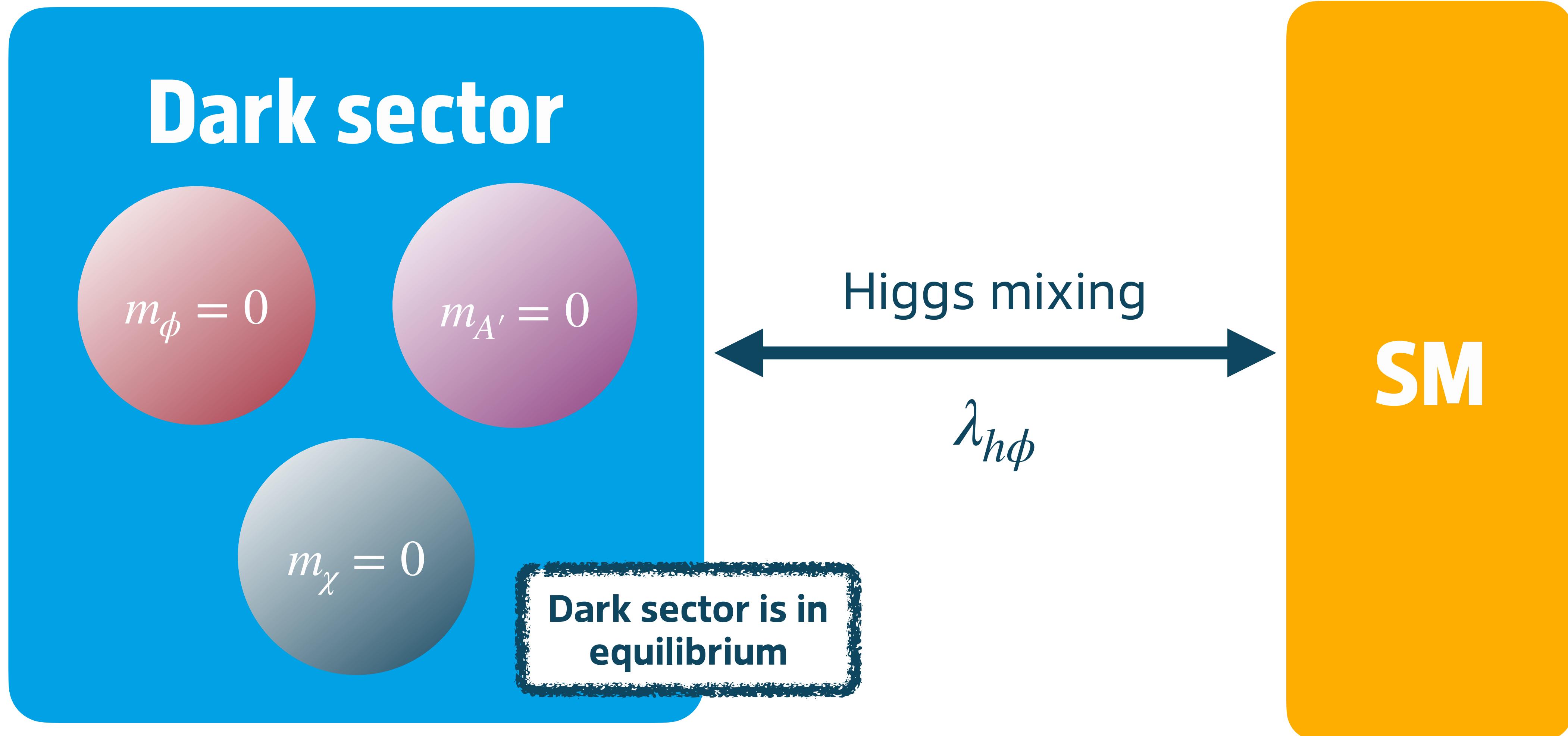


Teilen

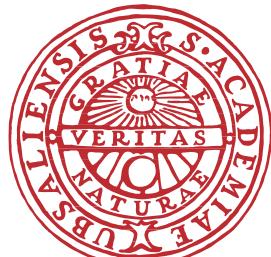
Speichern



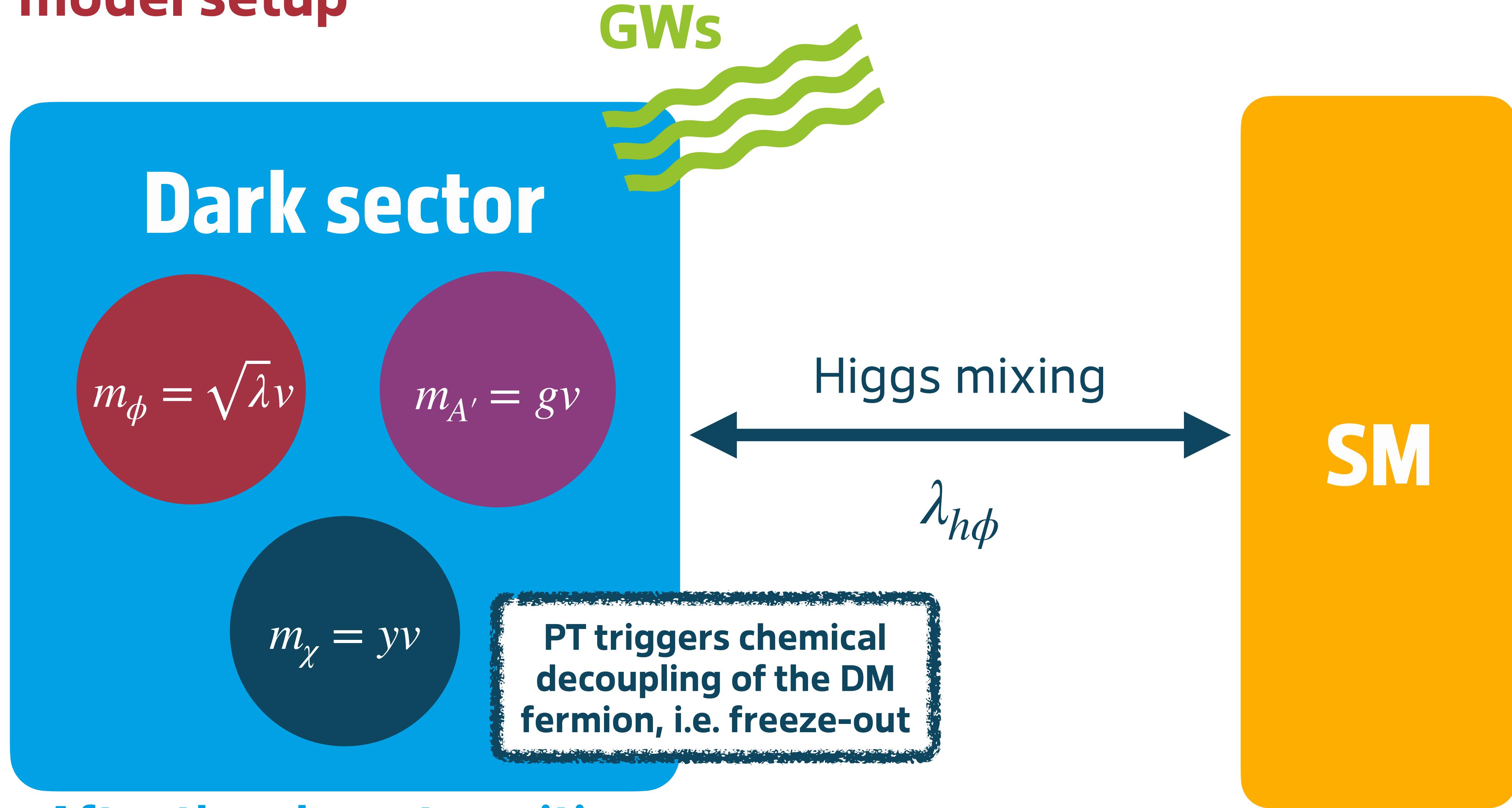
# Our model setup



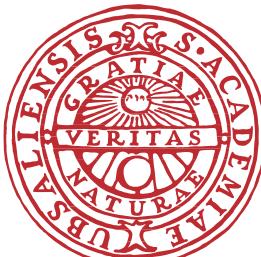
Before the phase transition



# Our model setup



After the phase transition



# A first glance at our punchline

**J**ournal of Cosmology and Astroparticle Physics  
An IOP and SISSA journal

RECEIVED: December 15, 2023  
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## Hunting WIMPs with LISA: correlating dark matter and gravitational wave signals

Torsten Bringmann ,<sup>a</sup> Tomás E. Gonzalo,<sup>b</sup> Felix Kahlhoefer,<sup>b</sup> Jonas Matuszak ,<sup>b,c</sup> and Carlo Tasillo ,<sup>d</sup>

<sup>a</sup>Department of Physics, University of Oslo, Box 1048, N-0316 Oslo, Norway

<sup>b</sup>Institute for Theoretical Particle Physics (TTP), Karlsruhe Institute of Technology (KIT), 76128 Karlsruhe, Germany

<sup>c</sup>Institute for Theoretical Particle Physics and Cosmology (TTK), RWTH Aachen University, D-52056 Aachen, Germany

<sup>d</sup>Deutsches Elektronen-Synchrotron DESY, Notkestr. 85, 22607 Hamburg, Germany

E-mail: [torsten.bringmann@fys.uio.no](mailto:torsten.bringmann@fys.uio.no), [tomas.gonzalo@kit.edu](mailto:tomas.gonzalo@kit.edu), [kahlhoefer@kit.edu](mailto:kahlhoefer@kit.edu), [jonas.matuszak@kit.edu](mailto:jonas.matuszak@kit.edu), [carlo.tasillo@desy.de](mailto:carlo.tasillo@desy.de)

**ABSTRACT:** The thermal freeze-out mechanism in its classical form is tightly connected to physics beyond the Standard Model around the electroweak scale, which has been the target of enormous experimental efforts. In this work we study a dark matter model in which freeze-out is triggered by a strong first-order phase transition in a dark sector, and show that this phase transition must also happen close to the electroweak scale, i.e. in the temperature range relevant for gravitational wave searches with the LISA mission. Specifically, we consider the spontaneous breaking of a U(1)' gauge symmetry through the vacuum expectation value of a scalar field, which generates the mass of a fermionic dark matter candidate that subsequently annihilates into dark Higgs and gauge bosons. In this set-up the peak frequency of the gravitational wave background is tightly correlated with the dark matter relic abundance, and imposing the observed value for the latter implies that the former must lie in the milli-Hertz range. A peculiar feature of our set-up is that the dark sector is not necessarily in thermal equilibrium with the Standard Model during the phase transition, and hence the temperatures of the two sectors evolve independently. Nevertheless, the requirement that the universe does not enter an extended period of matter domination after the phase transition, which would strongly dilute any gravitational wave signal, places a lower bound on the portal coupling that governs the entropy transfer between the two sectors. As a result, the predictions for the peak frequency of gravitational waves in the LISA band are robust, while the amplitude can change depending on the initial dark sector temperature.

**KEYWORDS:** cosmological phase transitions, dark matter theory, particle physics - cosmology connection, primordial gravitational waves (theory)

ARXIV EPRINT: [2311.06346](https://arxiv.org/abs/2311.06346)

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<https://doi.org/10.1088/1475-7516/2024/05/065>

JCAP05(2024)065

## Theorem:

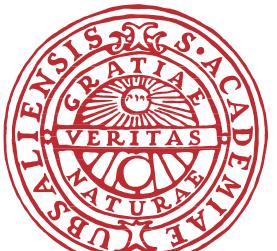
There is a correlation between the **GW peak frequency** and the **DM abundance**.

## Proof:

$f_{\text{peak}} \propto v$  and  $\Omega_{\text{DM}} \propto v^2$  for a transition with vacuum expectation value  $v$ .

## Lemma:

$\Omega_{\text{DM}} h^2 = 0.12 \implies f_{\text{peak}} \simeq \mathcal{O}(\text{mHz})$ . If DM freeze-out is triggered by a strong phase transition, it is observable by LISA.



# The miracle at work

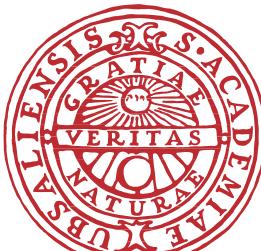
**Peak frequency:**  $f_{\text{peak}} \simeq 10 \text{ mHz} \left( \frac{\beta/H}{100} \right) \left( \frac{T^{\nu}}{1 \text{ TeV}} \right) \simeq 10 \text{ mHz} \left( \frac{\nu}{1 \text{ TeV}} \right)$

**DM abundance:**  $\Omega_{\text{DM}} h^2 \simeq 0.1 \frac{10^{-8} \text{ GeV}^{-2}}{\langle \sigma v \rangle} \propto \frac{\nu^2}{y^2}$

Assuming that dominant annihilation channel is  $\chi\chi \rightarrow \phi\phi$ :

$$\langle \sigma v \rangle \sim \frac{y^4}{m_\chi^2} \sim \frac{y^2}{\nu^2}$$

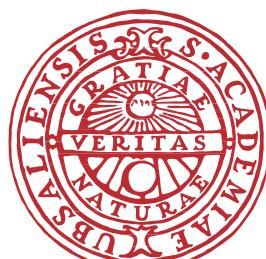
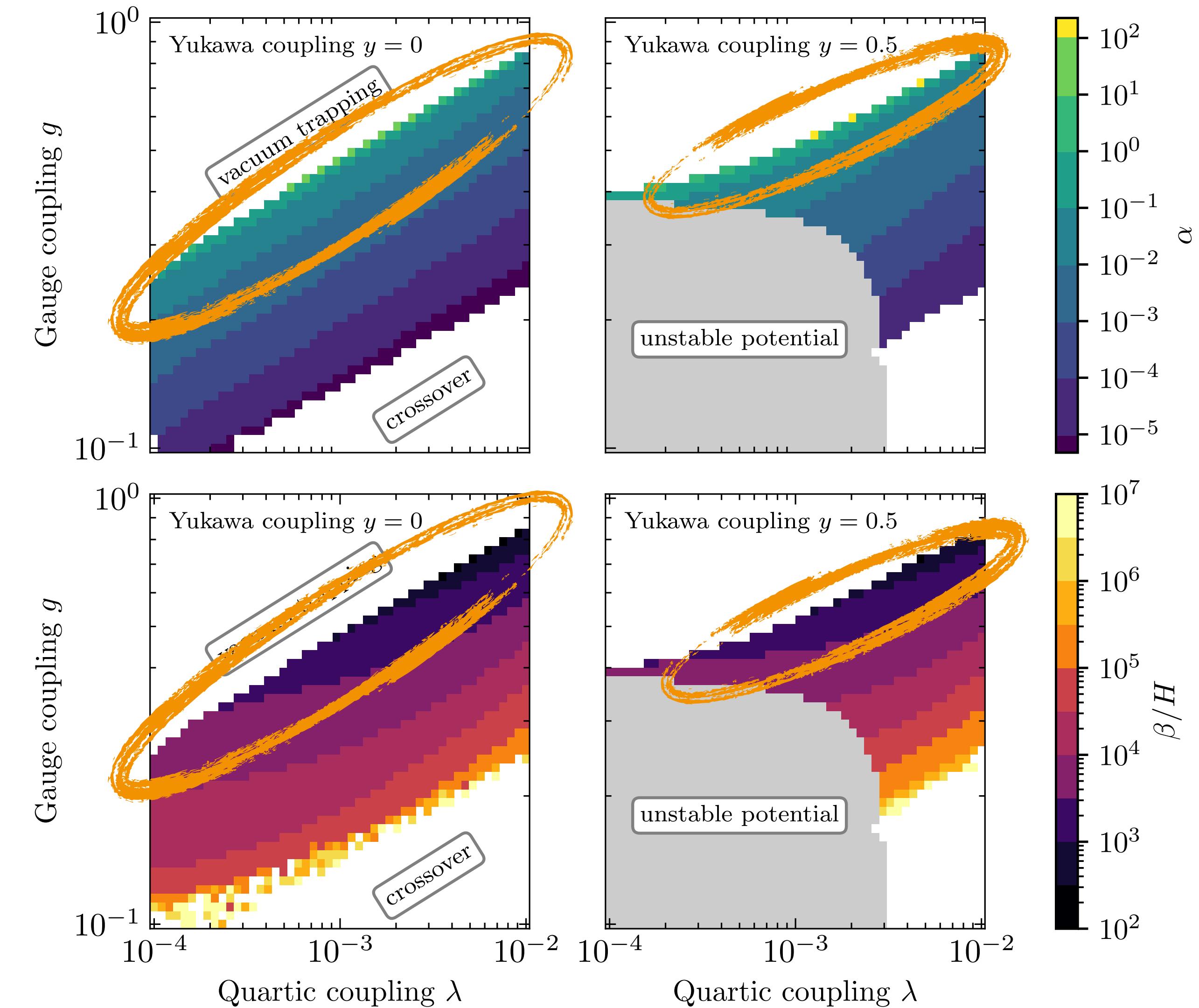
Since Yukawa coupling  $y$  is **a-priori** arbitrary: **no correlation expected...**



# Intermediate Yukawa couplings

**Strong-GW condition:**

Sizable couplings and  $m_\phi \lesssim m_{A'}$



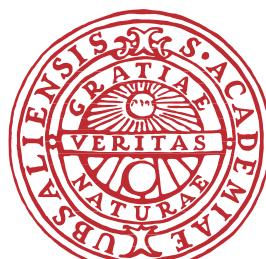
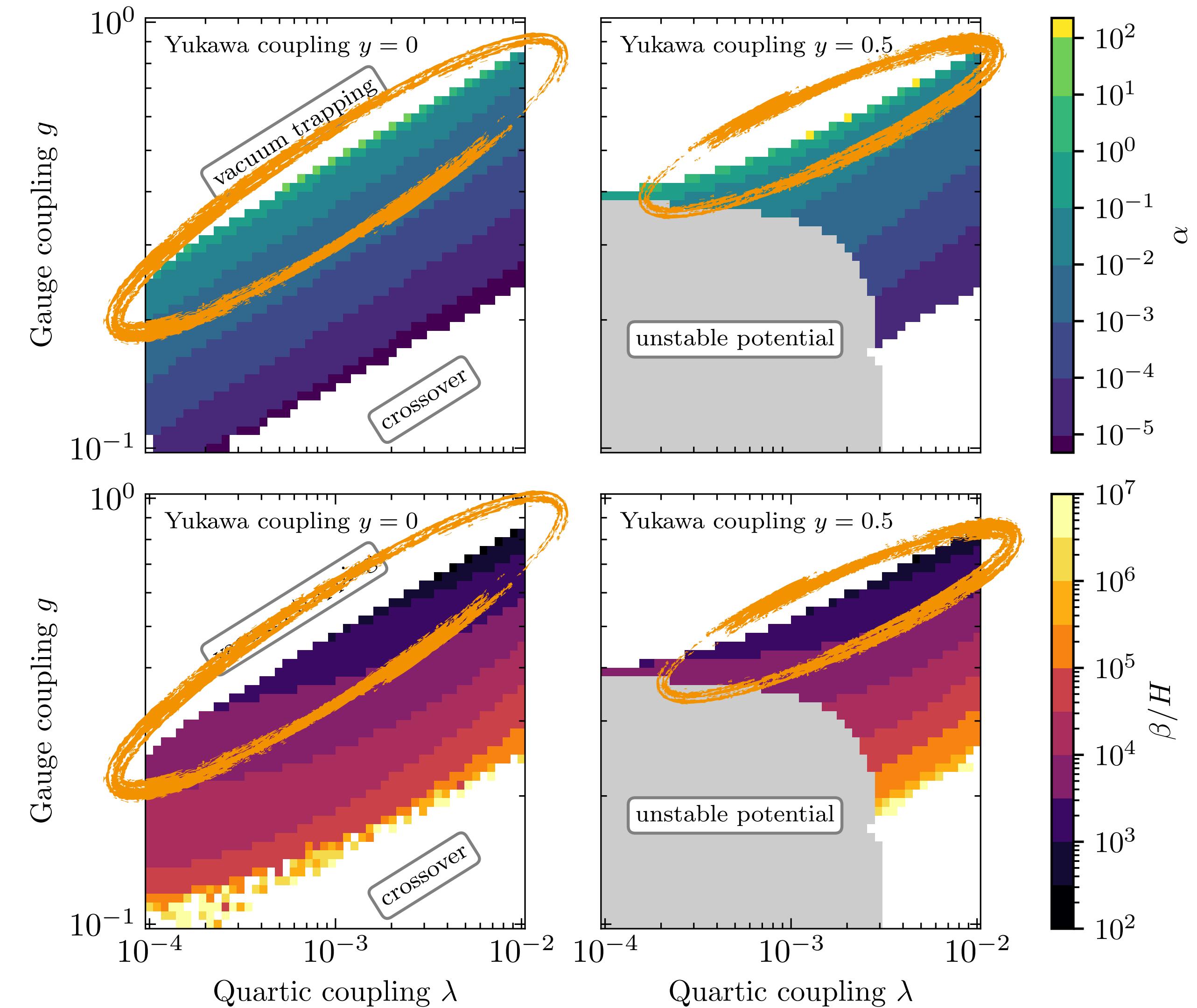
# Intermediate Yukawa couplings

**Strong-GW condition:**

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**Freeze-out condition:**

DM cannot be lightest dark sector state:  $m_\phi < m_\chi$  or  $m_{A'} < m_\chi$



# Intermediate Yukawa couplings

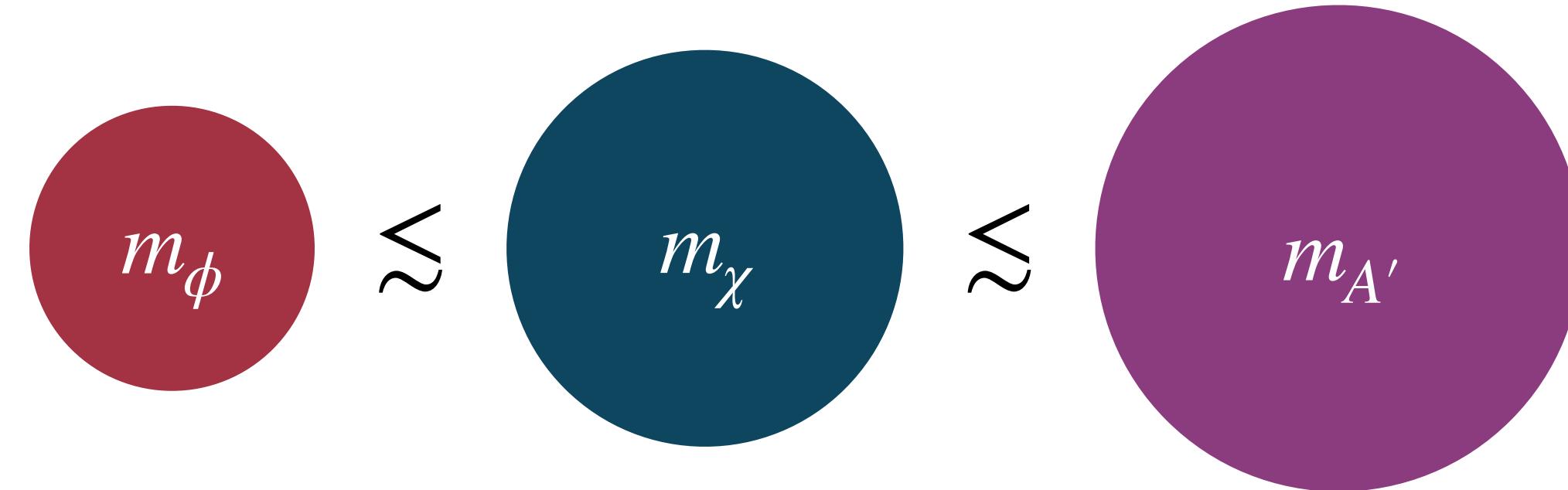
**Strong-GW condition:**

Sizable couplings and  $m_\phi \lesssim m_{A'}$

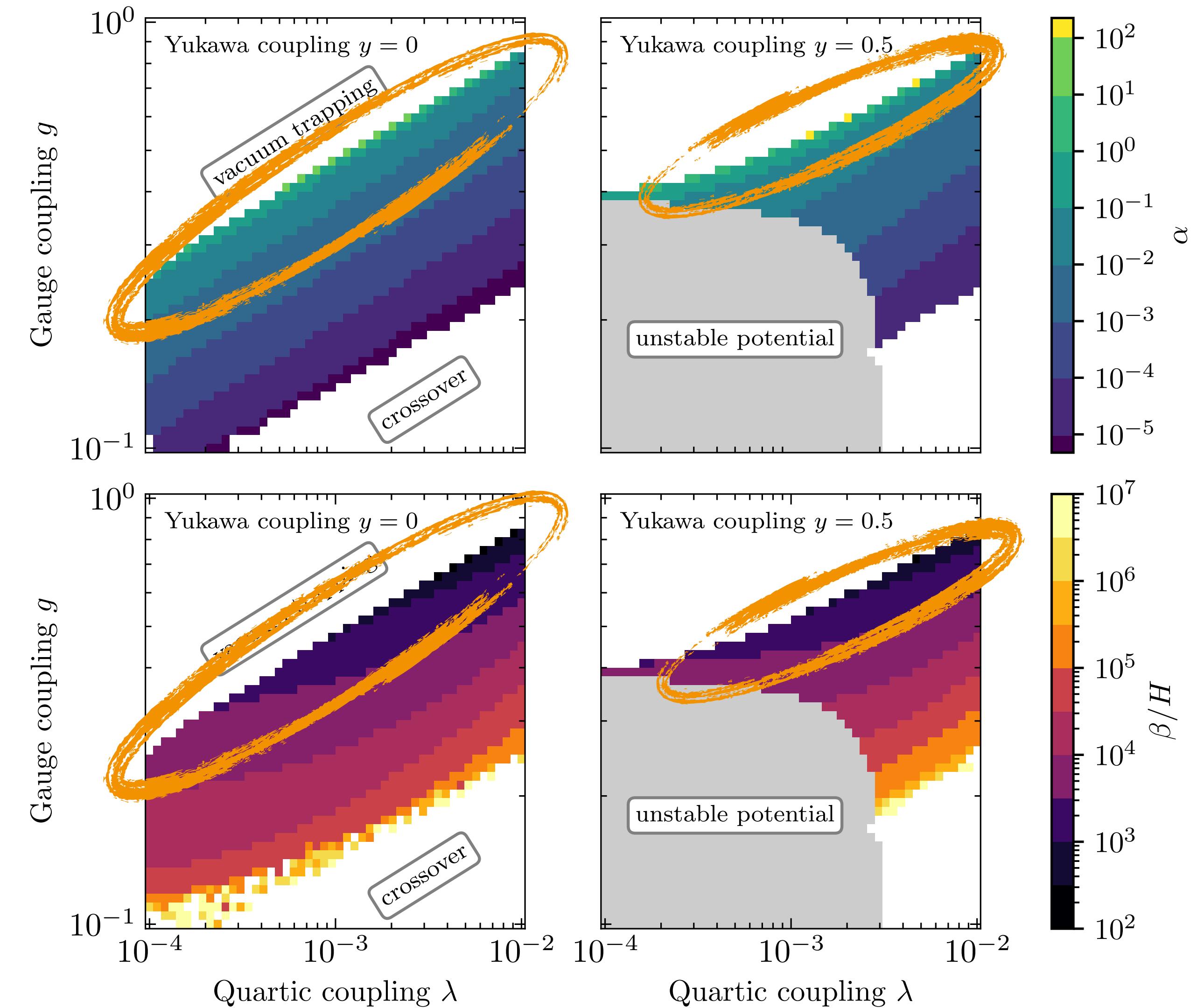
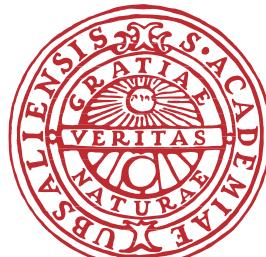
**Freeze-out condition:**

DM cannot be lightest dark sector state:  $m_\phi < m_\chi$  or  $m_{A'} < m_\chi$

**Conclusion:**



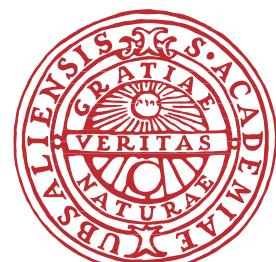
Yukawa couplings are bounded and  $\mathcal{O}(0.1)$ . Miracles can happen! 😊



# You shouldn't be convinced

**So far we skipped over several potential issues:**

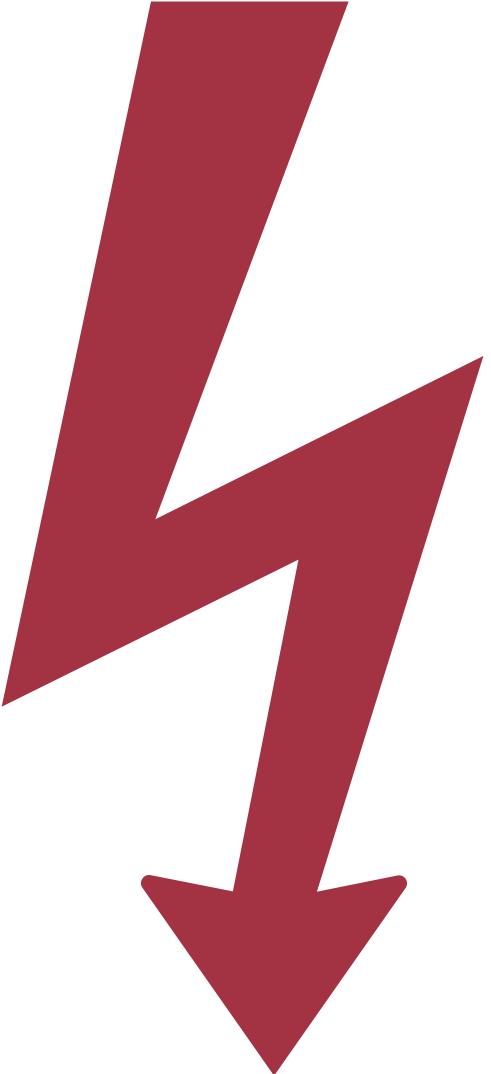
- Sizable Yukawa couplings vs. vacuum stability
- What about the  $\chi\chi \rightarrow A'A'$  and  $\chi\chi \rightarrow \phi A'$  annihilations?
- Influence of temperature ratio  $\xi = T_{\text{DS}}/T_{\text{SM}}$  on  $\Omega_{\text{GW}}(f)$  and  $\Omega_{\text{DM}}$ ?
- $\lambda_{h\phi}$ : Collider bounds? Early matter domination?



# You shouldn't be convinced

**So far we skipped over several potential issues:**

- Sizable Yukawa couplings vs. vacuum stability
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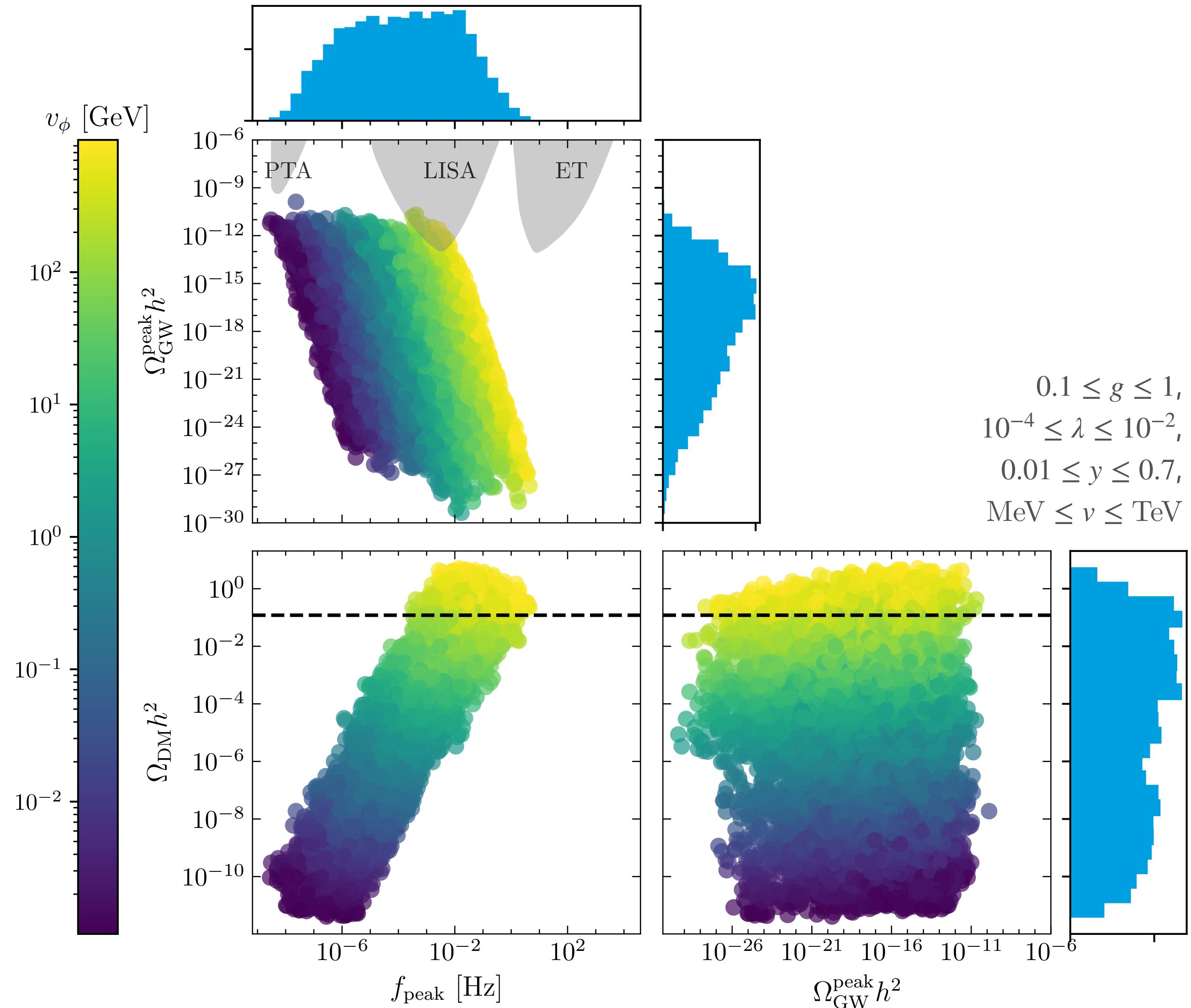


We performed full model scans\* over  $\lambda, g, y, v, \xi, \lambda_{h\phi}$  and confirmed the LISA miracle!

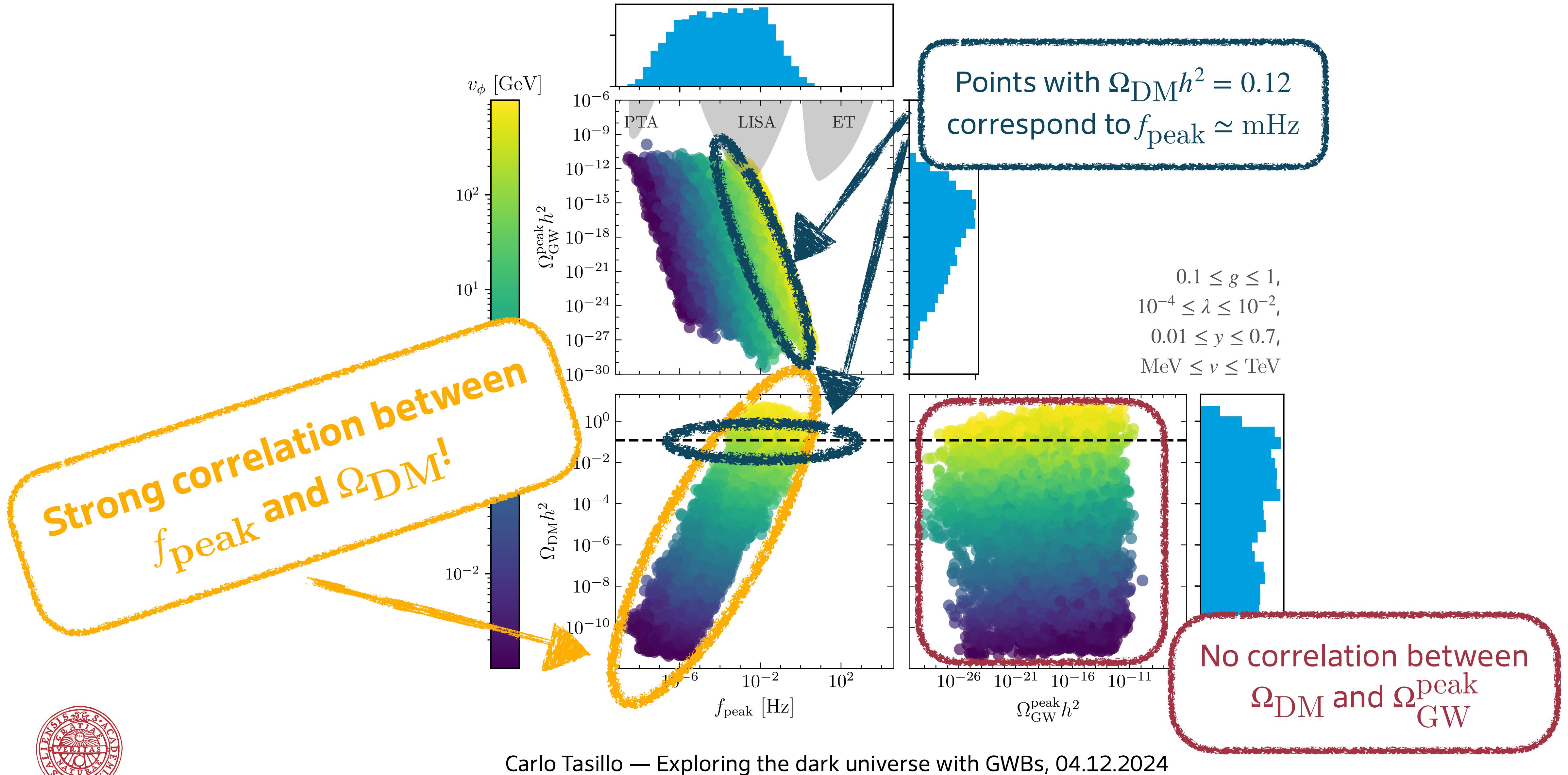
\* TransitionListener & DarkSUSY [Ertas+ 2109.06208, Bringmann+ 1802.03399]



# Results of our scans



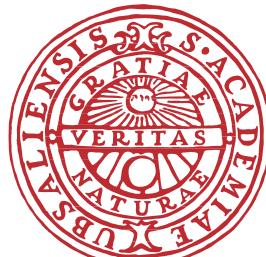
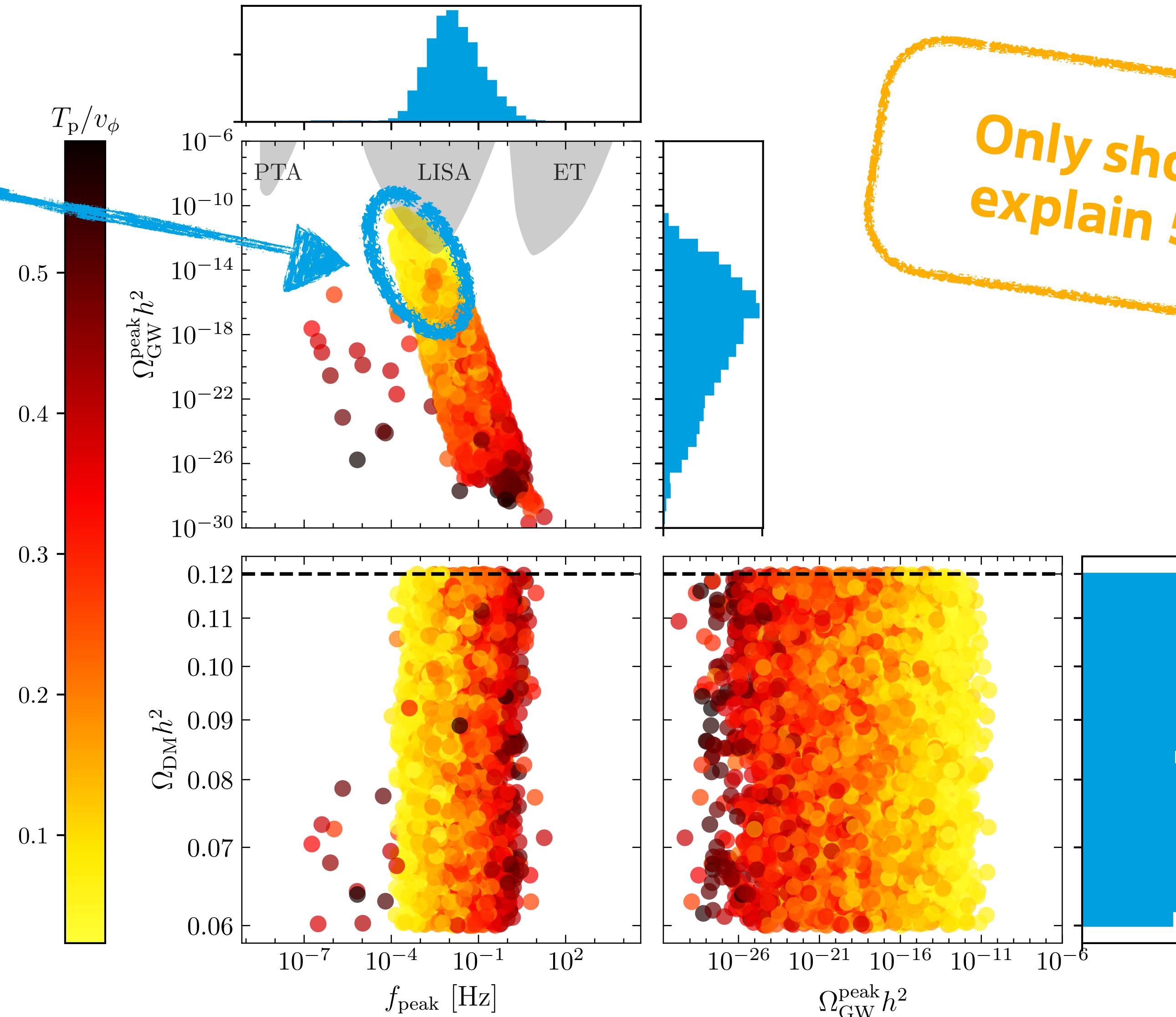
# Results of our scans



# Selection: observed DM abundance is explained

Strong supercooling

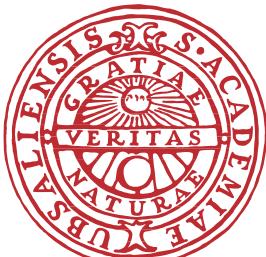
35% of points with strong supercooling and correct DM abundance are observable



# Summary

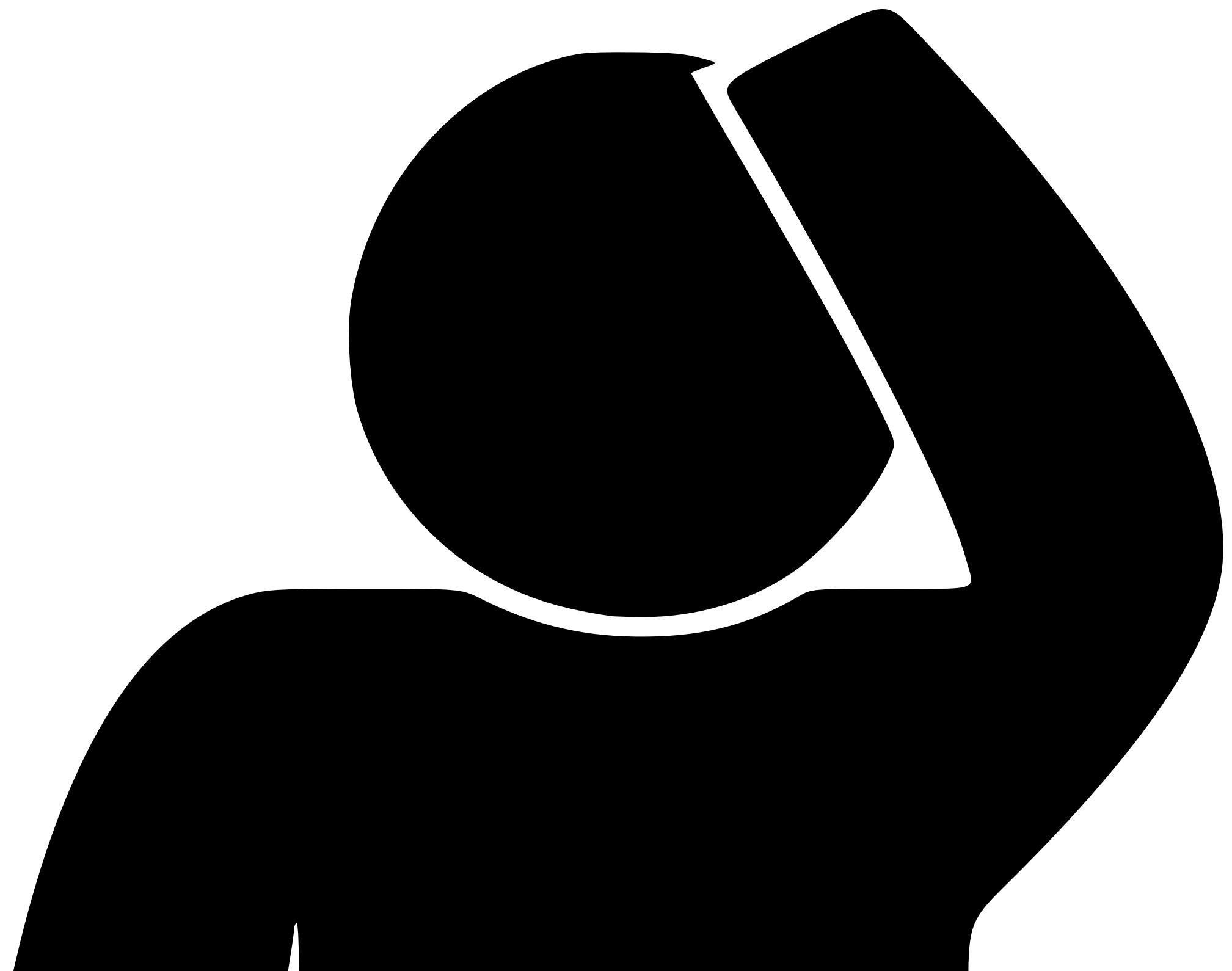


- We are only at the dawn of GW cosmology, but can already probe the pre-BBN universe!
- PTAs could have observed a dark sector phase transition or merging supermassive PBHs
  - ➡ Dark sector phase transitions cannot be too strong & quick: Need  $\alpha \lesssim 0.1$  and  $\beta/H \lesssim 10$  or (better) quick decays ( $\tau_\phi \lesssim 0.1$  s)
  - ➡ PBHs need to be clustered, no  $\mu$ -distortions at production
  - ➡ Look out for our next GAMBIT results
- A future LISA detection of a GW background would hint towards secluded DS freeze-out



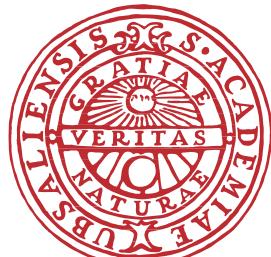
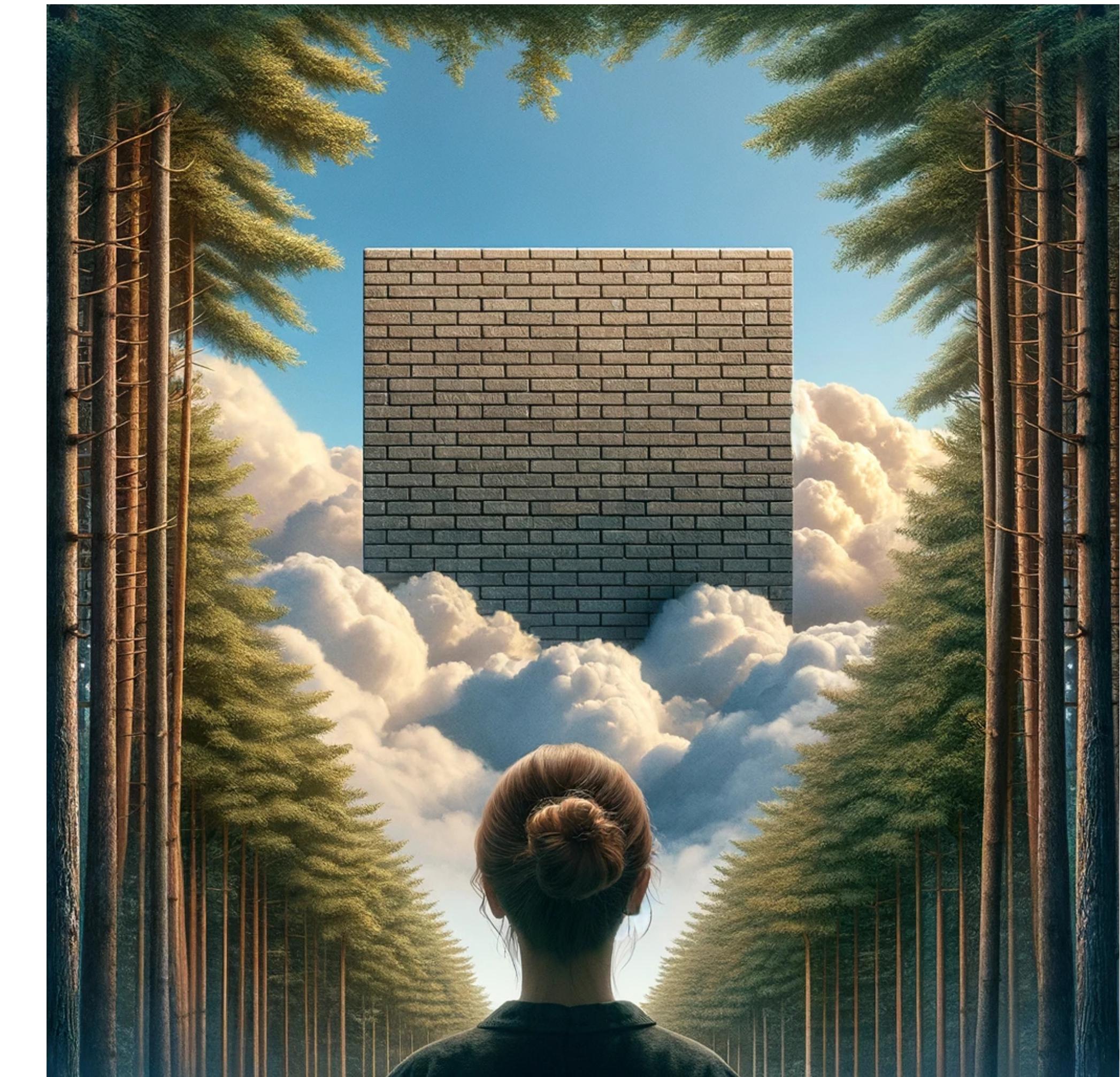
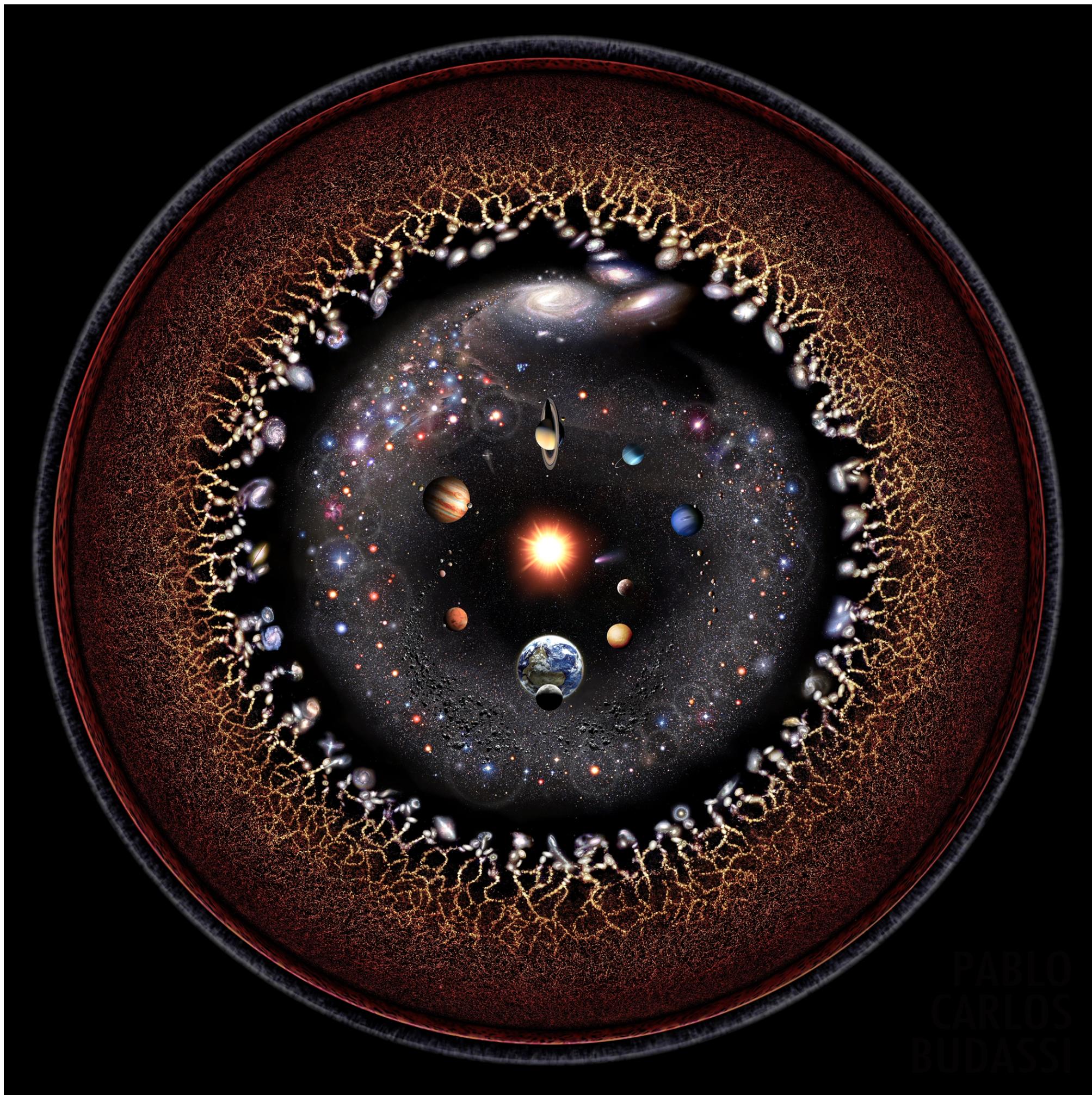
**Thank you very much  
for your attention!**

**Do you have any questions?**

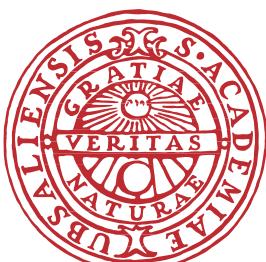
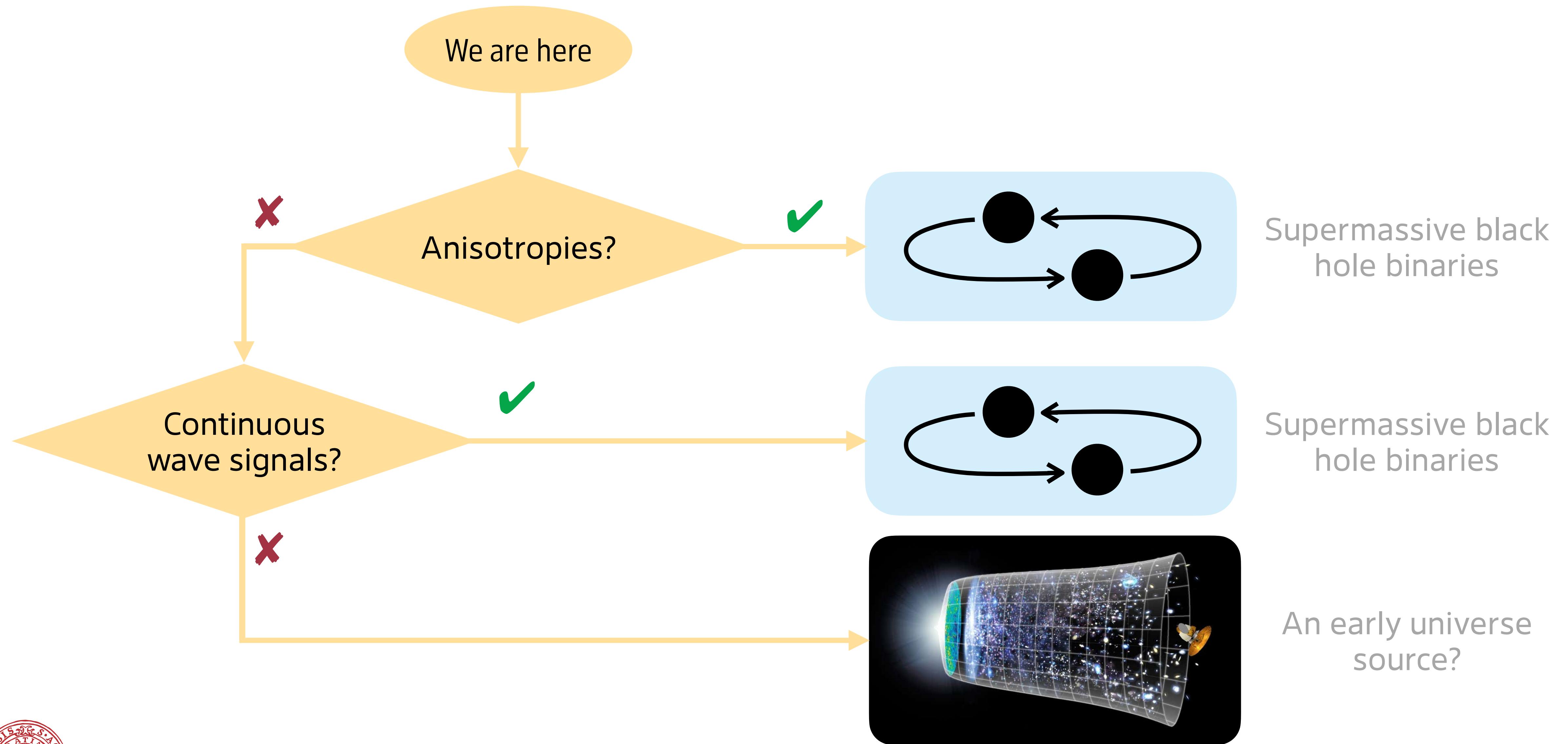


# Backup slides

# What do we know about the early universe?



# Quo vadis pulsar timing?



# GWB details

$$h^2 \Omega_{\text{GW}}(f) = \mathcal{R} h^2 \tilde{\Omega} \left( \frac{\kappa_{\text{sw}} \alpha}{\alpha + 1} \right)^2 \left( \frac{\beta}{H} \right)^{-1} \mathcal{Y} S(f)$$

$$\mathcal{R} h^2 = \Omega_\gamma h^2 \left( \frac{h_{\text{SM},0}}{h_{\text{tot,p}}} \right)^{4/3} \left( \frac{g_{\text{tot,p}}}{g_{\gamma,0}} \right) = 1.653 \cdot 10^{-5} \left( \frac{100}{h_{\text{tot,p}}} \right)^{4/3} \left( \frac{g_{\text{tot,p}}}{100} \right)$$

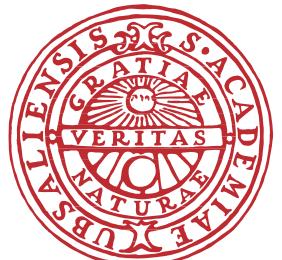
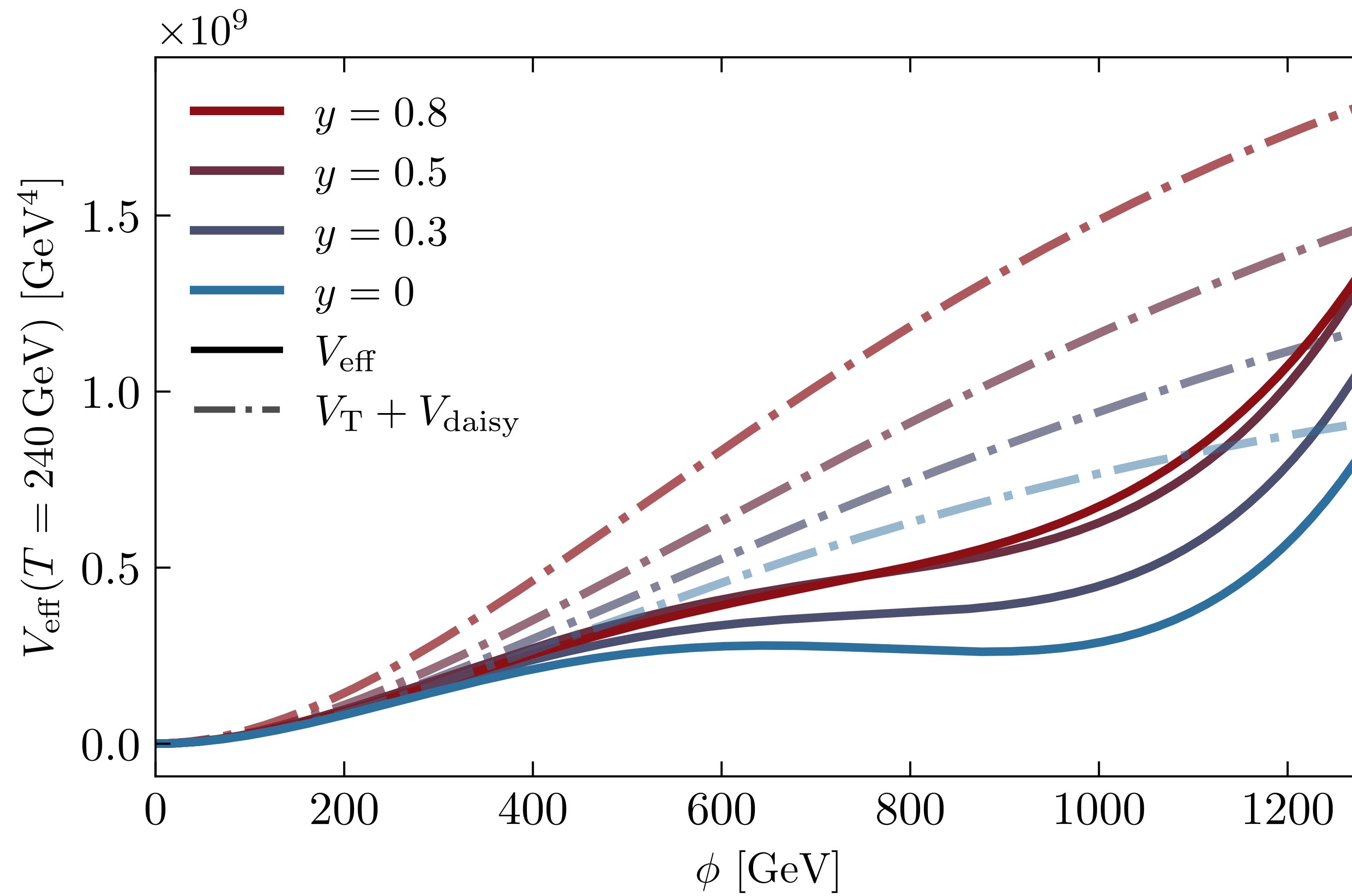
$$\mathcal{Y} = \min [1, \tau_{\text{sh}} H] \simeq \min \left[ 1, \frac{3.38}{\beta/H} \sqrt{\frac{1+\alpha}{\kappa_{\text{sw}} \alpha}} \right]$$

$$S(f) = \left( \frac{f}{f_{\text{peak}}} \right)^3 \left( \frac{7}{4 + 3(f/f_{\text{peak}})^2} \right)^{7/2}$$

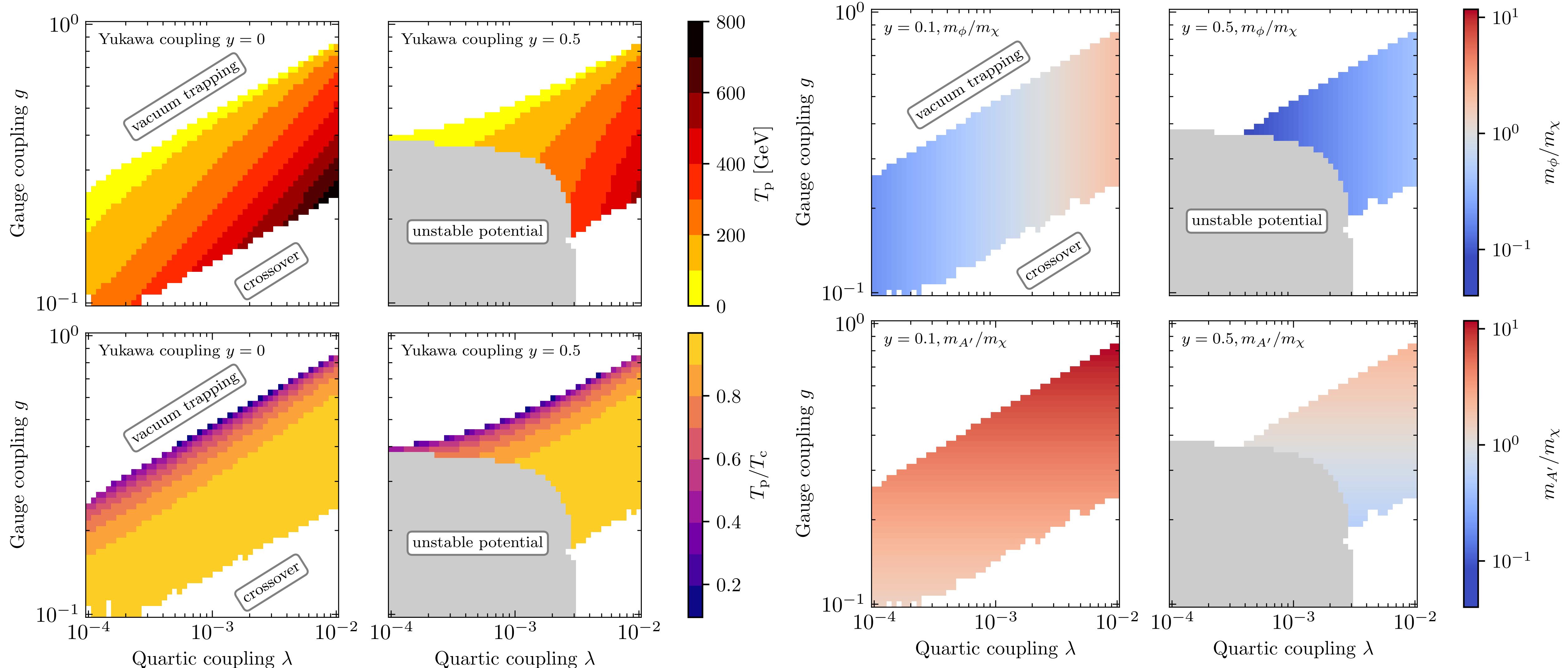
$$f_{\text{peak}} = 8.9 \text{ mHz} \left( \frac{T_p}{100 \text{ GeV}} \right) \left( \frac{\beta/H}{1000} \right) \left( \frac{g_{\text{tot,p}}}{100} \right)^{1/2} \left( \frac{100}{h_{\text{tot,p}}} \right)^{1/3}$$



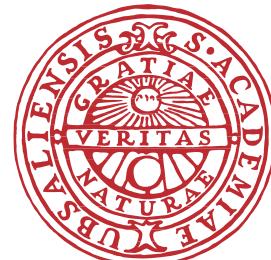
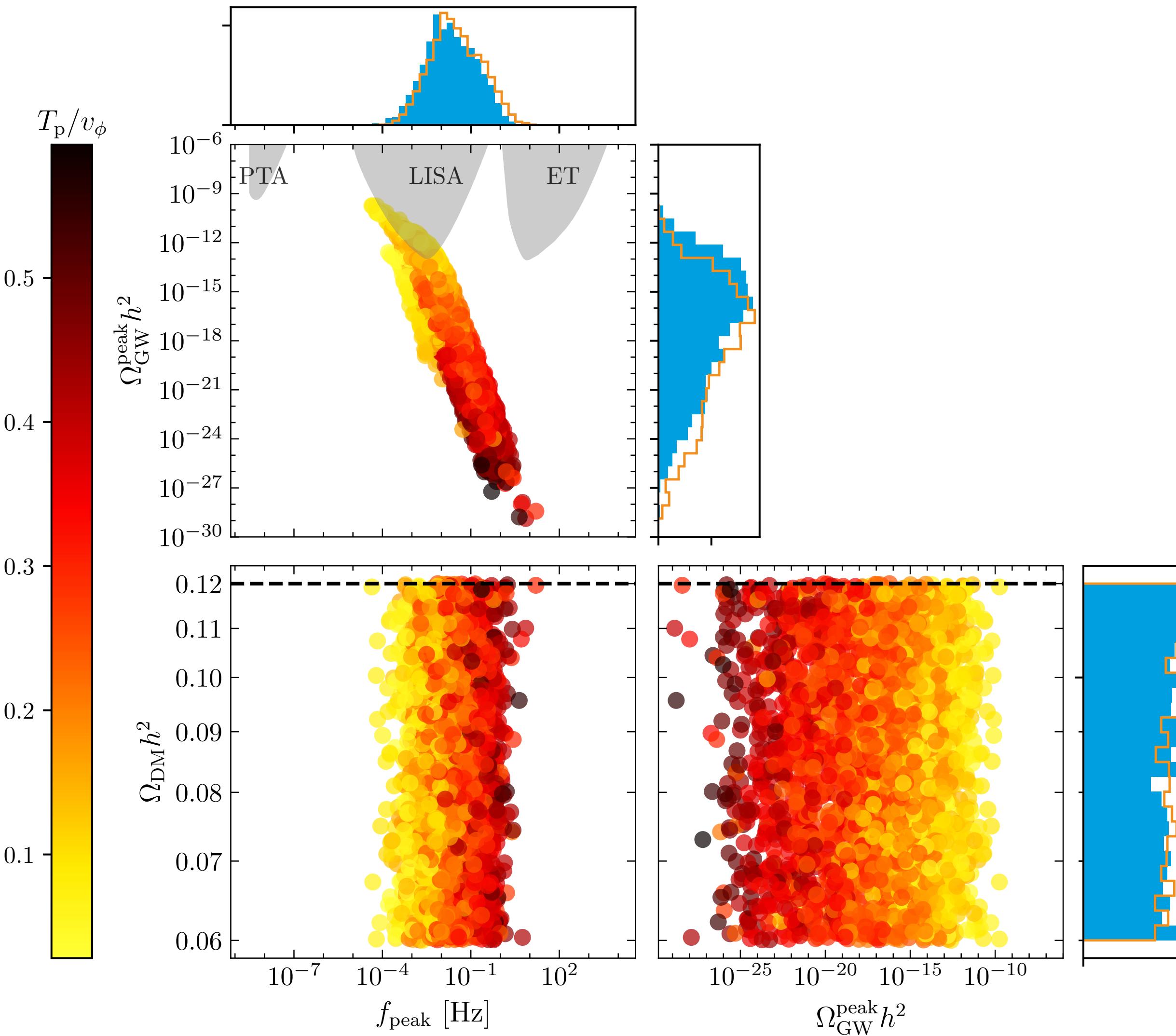
# Effect of Yukawa coupling on effective potential



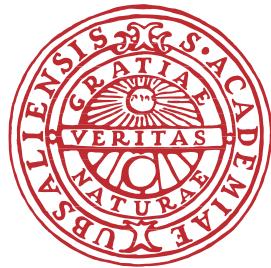
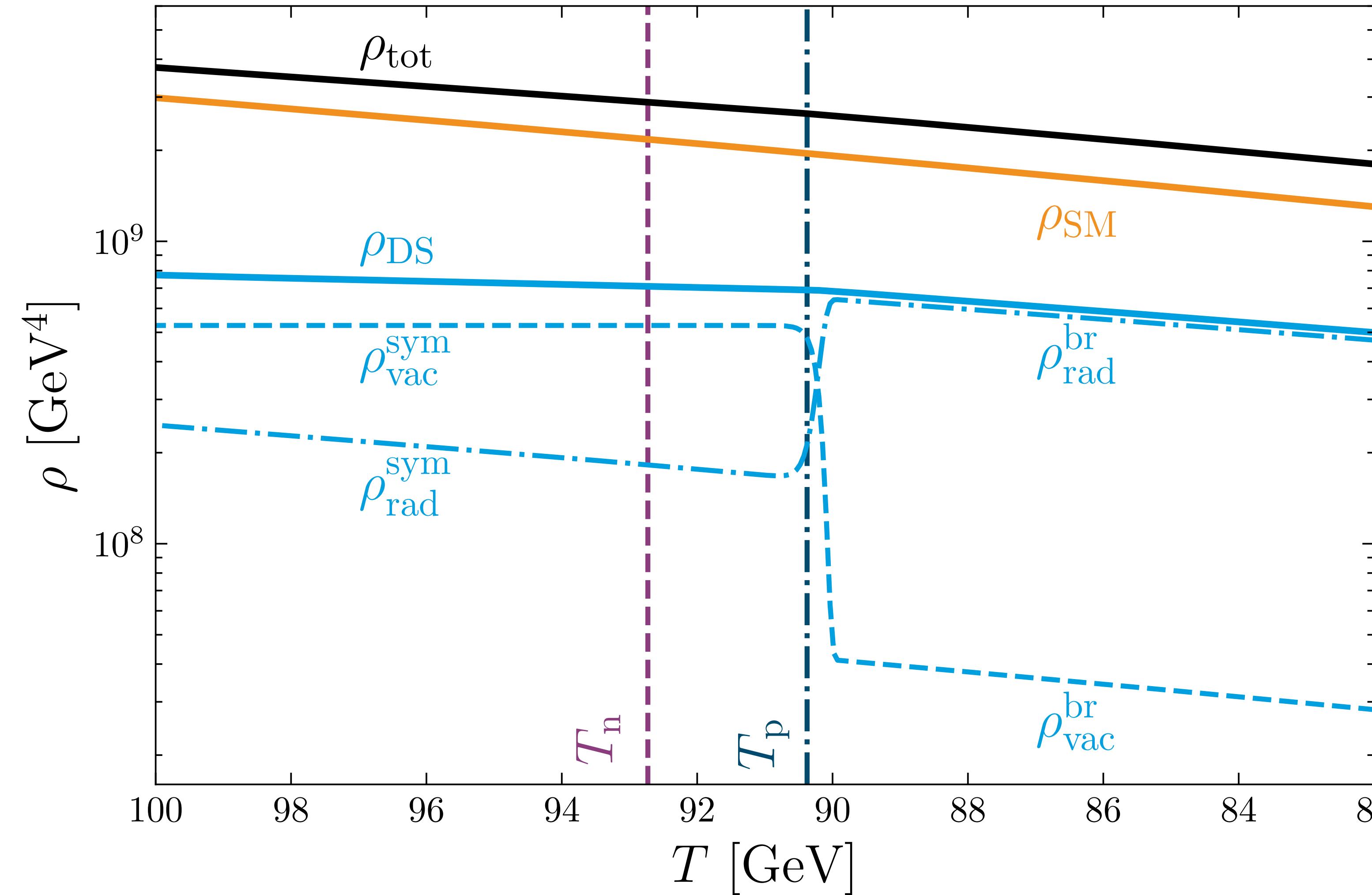
# Grid scans over couplings



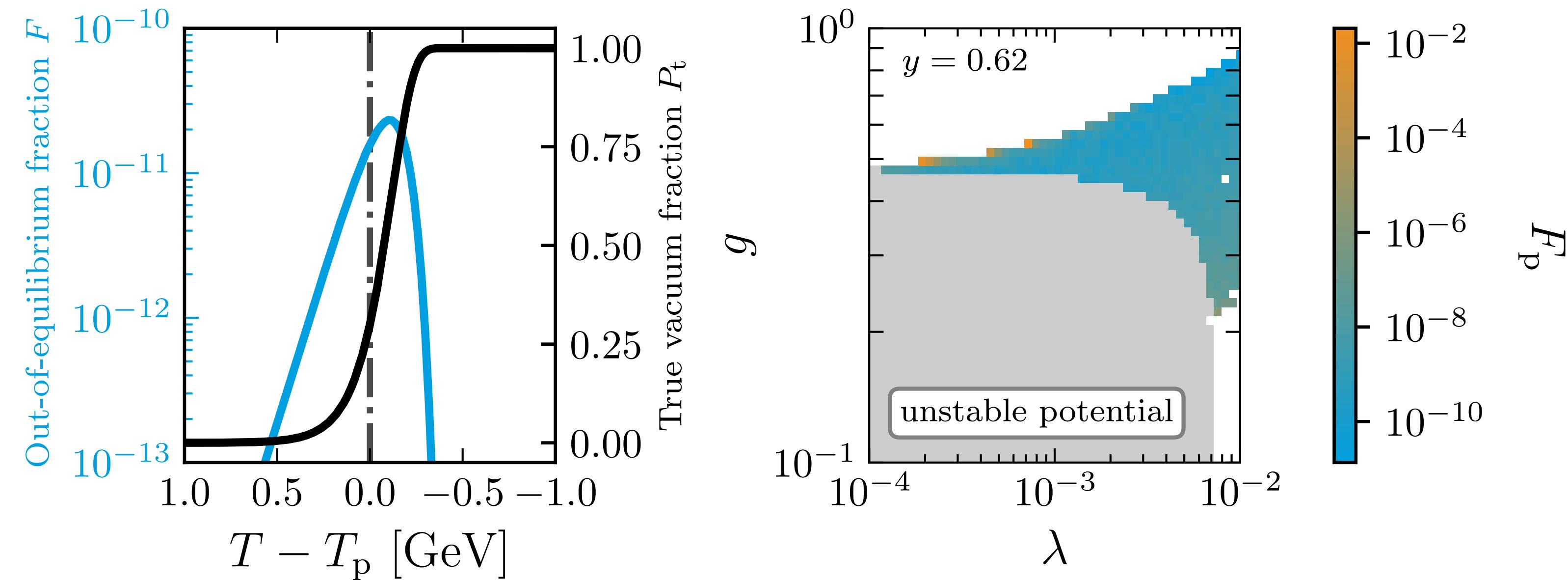
# Comparison with hot dark sector phase transition



# Evolution of energy densities



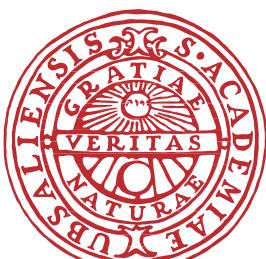
# Out-of-equilibrium fraction of the dark sector



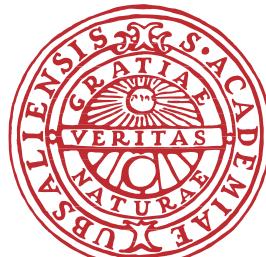
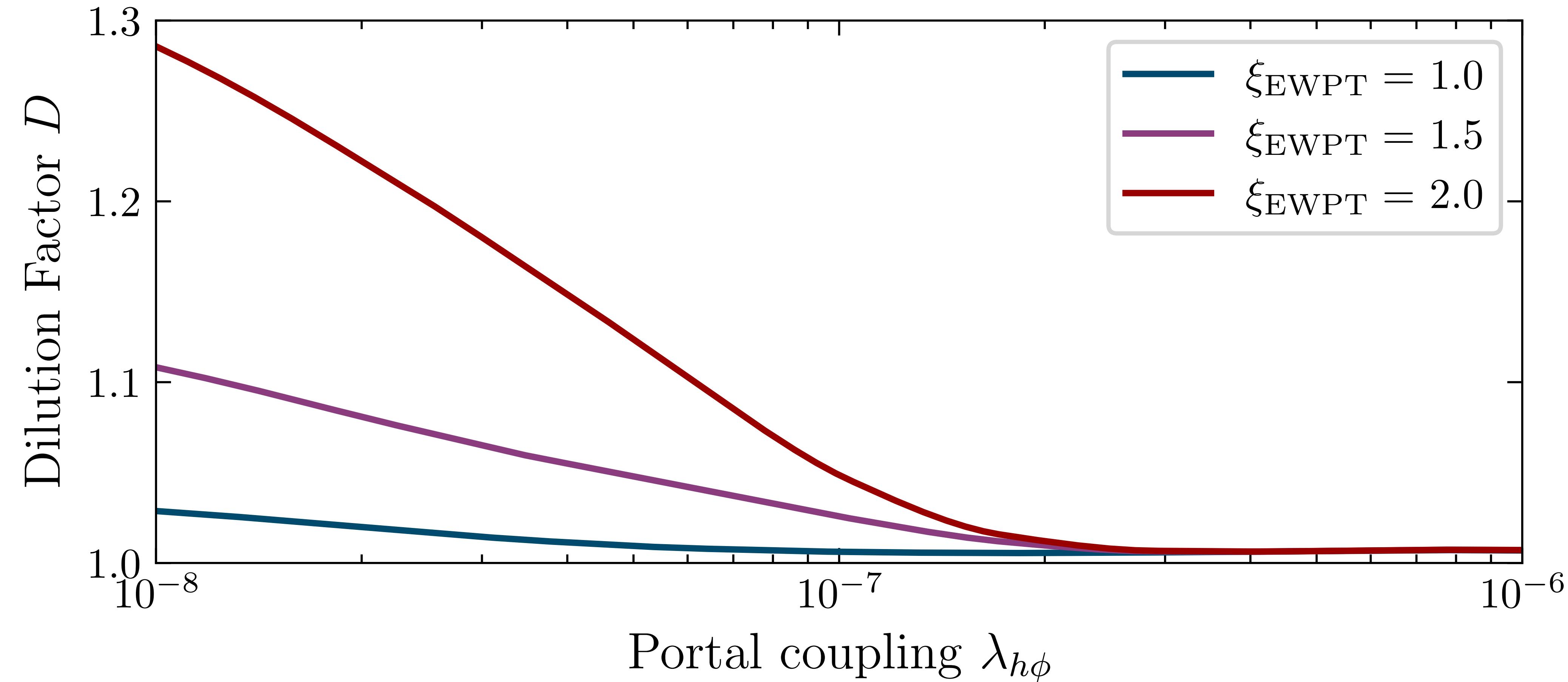
$$F(t) \equiv P(t - \tau) - P(t) > 0$$

$$\begin{aligned} F(t) &\approx \exp(-0.34e^{\beta(t-t_p-\tau)}) - \exp(-0.34e^{\beta(t-t_p)}) \\ &\approx \beta\tau e^{\beta(t-t_p)} \exp(-0.34e^{\beta(t-t_p)}) \leq 0.37\beta\tau. \end{aligned} \tag{4.6}$$

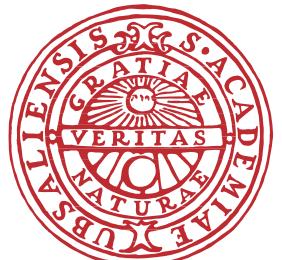
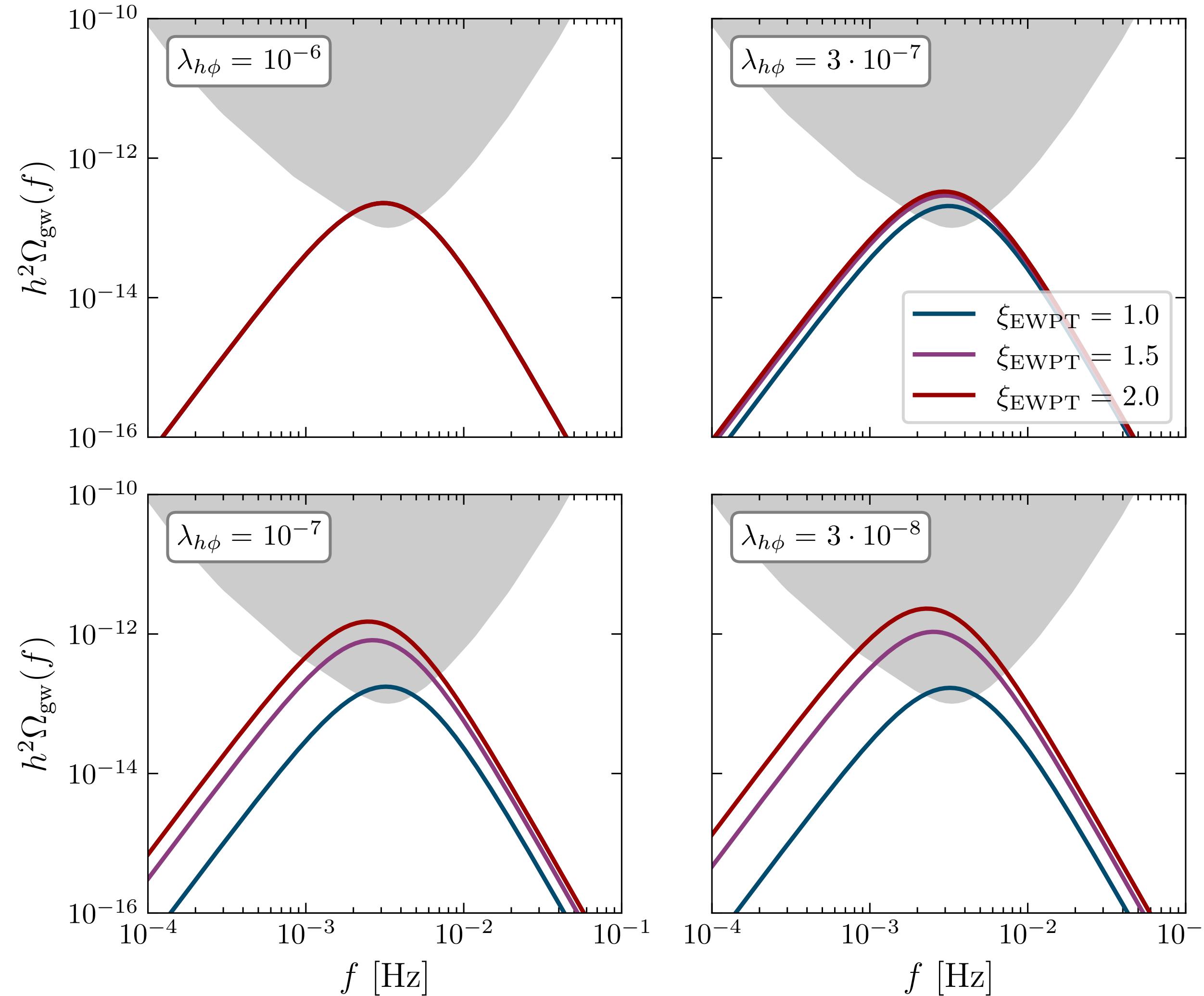
Here, the last term follows by inserting the time at which  $F(t)$  peaks, which is found to be  $t \approx t_p - 1.08/\beta$ . Alternatively, one can interpret  $F$  as the volume fraction of a shell around the bubbles with the width of the mean free path of the particles that just entered the bubbles.



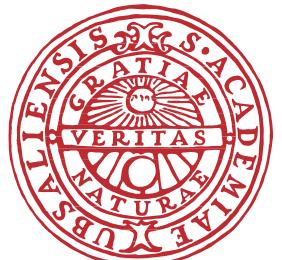
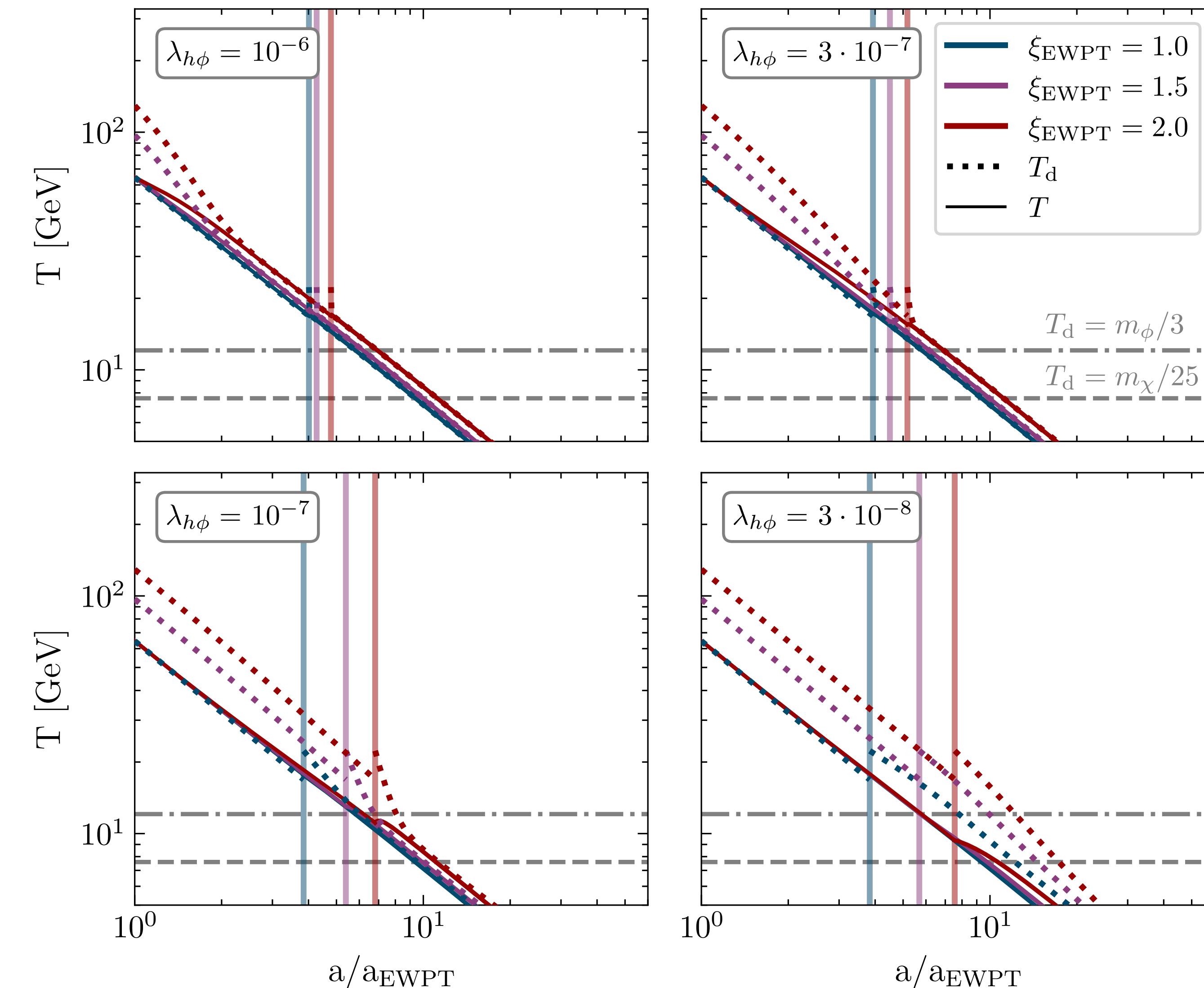
# Dilution effect



# Effect of $\lambda_{h\phi}$ and $\xi$



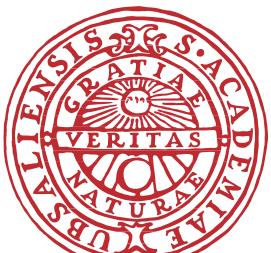
# Temperature evolution in the dark sector



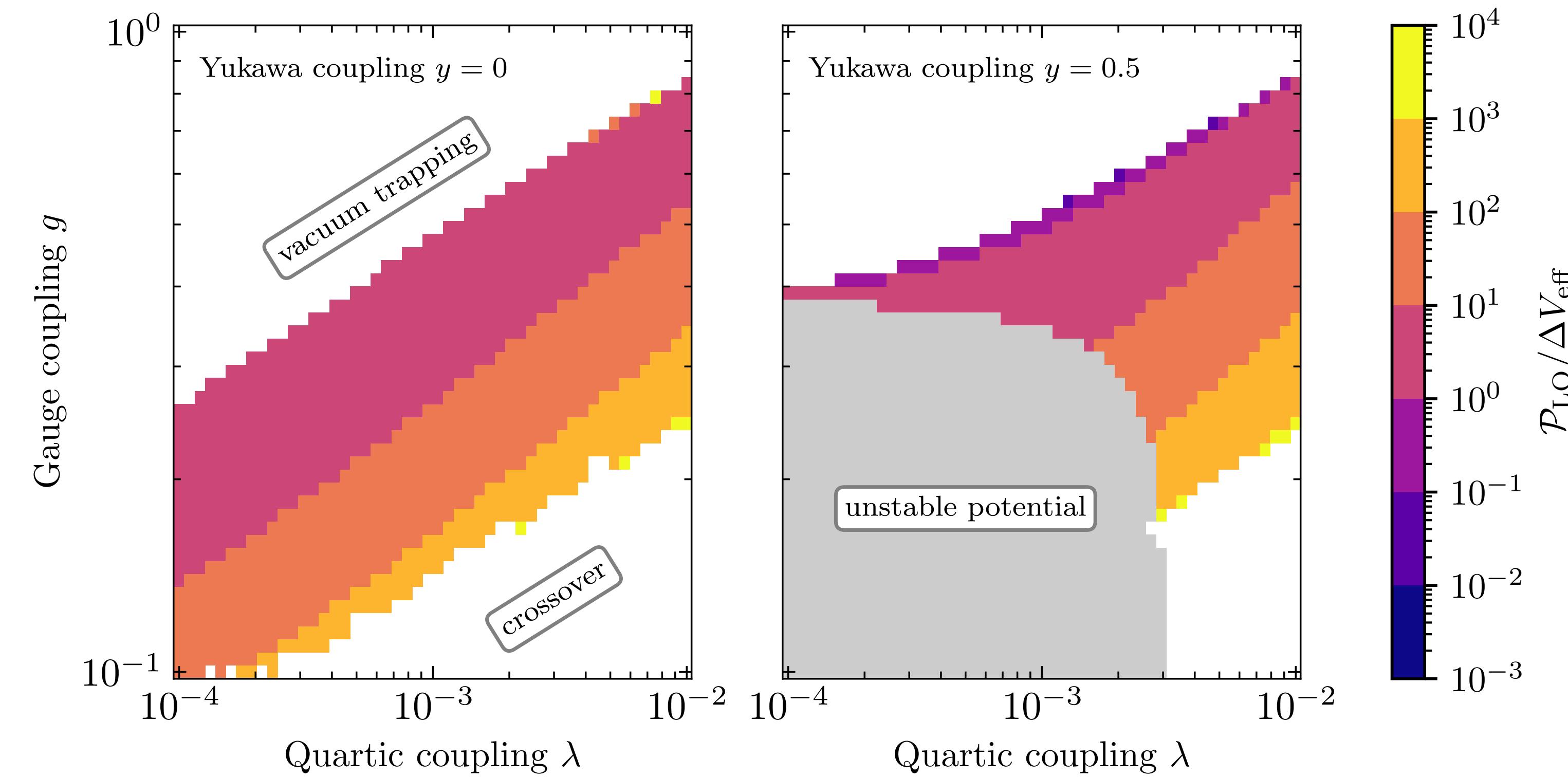
# Detection probabilities

	Fraction of parameter points observable by LISA $\xi_{\text{EWPT}} = 1, \lambda_{h\phi} = 10^{-6}$		$\xi_{\text{EWPT}} = 2, \lambda_{h\phi} = 10^{-7}$
Full sample	0.1%		0.5%
First-order PT	0.8%		3%
First-order PT + relic density	3%		8%
Strong supercooling	10%		21%
Strong supercooling + relic density	35%		69%

**Table 2.** Fraction of parameter points that predict an observable GW signal for LISA after imposing various selection requirements on the sample of points drawn from the parameter ranges discussed in section 2.5.



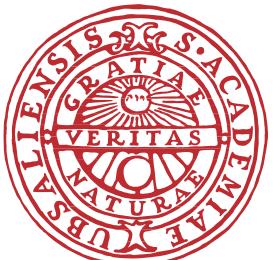
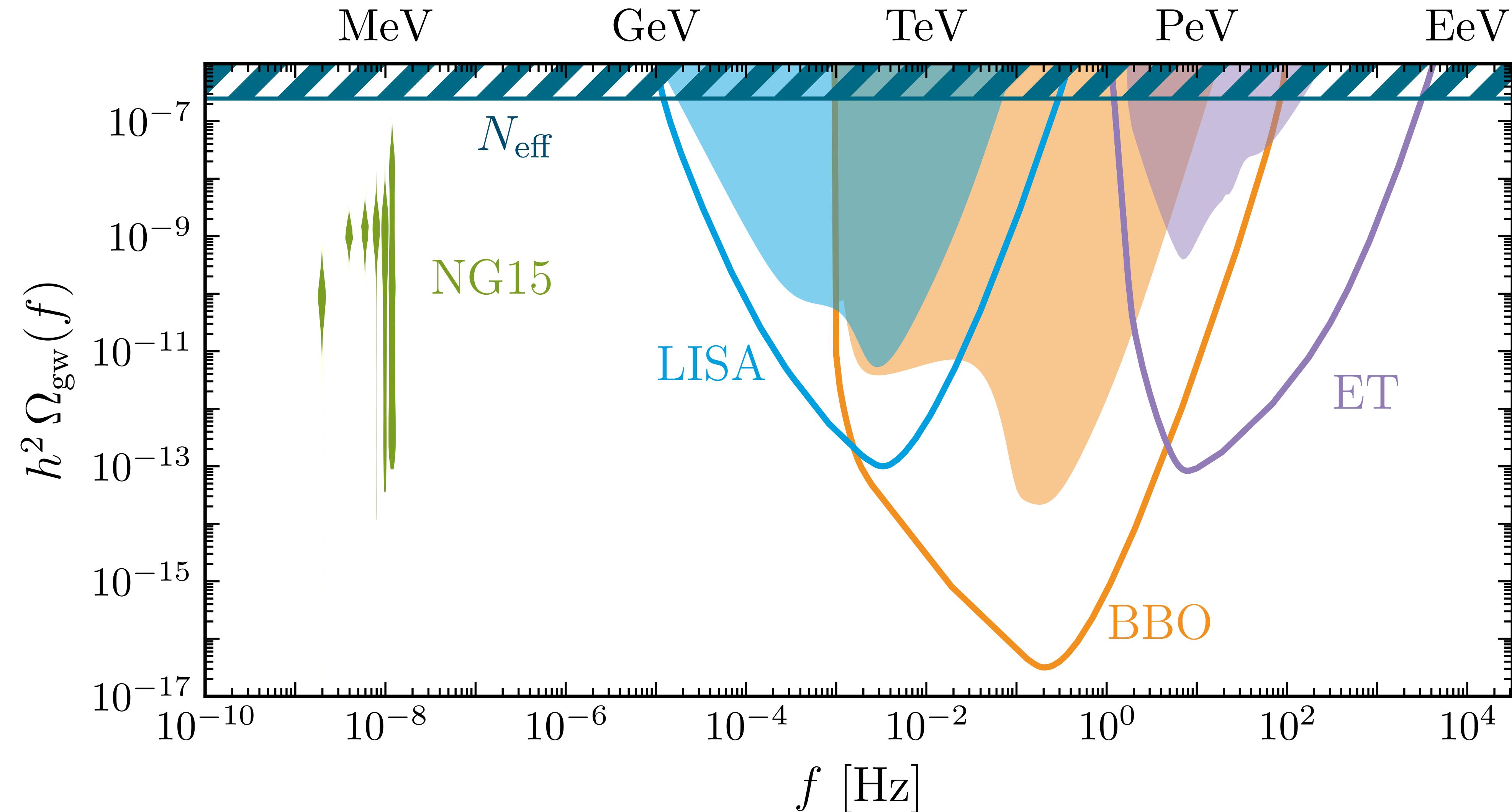
# Bödeker-Moore criterion



Bödeker-Moore criterion: 
$$\begin{cases} \Delta V_{\text{eff}} > \mathcal{P}_{\text{LO}} & \text{Relativistic bubble walls} \\ \Delta V_{\text{eff}} < \mathcal{P}_{\text{LO}} & \text{Non-relativistic bubble walls} \end{cases}$$



# Sensitivity for cosmic GW backgrounds



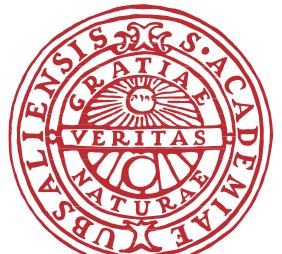
# Model details

$$\begin{aligned}\mathcal{L} = & |D_\mu \Phi|^2 - \frac{1}{4} A'_{\mu\nu} A'^{\mu\nu} + \mu^2 \Phi^* \Phi - \lambda (\Phi^* \Phi)^2 \\ & + \chi_L^\dagger i \not{D} \chi_L + \chi_R^\dagger i \not{D} \chi_R - y \Phi \chi_L^\dagger \chi_R - y \Phi^* \chi_R^\dagger \chi_L\end{aligned}$$

The tree-level scalar potential of our model has a minimum at  $v_\phi = \pm \sqrt{\mu^2/\lambda}$ . One can hence expand the complex field as  $\Phi = (v_\phi + \phi + i\varphi)/\sqrt{2}$ , where  $\phi$  and  $\varphi$  are real scalar fields. In addition, the chiral fermions  $\chi_L$  and  $\chi_R$  can be written as a Dirac fermion  $\chi$ . The Lagragian in eq. (2.1) can thus be re-written as

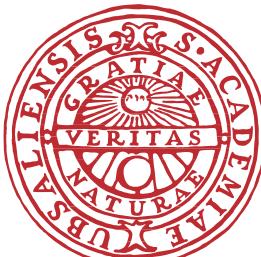
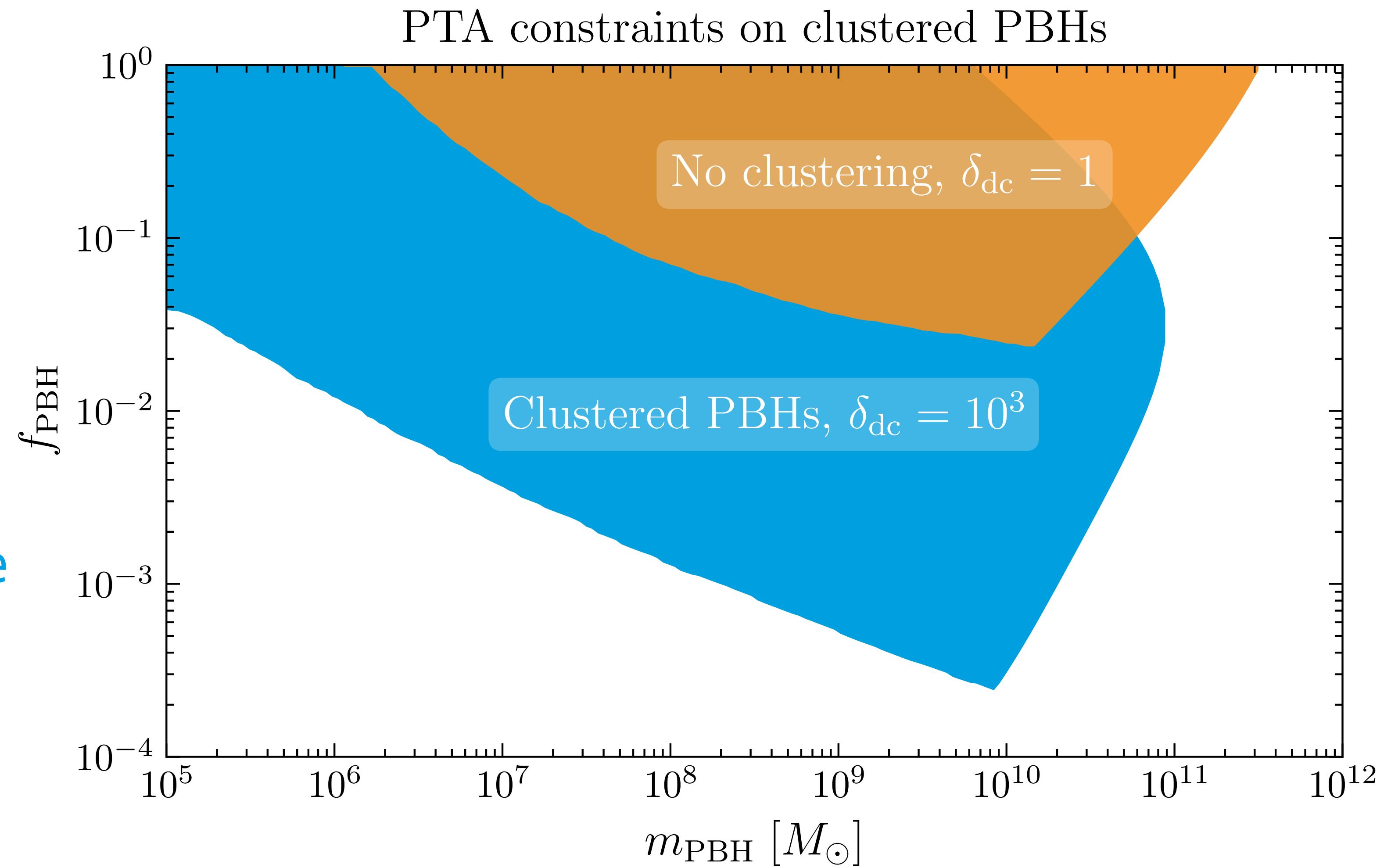
$$\begin{aligned}\mathcal{L} = & \frac{1}{2} \partial_\mu \phi \partial^\mu \phi + \frac{1}{2} \partial_\mu \varphi \partial^\mu \varphi - \frac{1}{4} A'_{\mu\nu} A'^{\mu\nu} - \frac{1}{2} m_\phi^2 \phi^2 + \frac{1}{2} m_{A'}^2 A'^2 \\ & - g A'_\mu [\varphi \partial^\mu \phi - \phi \partial^\mu \varphi - v_\phi \partial^\mu \varphi] + \frac{g^2}{2} \phi^2 A'^2_\mu + \frac{g^2}{2} \varphi^2 A'^2_\mu + g^2 v_\phi \phi A'^2_\mu \\ & - \lambda v_\phi \phi^3 - \lambda v_\phi \varphi^2 \phi - \frac{\lambda}{4} \phi^2 \varphi^2 - \frac{\lambda}{4} \phi^4 - \frac{\lambda}{4} \varphi^4 \\ & + i \bar{\chi} \not{D} \chi - m_\chi \bar{\chi} \chi + \frac{g}{2} \bar{\chi} A' \gamma^5 \chi - \frac{y}{\sqrt{2}} \phi \bar{\chi} \chi + i \frac{y}{\sqrt{2}} \varphi \bar{\chi} \gamma^5 \chi,\end{aligned}$$

$$m_\phi^2 = -\mu^2 + 3\lambda v_\phi^2 = 2\lambda v_\phi^2, \quad m_\varphi^2 = 0, \quad m_{A'}^2 = g^2 v_\phi^2, \quad m_\chi^2 = \frac{y^2}{2} v_\phi^2.$$

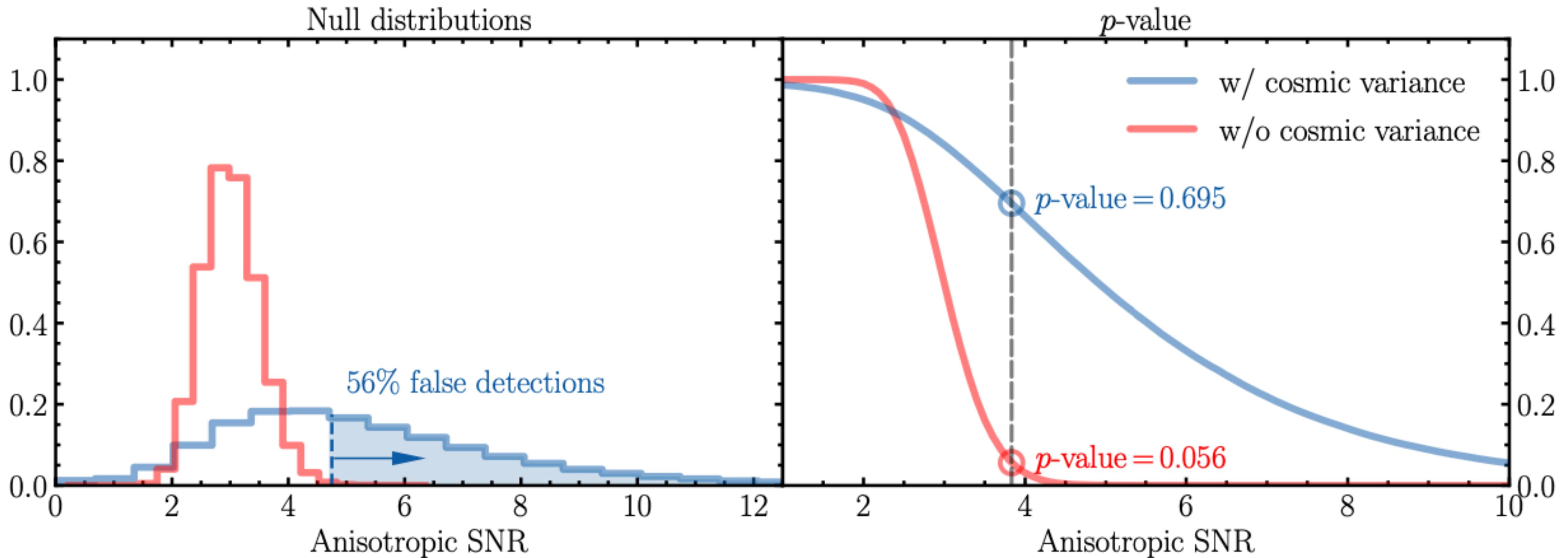


# In any case: Novel PBH bounds

In the shaded regions, the GW signal exceeds the measured PTA signal.



# The impact of cosmic variance on PTA anisotropy searches



2408.07741, Konstandin, Lemke, Mitridate, Perboni



# Gravitational waves from decaying sources in strong PTs

$$\Omega_{\text{GW}}(k) = 3 \mathcal{T}_{\text{GW}} \tilde{\Omega}_{\text{GW}} (H_*/\beta)^2 K_{\text{int}}^2 R_* \beta S(kR_*) , \quad (5.1)$$

where  $S(k)$  denotes the shape function of the spectrum that is normalized to  $\int d \ln k S(k) = 1$ , and  $K_{\text{int}}^2$  is the integrated kinetic energy fraction  $K^2$  over  $\tilde{t} \equiv t\beta$ , such that it reduces to  $K^2 \tau_{\text{sw}} \beta$  when  $K$  is constant, being  $\tau_{\text{sw}}$  the GW source duration. Therefore, Eq. (5.1) is a generalization of the parameterization used in the stationary UETC assumption previously tested with numerical simulations [40, 50, 52] and usually assumed for sound-wave sourcing of GWs [22, 51, 54, 59, 91, 92, 99] that predicts a linear growth with the GW source duration when  $K$  does not decay with time.

The most robust results (i.e., an almost independent value of  $\tilde{\Omega}_{\text{GW}}$  with the PT parameters) are obtained when the typical bubble separation  $R_*$ , which determines the length scale of fluid perturbations, is given by the front of the expanding bubbles [22]

$$\beta R_* = (8\pi)^{1/3} \max(v_w, c_s) , \quad (5.2)$$

where  $1/\beta$  parameterizes the duration of the PT,  $v_w$  is the wall velocity, and  $c_s$  the speed of sound. This way, the residual dependence on the wall velocity in  $\tilde{\Omega}_{\text{GW}}$  is quite limited and we estimate from our numerical simulations values for the GW efficiency  $\tilde{\Omega}_{\text{GW}} \sim \mathcal{O}(10^{-2})$  for a range of PTs [see Fig. 7 and Eq. (4.9)],

$$10^2 \tilde{\Omega}_{\text{GW}} = \begin{cases} 1.04^{+0.81}_{-0.67}, & \text{for } \alpha = 0.0046; \\ 1.64^{+0.29}_{-0.13}, & \text{for } \alpha = 0.05; \\ 3.11^{+0.25}_{-0.19}, & \text{for } \alpha = 0.5, \end{cases} \quad (5.3)$$

$$K_{\text{int}}^2(b, \tau_{\text{sw}}) \rightarrow \mathcal{K}_0^2 \beta t_* \frac{(1 + \tau_{\text{sw}}/t_*)^{1-2b} - 1}{1 - 2b} , \quad (5.6)$$

when one uses the power-law fit for  $K(\tilde{t})$  and assumes that the GW production roughly starts at the time  $\tilde{t}_* \simeq \tilde{t}_0 \simeq 10$  (note that the actual value of  $\tilde{t}_0$  only appears as a consequence of our particular fit). It is unclear what should be the final time of GW sourcing in these cases, as the simulations seem to already be modelling the non-linear regime, so we leave  $\tilde{\tau}_{\text{sw}}$  as a free parameter. We note that this is an indication that the GW spectrum might still grow once that non-linearities develop in the fluid, such that the use of Eq. (5.5) would in general underestimate the GW production. We compare in Fig. 8 the numerical dependence of the GW amplitude with the source duration  $\tilde{\tau}_{\text{sw}}$  found in the simulations to the one obtained using Eq. (5.6), extending the analytical fit beyond the time when the simulations end.

As a final remark on the integrated GW amplitude, we note that so far Universe expansion has been ignored, which is not justified for long source durations. Taking into account that the fluid equations are conformal invariant after the PT if the fluid is radiation-dominated, we can apply the results from our fluid simulations in Minkowski space-time to an expanding Universe, as long as the PT duration is short ( $\beta/H_* \gg 1$ ) even if the GW source duration is not short (see discussion in Sec. 2.6). Then, as a proxy to estimate the effect of the Universe expansion, we can use the following value for  $K_{\text{int}}^2$  [see Eq. (2.23)]

$$K_{\text{int}}^2 \rightarrow \mathcal{K}_0^2 \Upsilon_b(\tau_{\text{sw}}) (\beta/H_*) , \quad (5.7)$$

which generalizes the suppression factor  $\Upsilon = H_* \tau_{\text{sw}} / (1 + H_* \tau_{\text{sw}})$  when the source does not decay [89, 91] to any decay rate  $b$  using Eq. (2.24) for the presented power-law decay fit of  $K(\tilde{t})$ . We also compare in Fig. 8 the expected evolution of the GW amplitude with the source

2409.03651, Caprini, Jinno, Konstandin, Roper Pol, Rubira, Stomberg

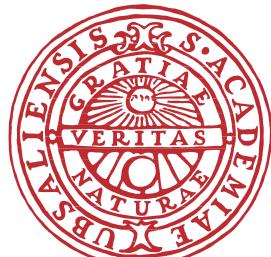
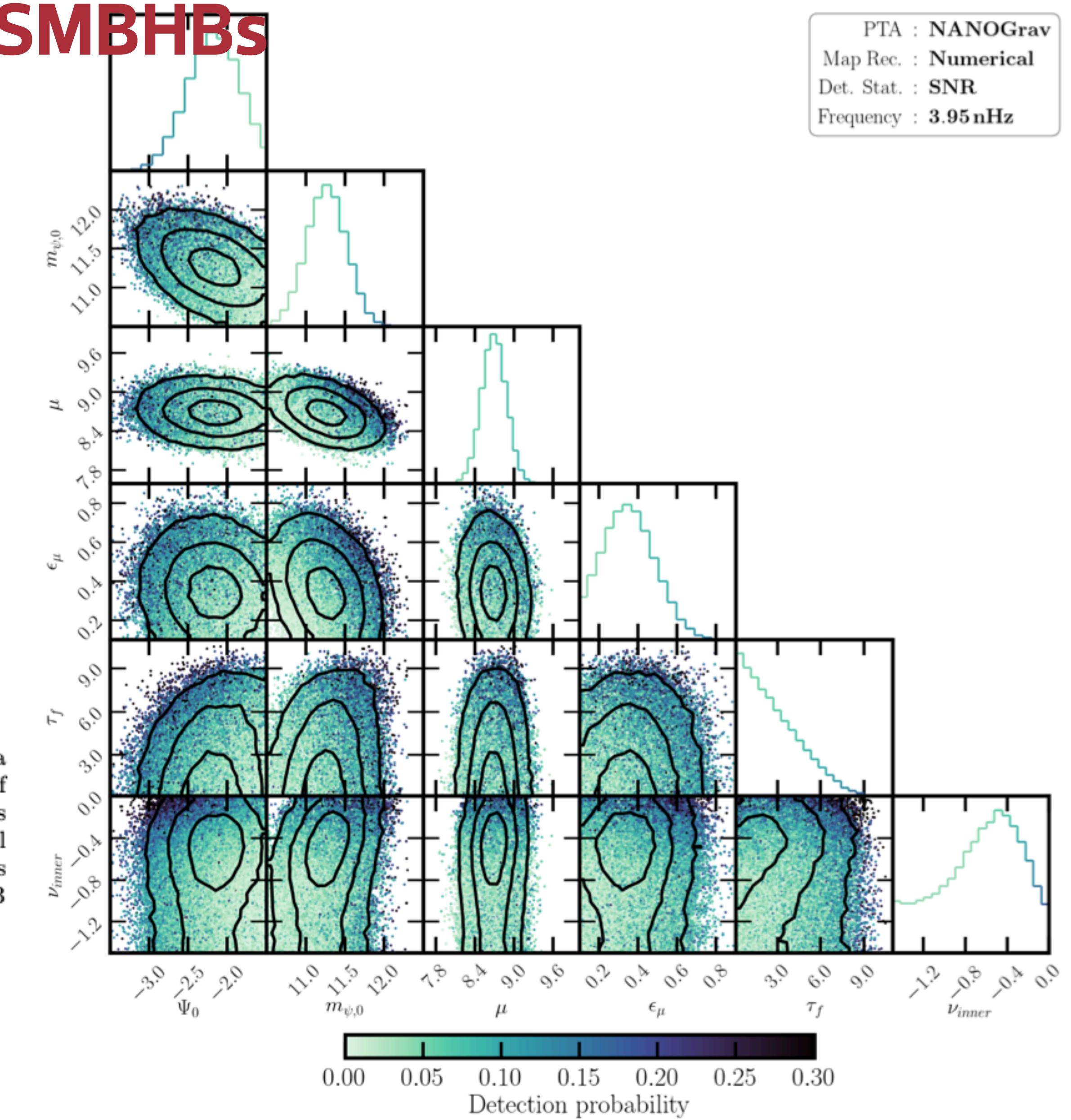


# Detecting GW anisotropies from SMBHBs

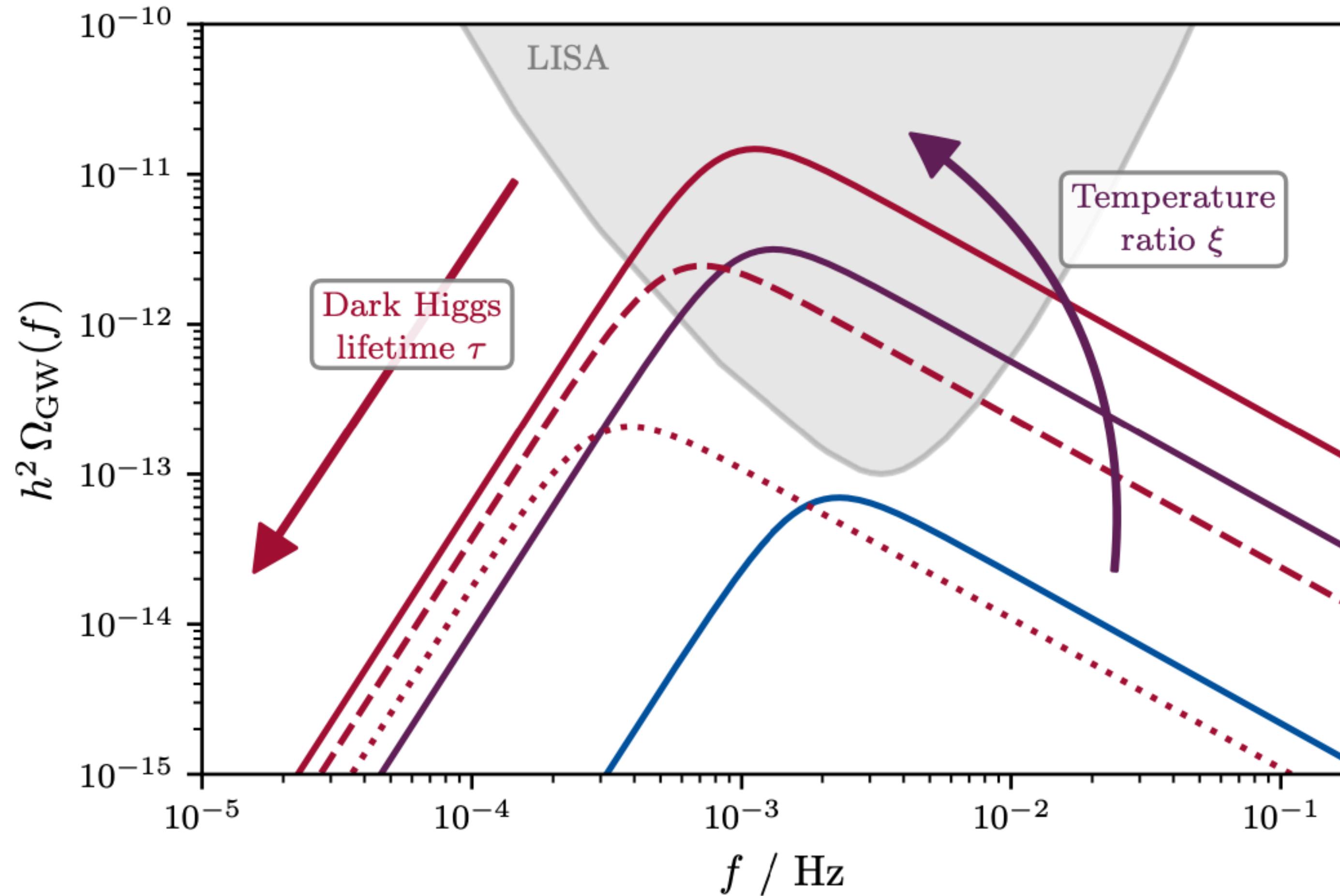
PTA : NANOGrav  
 Map Rec. : Numerical  
 Det. Stat. : SNR  
 Frequency : 3.95 nHz

We find that a PTA with the noise characteristics of the NANOGrav 15-year data set had only a 2% – 11% probability of detecting SMBHB-generated anisotropies, depending on the properties of the SMBHB population. However, we estimate that for the IPTA DR3 data set these probabilities will increase to 4% – 28%, putting more pressure on the SMBHB interpretation in case of a null detection. We also identify SMBHB populations that are more likely to produce detectable levels of anisotropies. This information could be used together with the spectral properties of the GWB to characterize the SMBHB population.

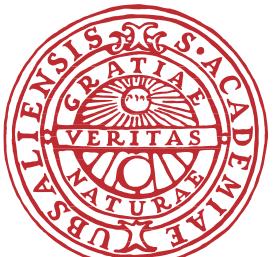
2407.08705, Lemke, Mitridate, Gersbach



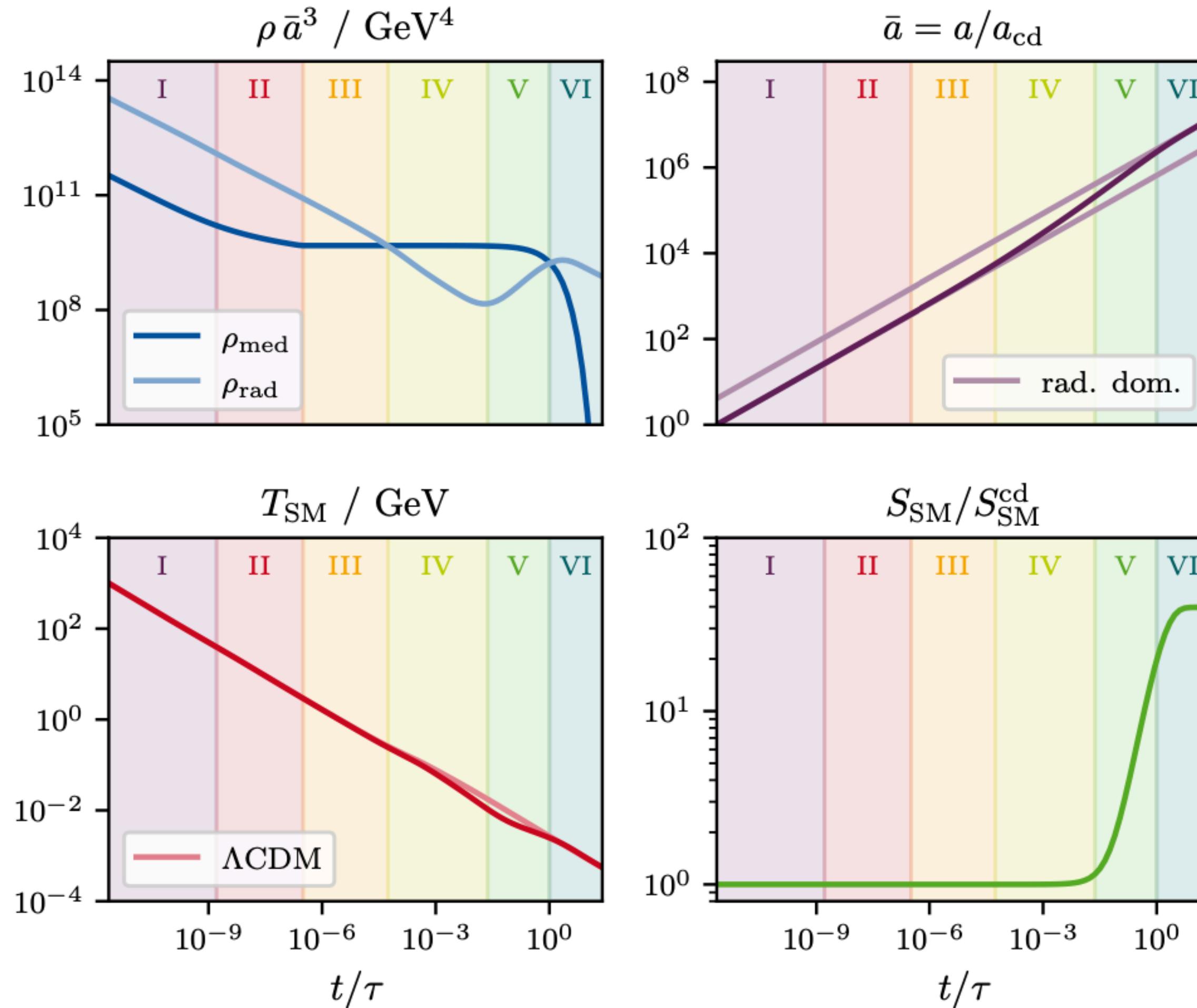
# Turn up the volume



2109.06208, Ertas, Kahlöfer, CT

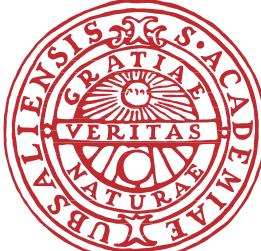


# Turn up the volume

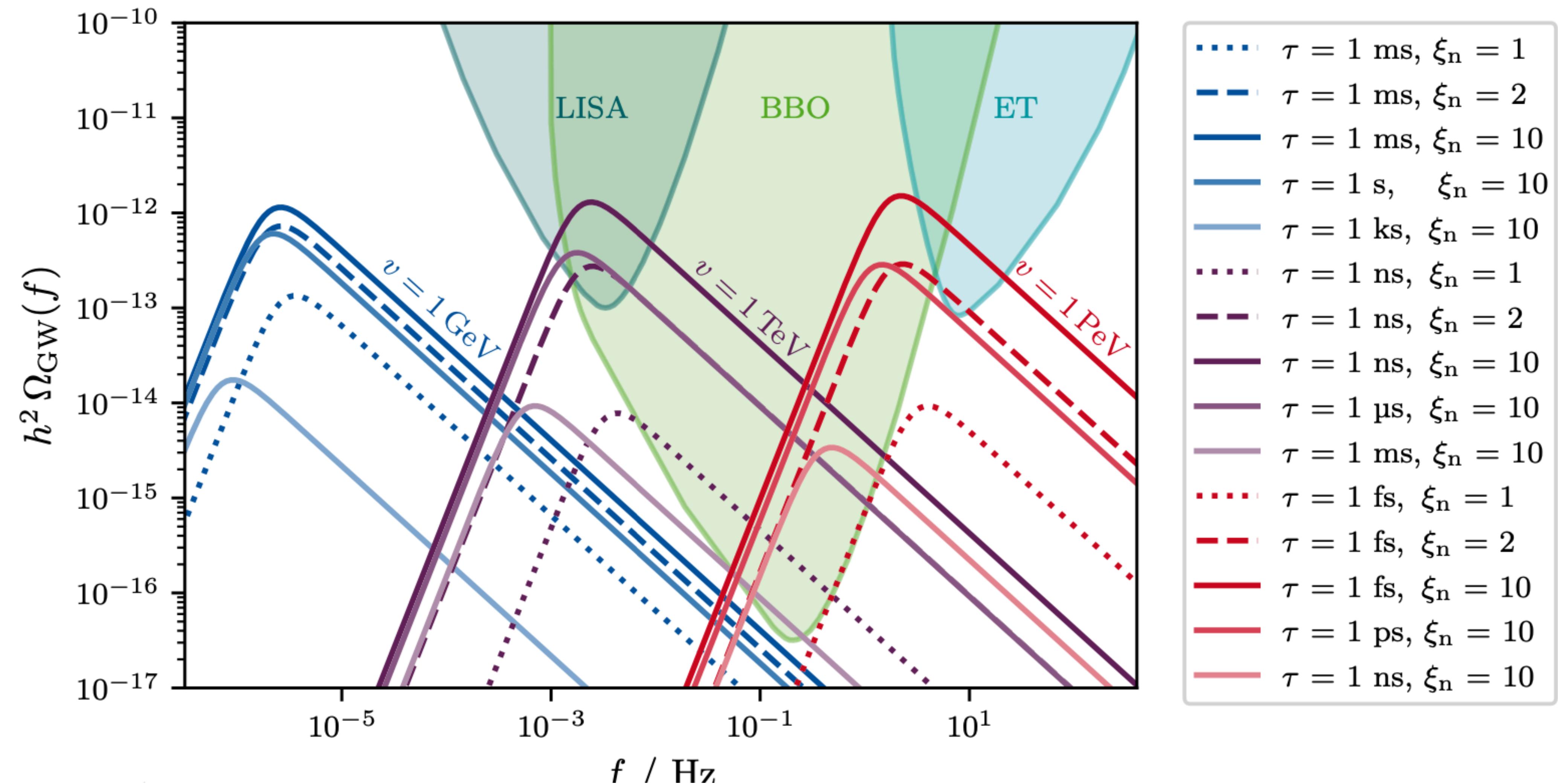


**Figure 5.** Time evolution of the comoving energy densities  $\rho \bar{a}^3$  of the mediator species and the SM radiation (top-left), the normalized scale factor  $\bar{a}$  (top-right), the temperature  $T_{\text{SM}}$  of the SM bath (bottom-left), as well as its entropy  $S_{\text{SM}}/S_{\text{SM}}^{\text{cd}}$  (bottom-right). The evolution can be divided into the following phases: relativistic mediator (I), cannibalism (II), non-relativistic mediator (III), early matter domination (IV), entropy injection (V), and decay (VI). See text for details.

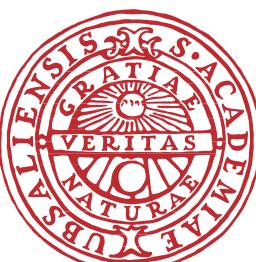
2109.06208, Ertas, Kahlhöfer, CT



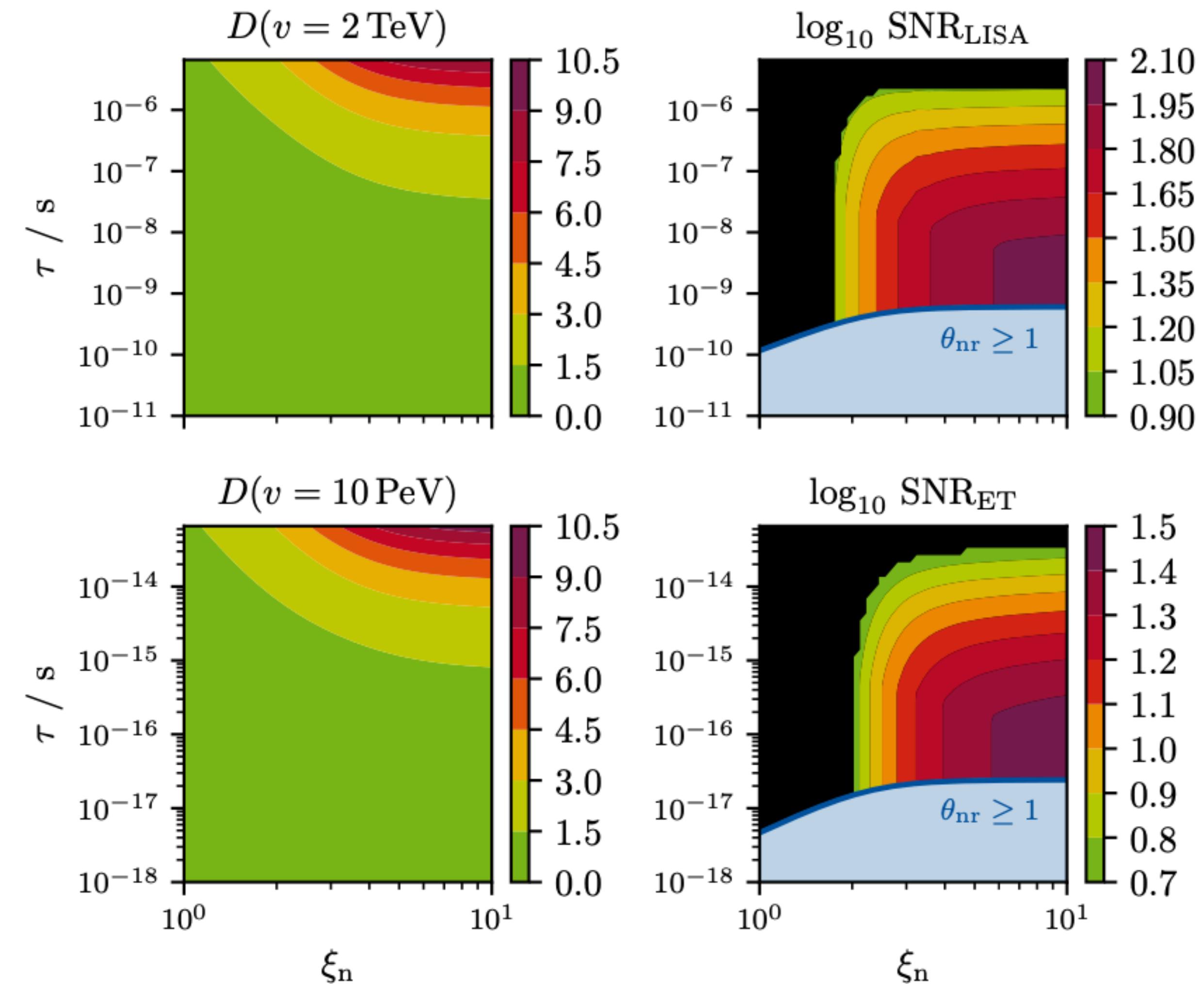
# Turn up the volume



2109.06208, Ertas, Kahlöfer, CT

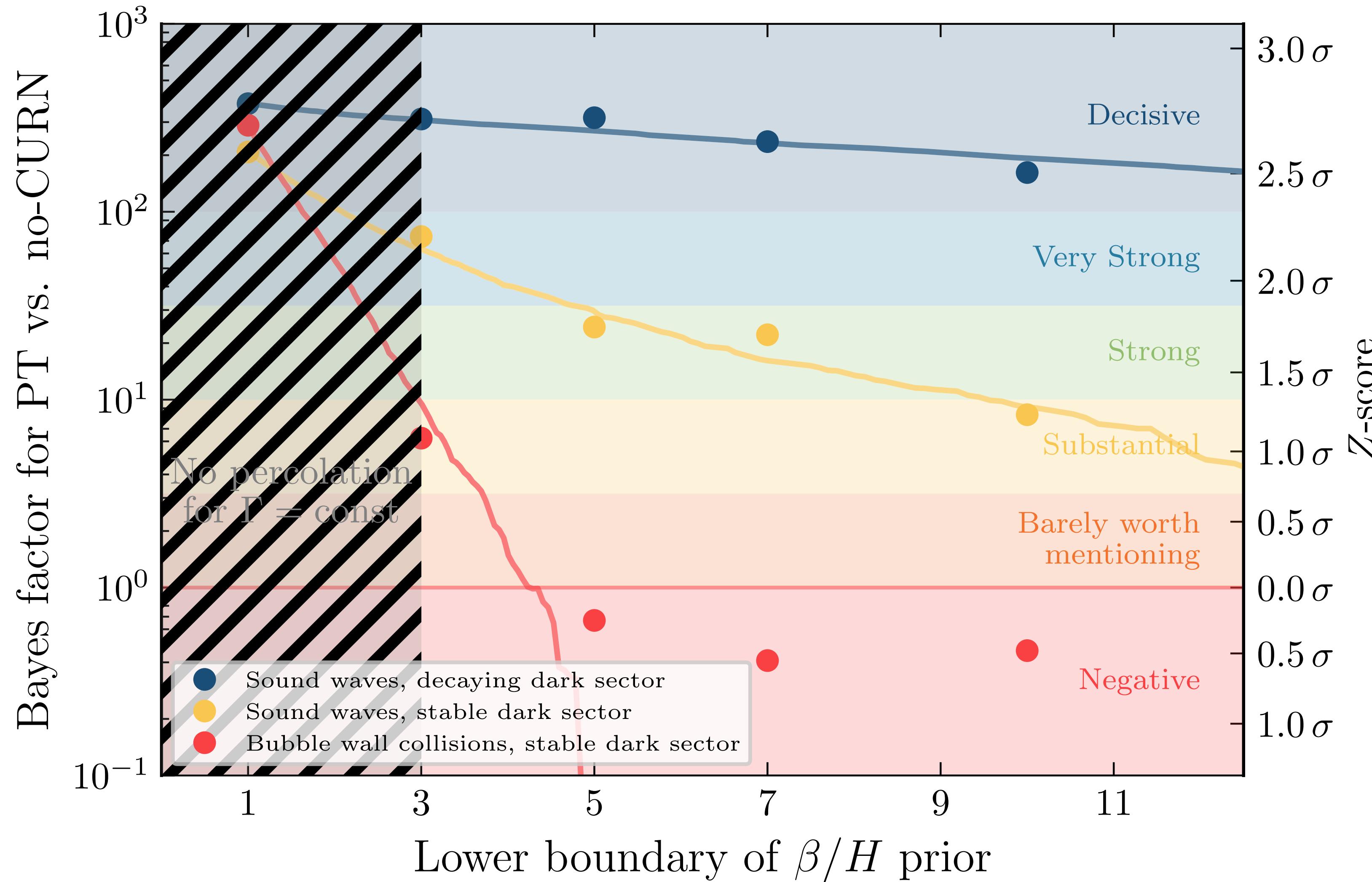


# Turn up the volume



2109.06208, Ertas, Kahlhöfer, CT

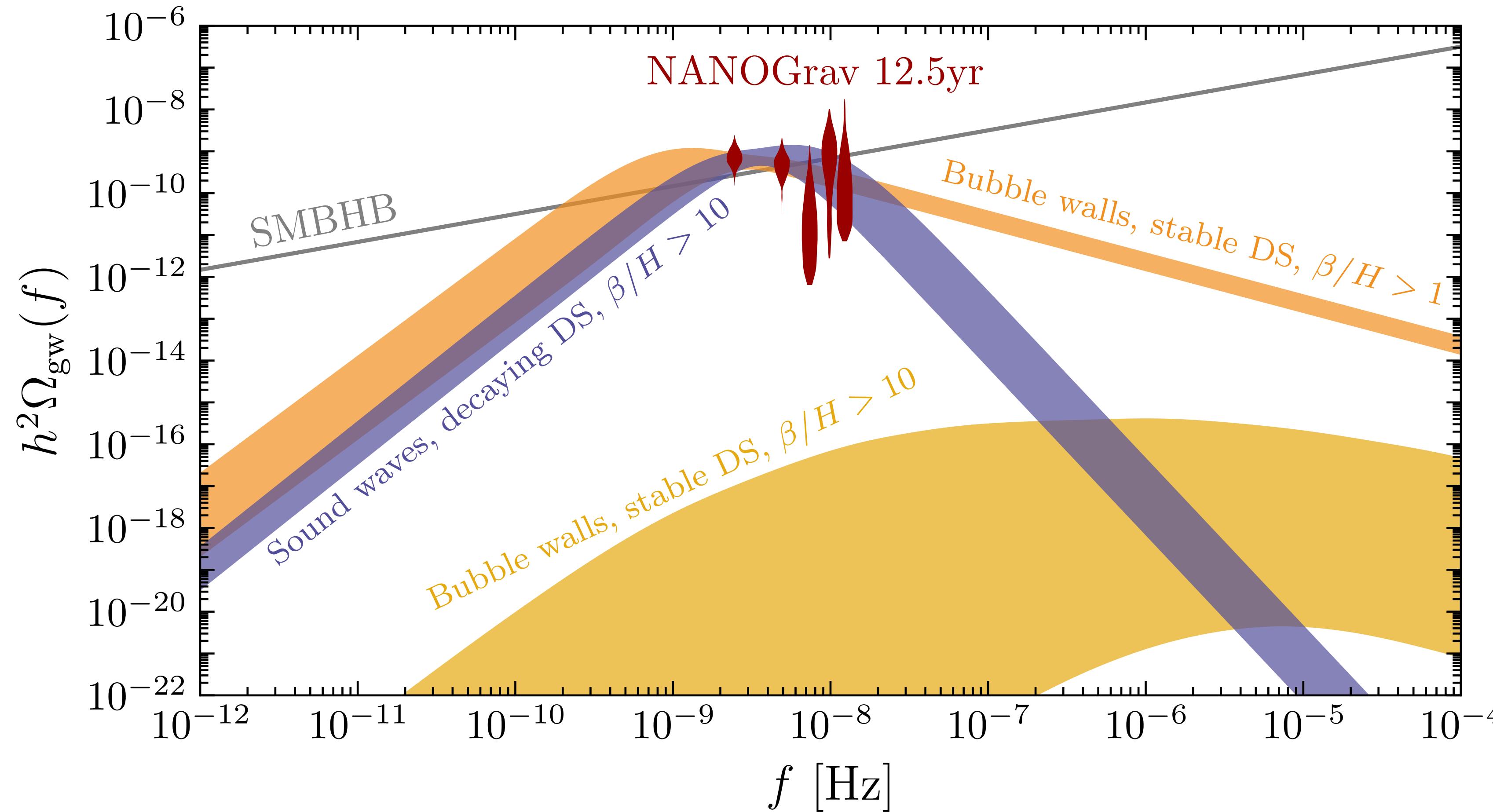
# Do PTAs observe a dark sector phase transition?



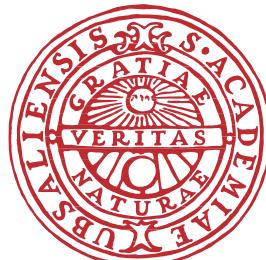
2306.09411, Bringmann, Depta, Konstandin, Schmidt-Hoberg, CT



# Do PTAs observe a dark sector phase transition?



2306.09411, Bringmann, Depta, Konstandin, Schmidt-Hoberg, CT



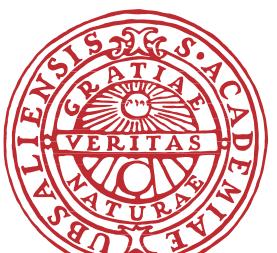
# Number of effective degrees of freedom at BBN

$$\rho_\nu + \rho_{\text{extra}} \equiv N_{\text{eff}} \times \frac{7}{8} \left( \frac{4}{11} \right)^{4/3} \rho_\gamma , \quad (2.29)$$

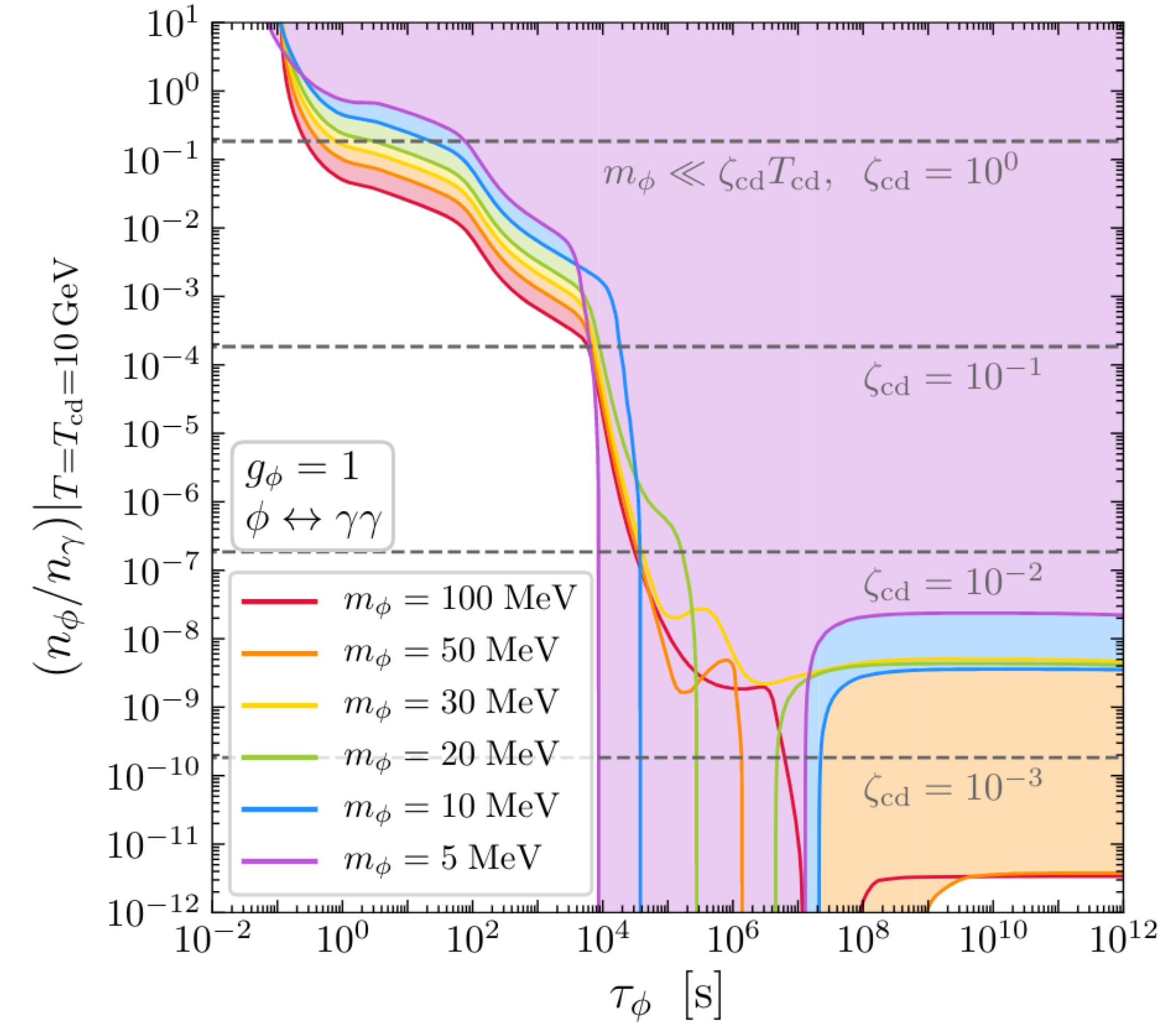
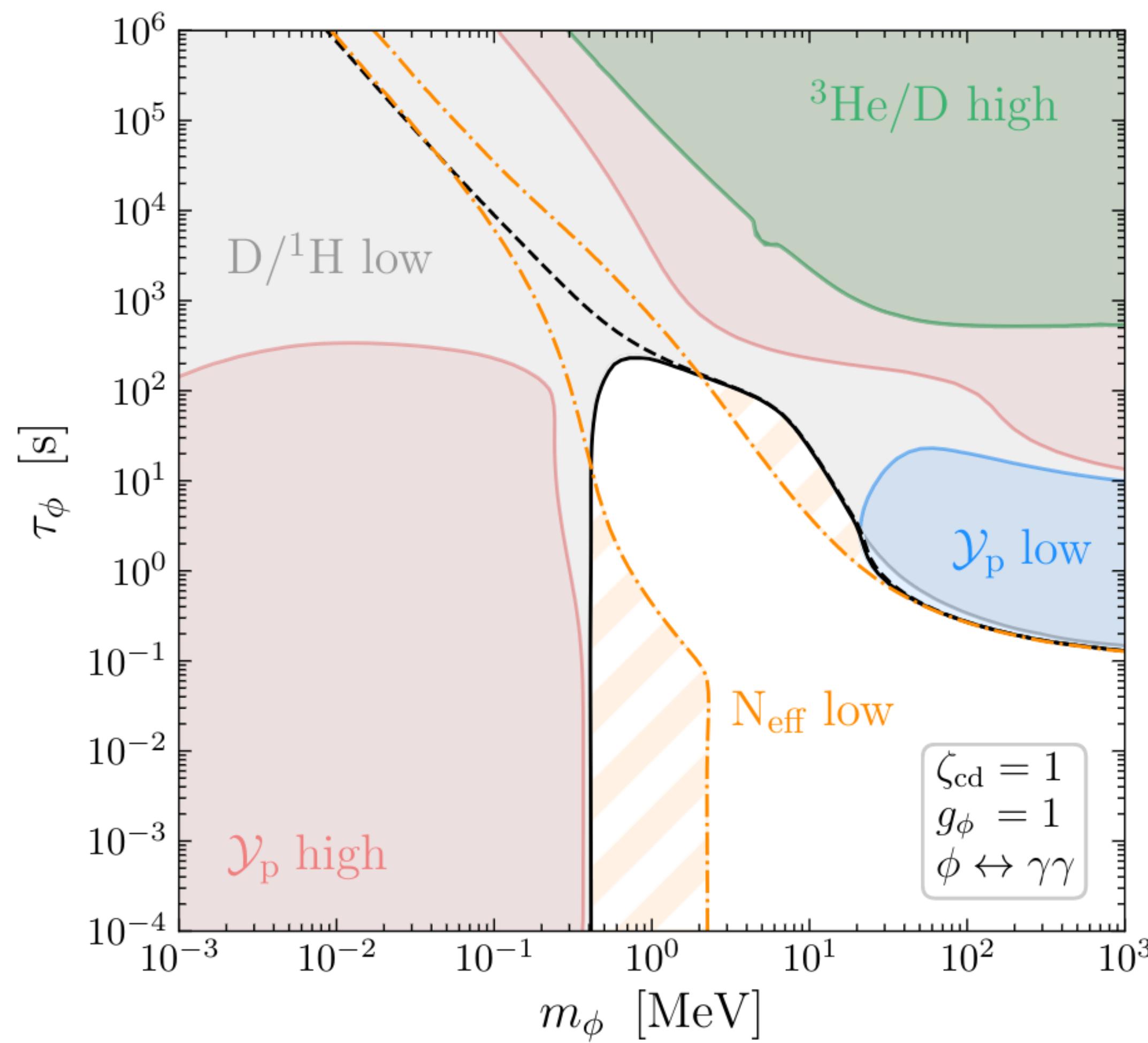
such that the extra energy can be expressed as<sup>7</sup>

$$\rho_{\text{extra}} = \Delta N_{\text{eff}} \times \frac{7}{8} \left( \frac{4}{11} \right)^{4/3} \rho_\gamma \quad \text{where} \quad \Delta N_{\text{eff}} \equiv N_{\text{eff}} - N_{\text{eff}}^{\text{SM}} . \quad (2.30)$$

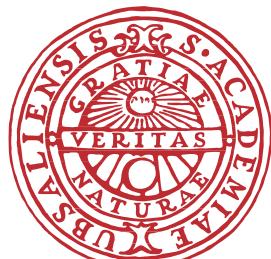
$$T_f \simeq \left( \frac{\pi^2}{45} \right)^{1/6} \left[ 1 + \frac{7}{8} \left( \frac{4}{11} \right)^{4/3} N_{\text{eff}} \right]^{1/6} \frac{1}{(G_F^2 m_{\text{Pl}})^{1/3}} .$$



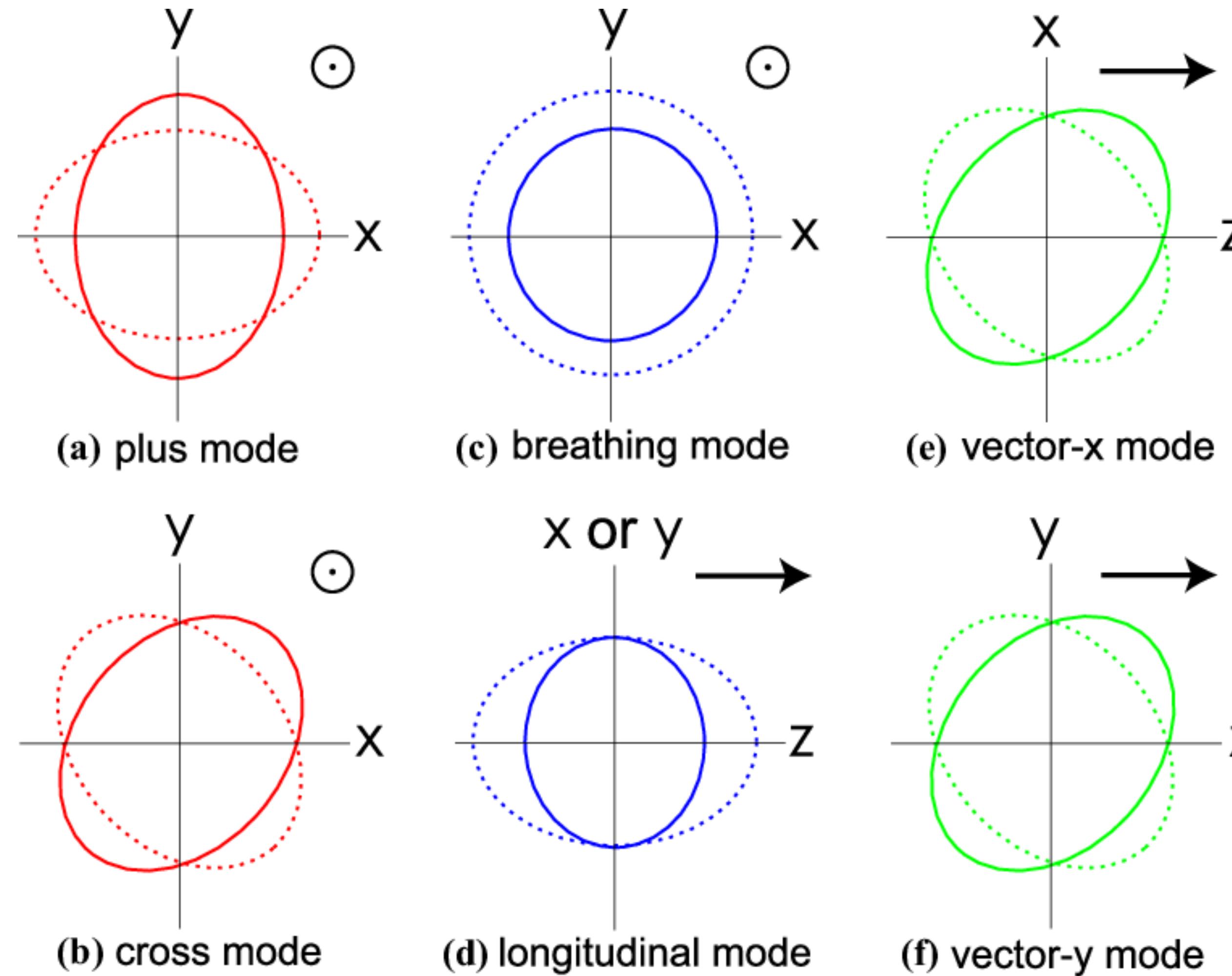
# BBN limits on MeV-scale electromagnetic scalar decays



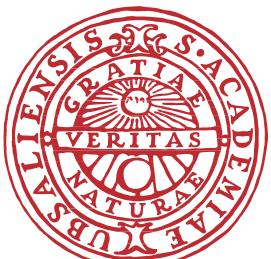
[Paul Frederik Depta]



# Polarization of a GW

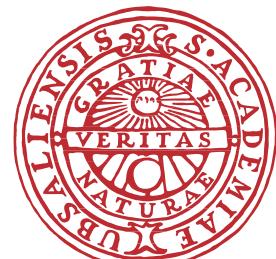


[Stephen Taylor et al., 2019]



# Cosmological perturbation theory

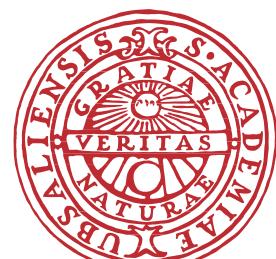
$$\Delta\Phi - 3\mathcal{H}(\Phi' - \mathcal{H}\Psi) = -2\frac{a^2\delta\rho}{m_{\text{Pl}}^2}, \quad \Delta(\Phi + \Psi) = -\frac{a^2\Delta\sigma}{m_{\text{Pl}}^2},$$
$$\Delta\Xi_i = -\frac{2a^2S_i}{m_{\text{Pl}}^2} \quad \text{and} \quad (h_{ij}^{\text{TT}})'' + 2\mathcal{H}(h_{ij}^{\text{TT}})' - \Delta h_{ij}^{\text{TT}} = \frac{2a^2\sigma_{ij}^{\text{TT}}}{m_{\text{Pl}}^2}.$$



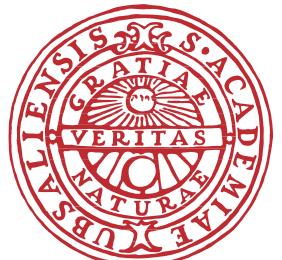
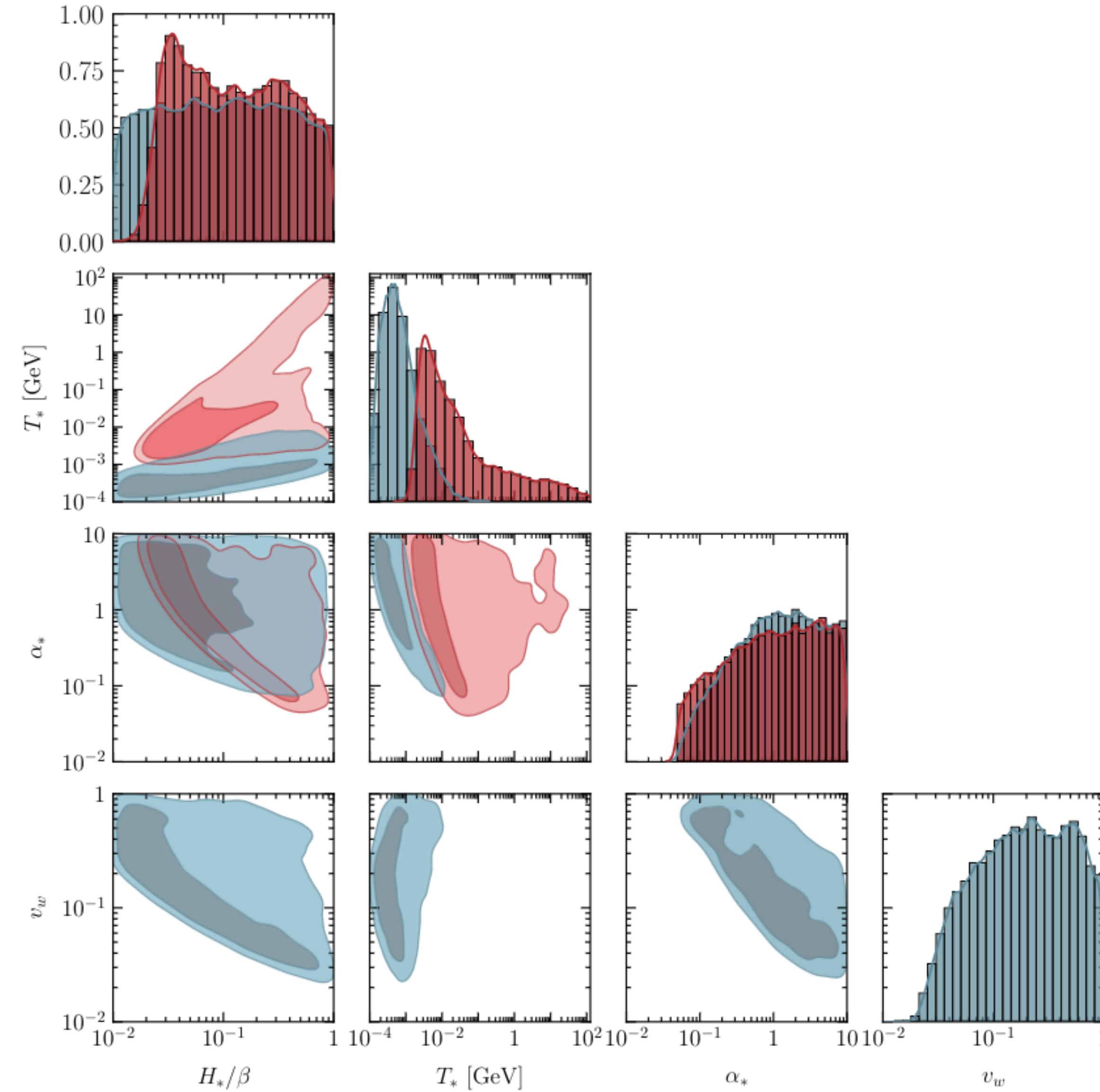
# Conversion of different GW spectra

$$h^2\Omega_{\text{gw}}(f) = \frac{4\pi^2}{3H_{100}^2} f^3 S_h(f) = \frac{2\pi^2}{3H_{100}^2} f^2 h_c^2(f)$$

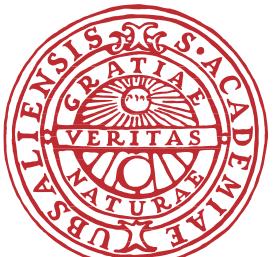
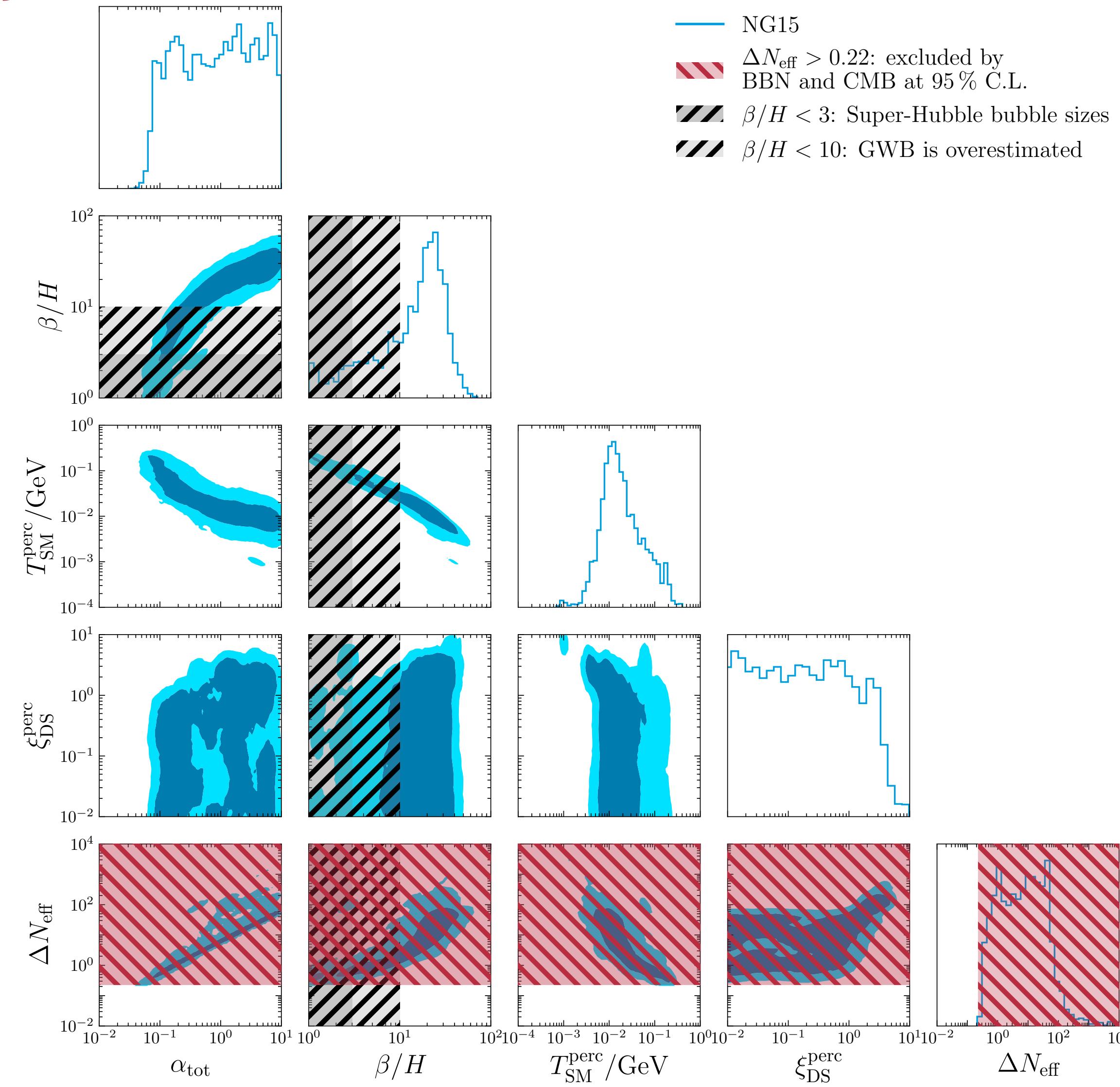
$$S_h(f) \simeq 10^{-36} \left( \frac{\text{Hz}}{f} \right)^3 \frac{h^2\Omega_{\text{gw}}(f)}{\text{Hz}}$$
$$h_c(f) \simeq 10^{-18} \left( \frac{\text{Hz}}{f} \right) \sqrt{h^2\Omega_{\text{gw}}(f)}$$



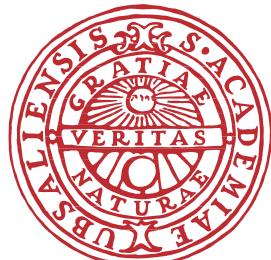
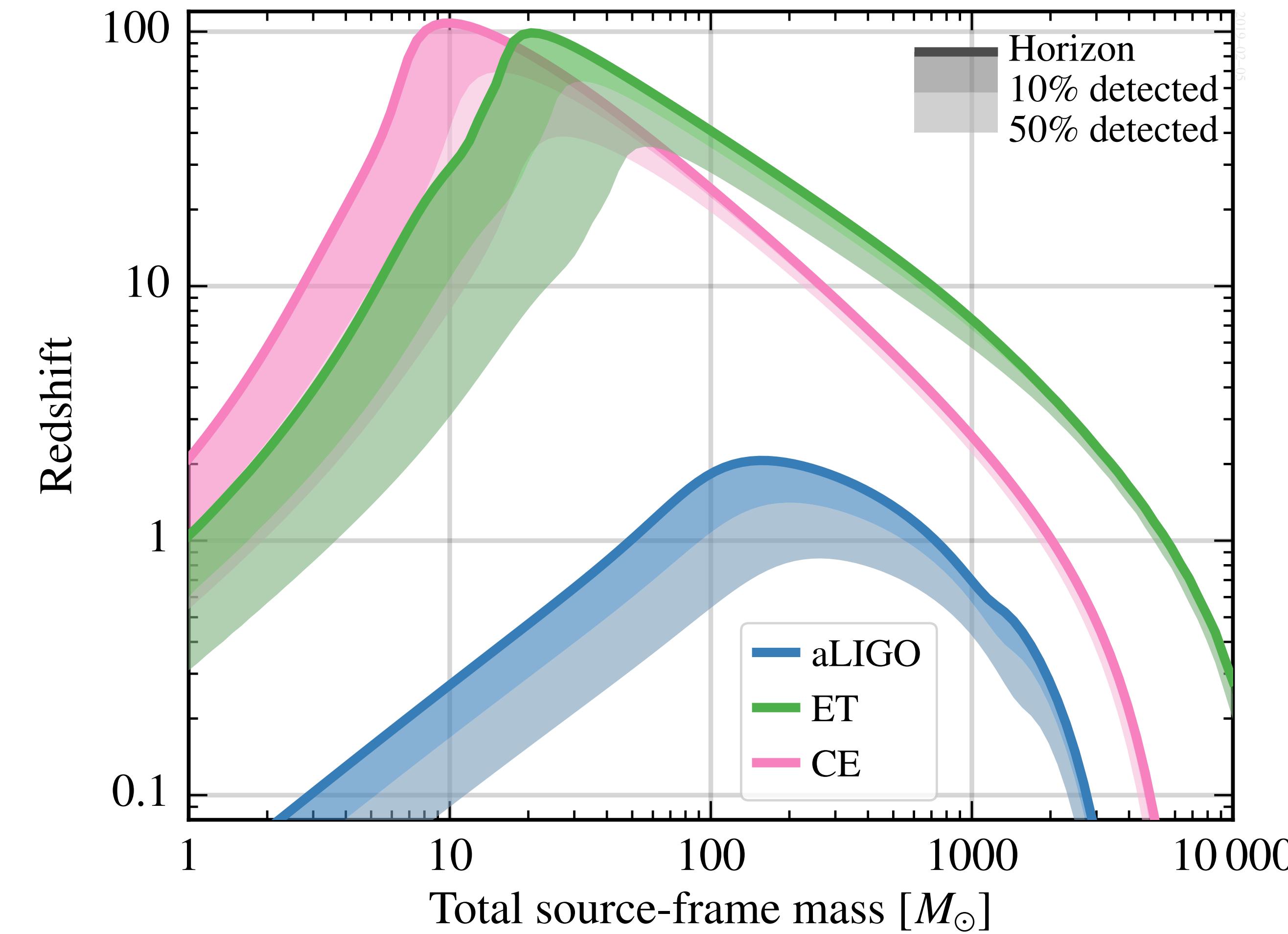
# NANOGrav 15yr new physics analysis



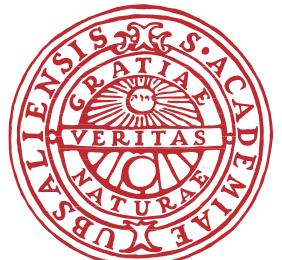
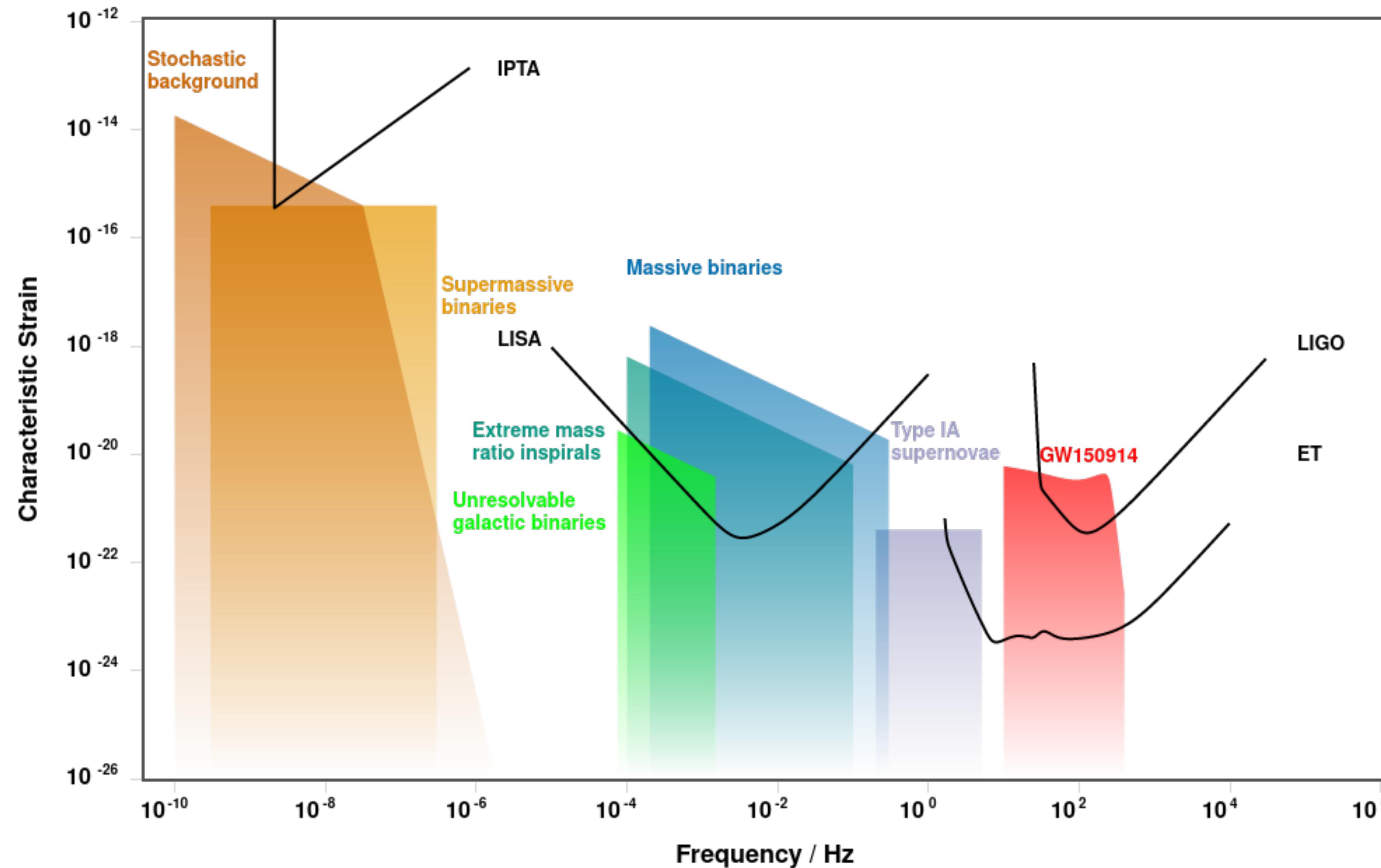
# NANOGrav 15yr with BBN and CMB limits



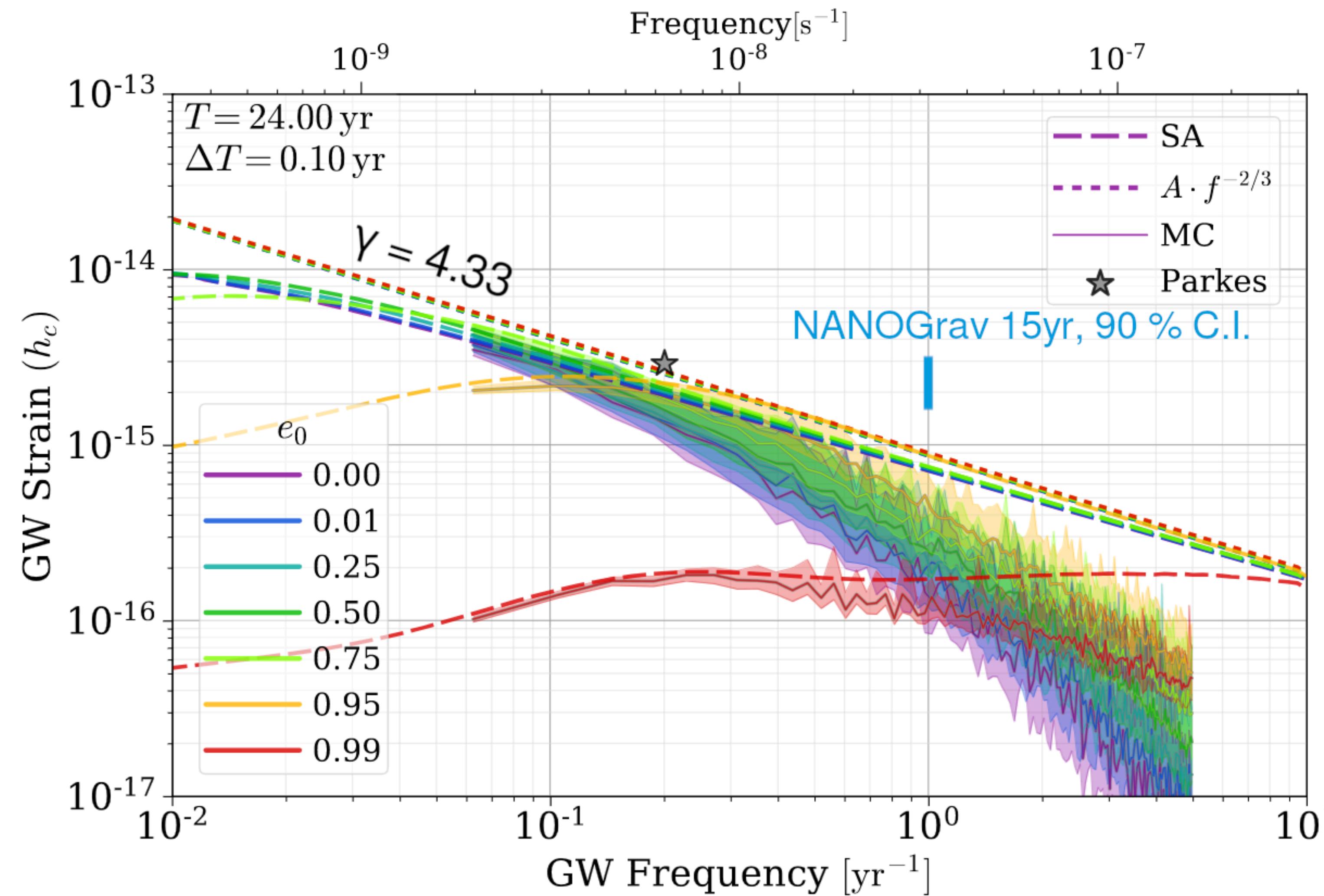
# Einstein Telescope science case



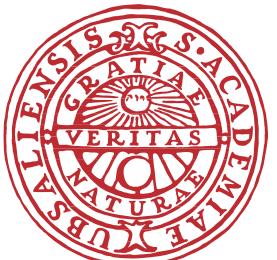
# GW spectrum in characteristic strain



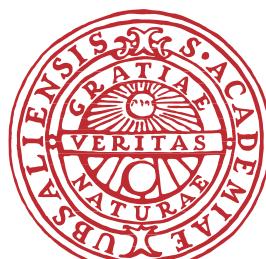
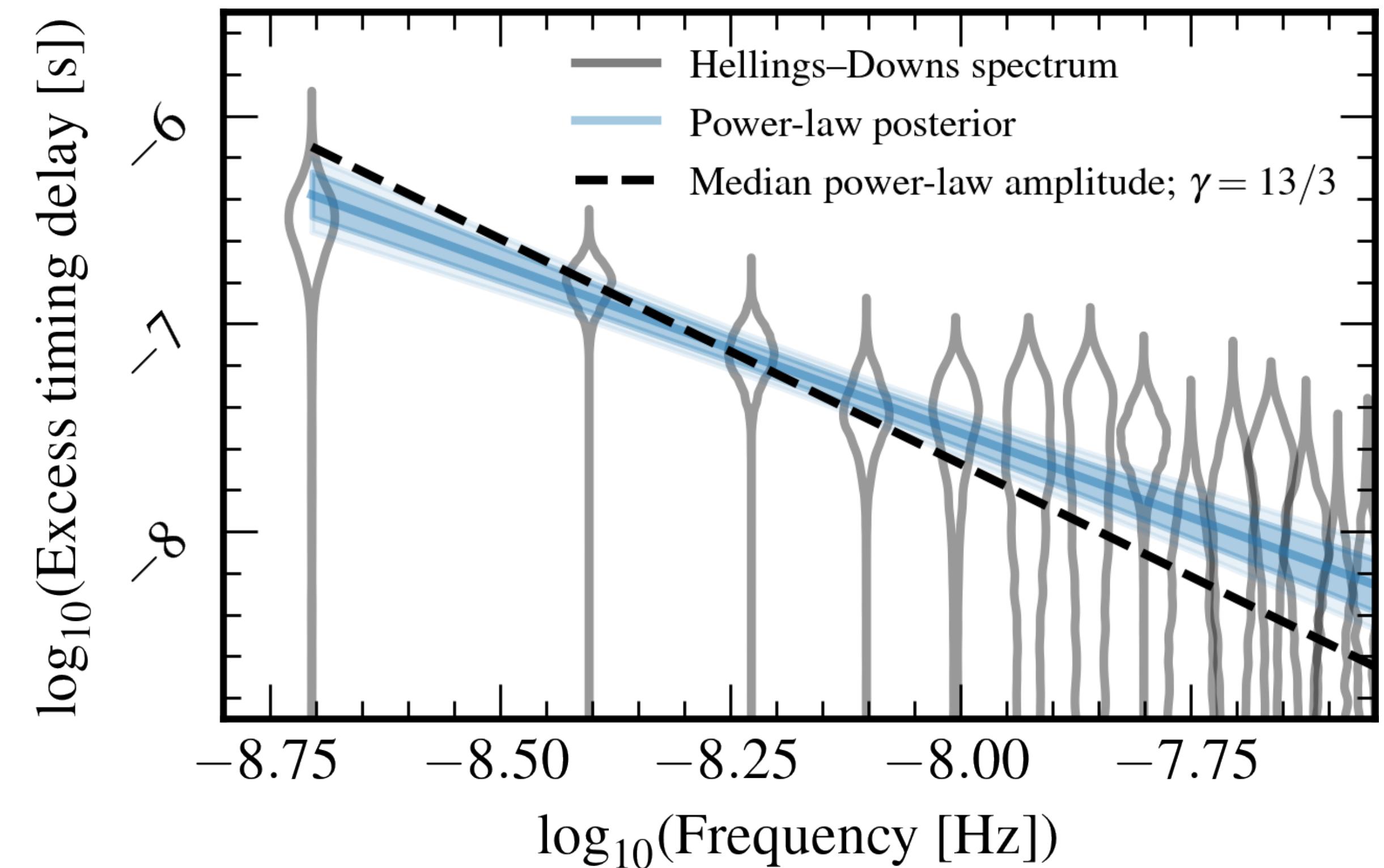
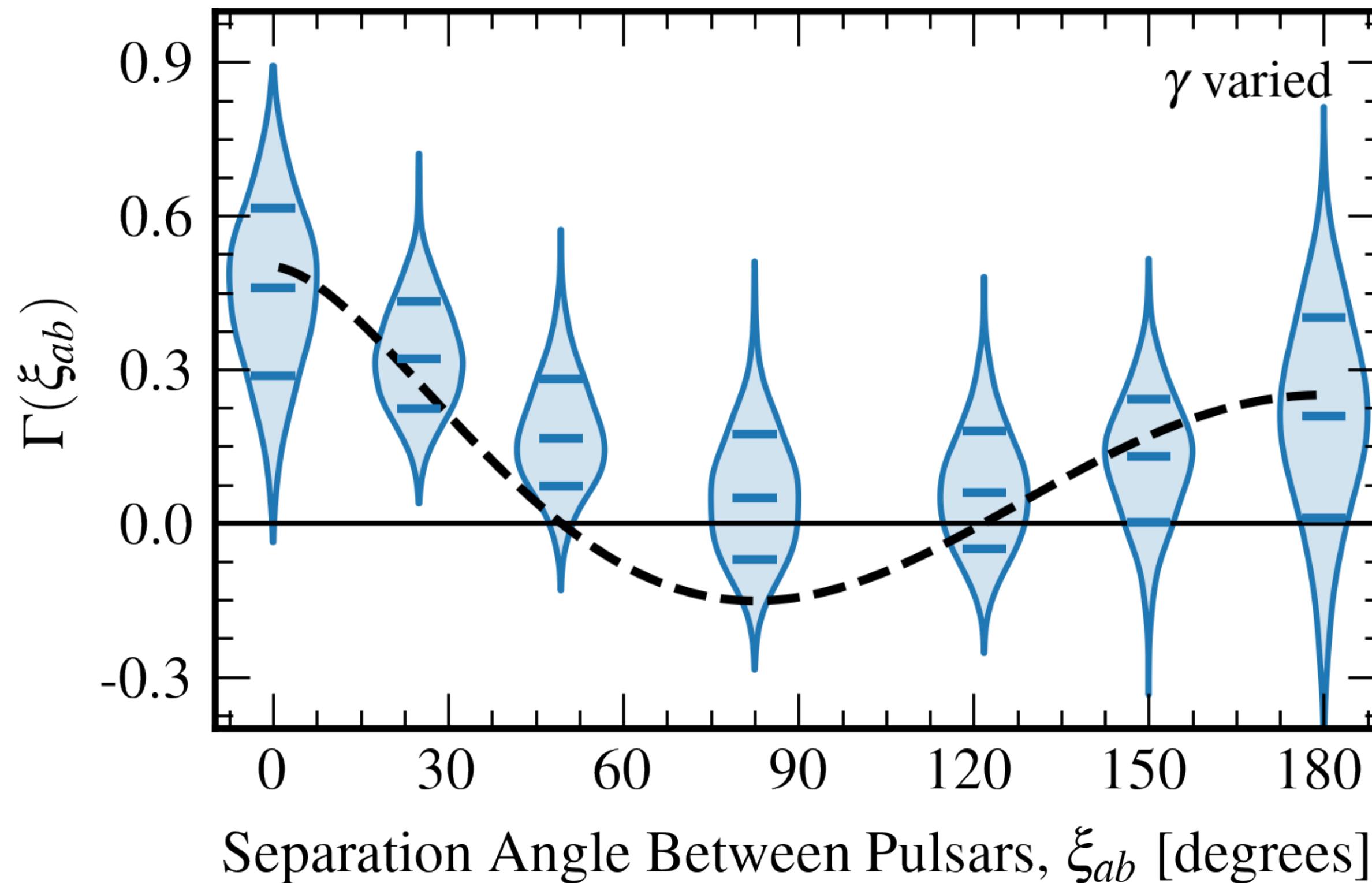
# Influence of eccentricity on SMBHB signals



[Adapted from: Kelley+, 1702.02180]



# NANOGrav 15yr data analysis



# Daisy-improved effective potential

**SUMMARY** Summing all discussed terms together, we obtain the one-loop, daisy-resummed effective potential of the QFT defined by the Lagrangian in eq. (4.6) [170]

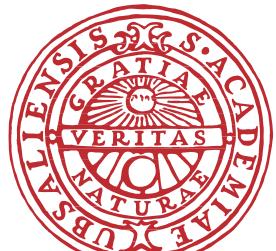
$$V_{\text{eff}}(\phi, T_d) = V_{\text{tree}} + V_{\text{CW}} + V_{\text{ct}} + V_T + V_{\text{daisy}} \quad (4.22)$$

with the individual contributions

$$\begin{aligned} V_{\text{CW}}(\phi) &= \sum_{a=\phi,\varphi,A',\chi} \eta_a n_a \frac{m_a^4(\phi)}{64\pi^2} \left[ \ln \frac{m_x^2(\phi)}{v_\phi^2} - C_a \right], \\ V_T(\phi, T_d) &= \frac{T^4}{2\pi^2} \sum_{a=\phi,\varphi,A',\chi} \eta_a n_a J_{\eta_a} \left( \frac{m_a^2(\phi)}{T_d^2} \right), \\ V_{\text{daisy}}(\phi, T_d) &= -\frac{T_d}{12\pi} \sum_{b=\phi,\varphi,A'_L} n_b \left[ (m_b^2 + \Pi_b(T_d))^{3/2} - (m_b^2)^{3/2} \right], \\ V_{\text{ct}}(\phi) &= -\frac{\delta\mu^2}{2}\phi^2 + \frac{\delta\lambda}{4}\phi^4 \end{aligned}$$

$$\begin{aligned} \text{with } \Pi_\phi &= \Pi_\varphi = \left( \frac{\lambda}{3} + \frac{y^2}{12} + \frac{g^2}{4} \right) T_d^2, \quad \Pi_{A'} = \frac{3}{4} g^2 T_d^2, \\ \delta\mu^2 &= \left[ \frac{3}{2\phi} \frac{dV_{\text{CW}}(\phi)}{d\phi} - \frac{1}{2} \frac{d^2V_{\text{CW}}(\phi)}{d\phi^2} \right] \Big|_{\phi=v_\phi}, \\ \text{and } \delta\lambda &= \left[ \frac{1}{2\phi^3} \frac{dV_{\text{CW}}(\phi)}{d\phi} - \frac{1}{2\phi^2} \frac{d^2V_{\text{CW}}(\phi)}{d\phi^2} \right] \Big|_{\phi=v_\phi}. \end{aligned} \quad (4.23)$$

As above,  $n_a$  are the dofs of the fields coupled to  $\phi$ ,  $\eta_x$  is  $+1$  ( $-1$ ) for bosons (fermions),  $C_a = 3/2$  ( $5/6$ ) are the renormalization constants for scalars and fermions (gauge bosons), and  $J_{\eta_a}$  are the thermal functions as defined in eq. (4.15).



# Computation of the bounce

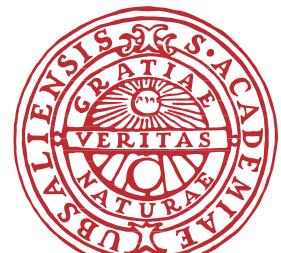
$$\Gamma(t) = A(T_d) \exp \left[ -\frac{S_3(T_d)}{T_d} \right]$$

$$S_3(T_d) \equiv S_3[\phi_b(\mathbf{x}; T_d)] = \int d^3x \left[ \frac{(\nabla \phi_b)^2}{2} + V_{\text{eff}}(\phi_b, T_d) \right]$$

$$\frac{\partial^2 \phi}{\partial r^2} + \frac{2}{r} \frac{\partial \phi}{\partial r} = \frac{dV_{\text{eff}}(\phi, T_d)}{d\phi} \equiv V'_{\text{eff}}(\phi, T_d).$$

$$\left. \frac{S_3(T_d)}{T_d} \right|_{T_{d,n}=\xi_n T_n} \simeq 146 - 2 \ln \left( \frac{g_*(T_n)}{100} \right) - 4 \ln \left( \frac{T_n}{100 \text{ GeV}} \right)$$

$$I(T) = \frac{4\pi}{3} v_w^3 \int_T^{T_c} dT' \frac{\Gamma(T')}{T'^4 H(T')} \left( \int_T^{T'} \frac{dT''}{H(T'')} \right)^3$$



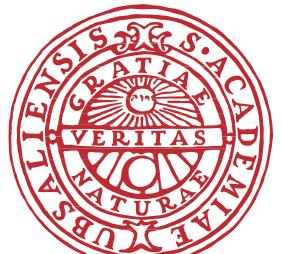
# GWs from PBH mergers

$$\Omega_{\text{gw}}(f) = \frac{f}{\rho_{\text{crit}}} \int_0^{t_0} dt_r \left( R(t_r + \tau_{f_r}) \frac{dE_{\text{gw}}^r}{df_r} \right)_{f_r=(1+z)f}$$

$$\frac{dE_{\text{gw}}^r}{df_r} \simeq \frac{(\pi G)^{2/3} m_{\text{PBH}}^{5/3}}{3 \times 2^{1/3}} \begin{cases} f_r^{-1/3} & f_r < f_1 \\ \frac{f_r^{2/3}}{f_1} & f_1 \leq f_r < f_2 \\ \frac{f_r^2 f_4^4}{f_1 f_2^{4/3} [4(f_r - f_2)^2 + f_4^2]^2} & f_2 \leq f_r < f_3 \\ 0 & f_3 \leq f_r . \end{cases} \quad (7.3)$$

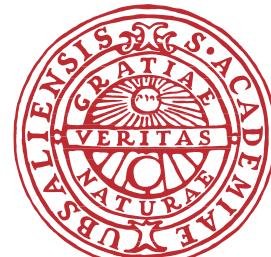
$$dn_3(x, y) = \frac{n_{\text{PBH}}}{2} e^{-N_{\text{PBH}}(y)} (4\pi n_{\text{PBH}} \delta_{\text{dc}})^2 x^2 y^2 dx dy$$

$$\begin{aligned} R(t_r) &= \int_0^{\tilde{x}} dx \int_x^\infty dy \frac{\partial^2 n_3}{\partial x \partial y} \delta(t_r - \tau(x, y)) \\ &= \frac{9 \tilde{N}_{\text{PBH}}^{53/37}}{296\pi \delta_{\text{dc}} \tilde{x}^3 \tilde{\tau}} \left( \frac{t_r}{\tilde{\tau}} \right)^{-34/37} \\ &\quad \times \left( \Gamma \left[ \frac{58}{37}, \tilde{N}_{\text{PBH}} \left( \frac{t_r}{\tilde{\tau}} \right)^{3/16} \right] - \Gamma \left[ \frac{58}{37}, \tilde{N}_{\text{PBH}} \left( \frac{t_r}{\tilde{\tau}} \right)^{-1/7} \right] \right) \end{aligned}$$



# Expected number of PBH pairs contributing to GWB

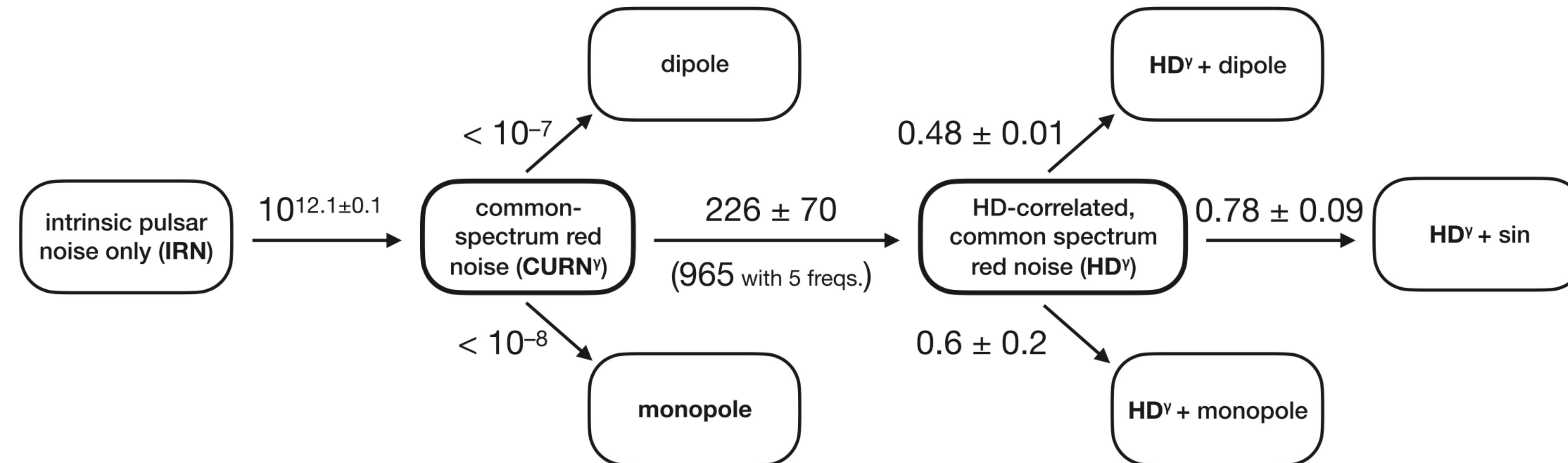
$$\bar{N}(f_-, f_+) = \int_{f_-}^{f_+} \frac{df}{f} \int_0^\infty dz \frac{8}{3} \tau_{f_r} \frac{4\pi [d_c(z)]^2}{H(z)} R(t_r(z) - \tau_{f_r})$$



# Evidence in favor of a stochastic GW background

THE ASTROPHYSICAL JOURNAL LETTERS, 951:L8 (24pp), 2023 July 1

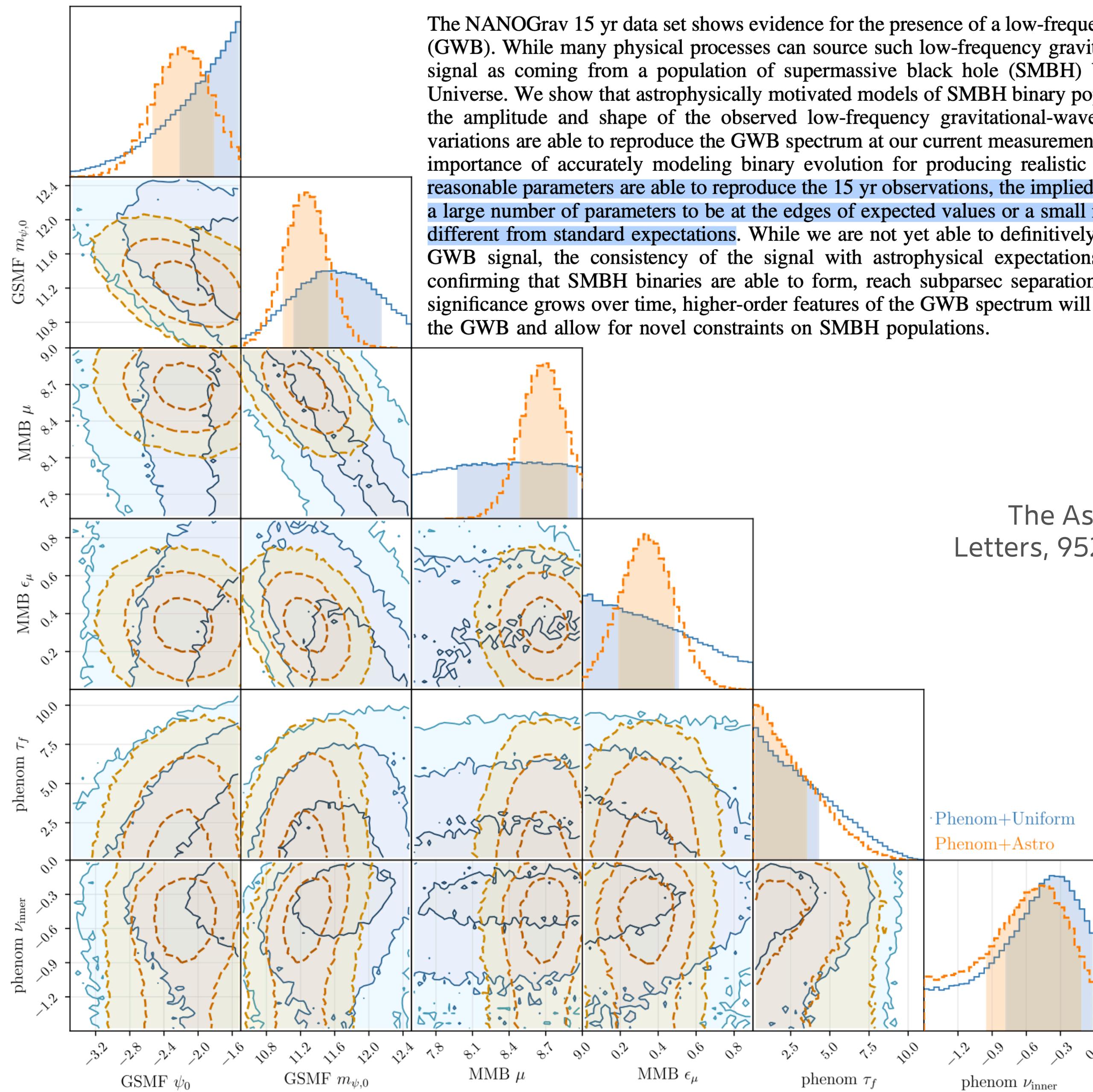
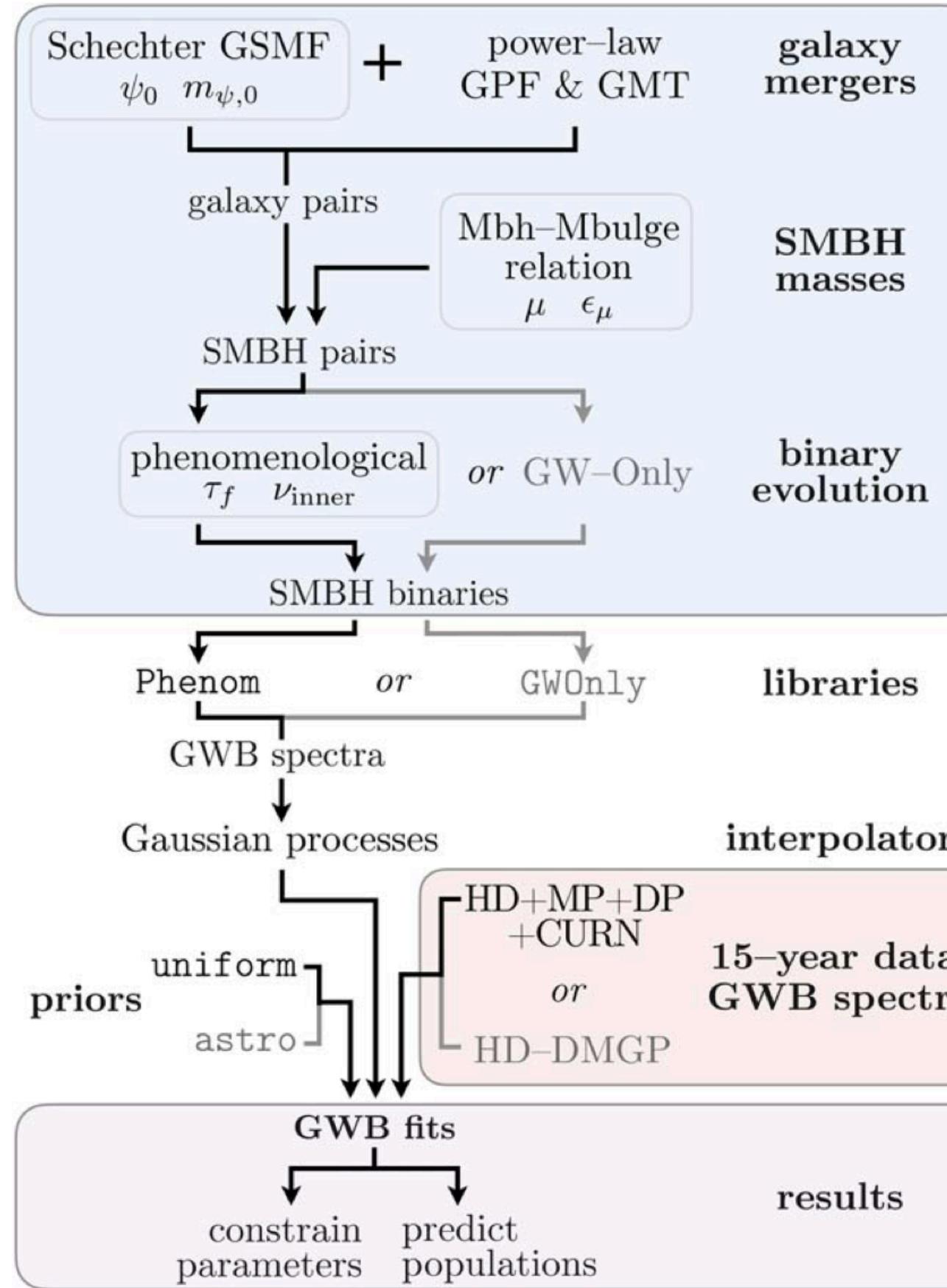
Agazie et al.



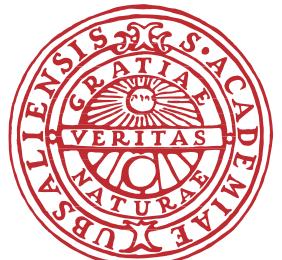
**Figure 2.** Bayes factors between models of correlated red noise in the NANOGrav 15 yr data set (see Section 5.3 and Appendix B). All models feature variable- $\gamma$  power laws.  $CURN^\gamma$  is vastly favored over IRN (i.e., we find very strong evidence for common-spectrum excess noise over pulsar intrinsic red noise alone);  $HD^\gamma$  is favored over  $CURN^\gamma$  (i.e., we find evidence for Hellings–Downs correlations in the common-spectrum process); dipole and monopole processes are strongly disfavored with respect to  $CURN^\gamma$ ; adding correlated processes to  $HD^\gamma$  is disfavored. While the interpretation of “raw” Bayes factors is somewhat subjective, they can be given a statistical significance within the hypothesis-testing framework by computing their background distributions and deriving the  $p$ -values of the observed factors, e.g., Figure 3.



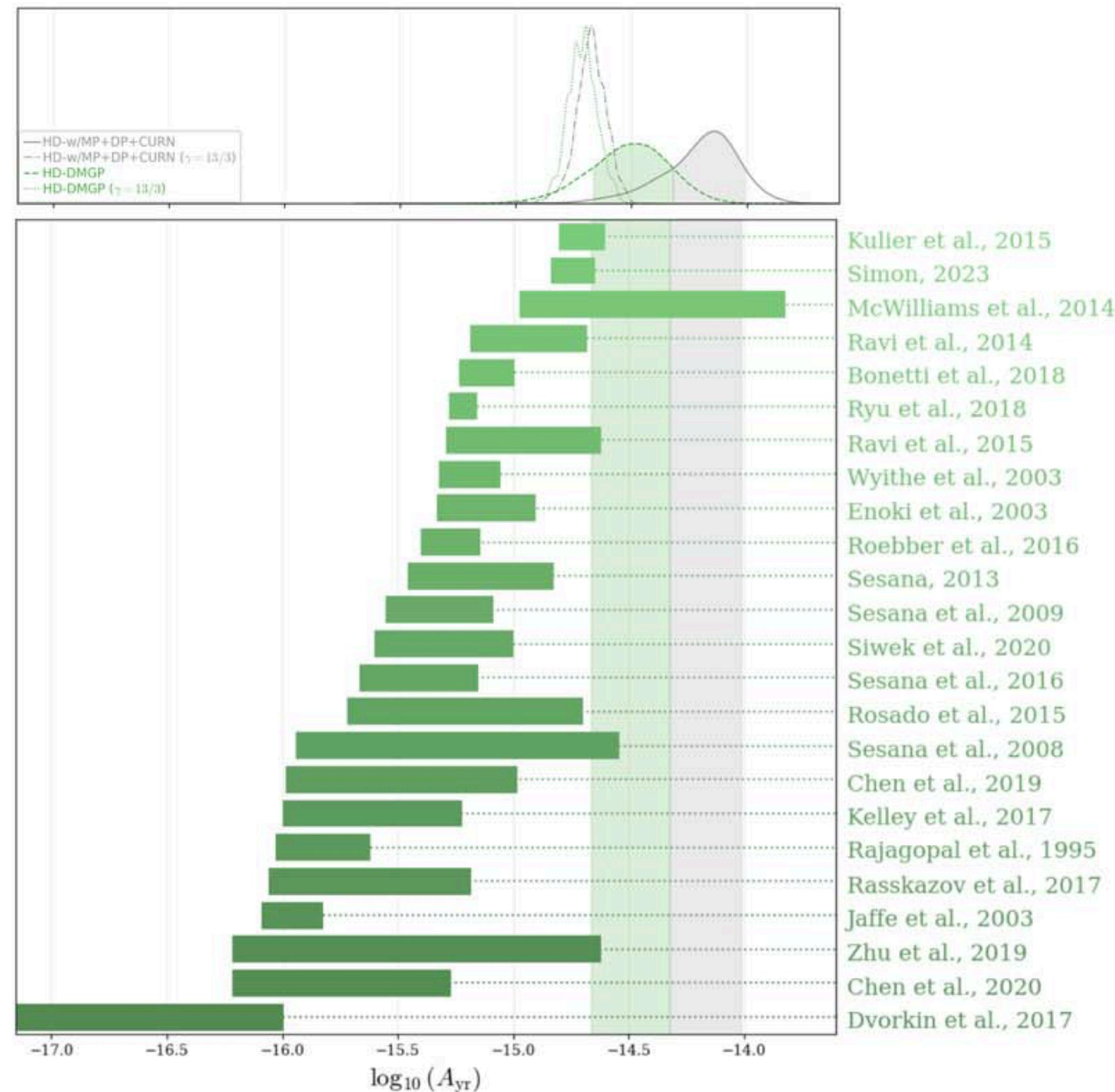
# Modeling SMBHBs



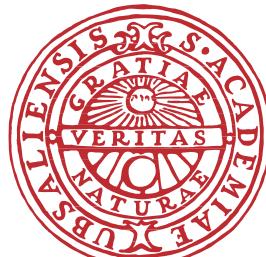
The Astrophysical Journal  
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August 1



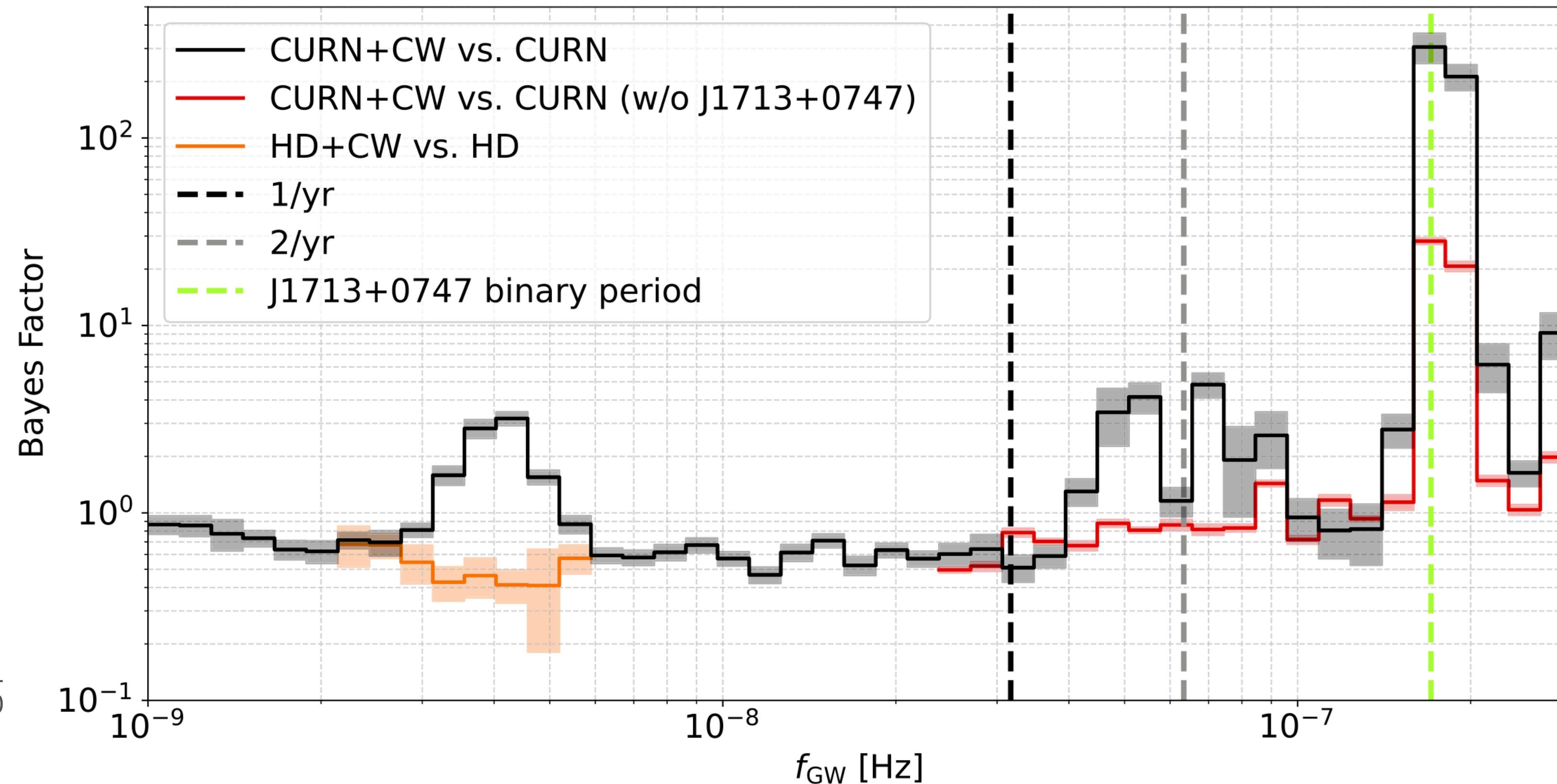
# Uncertainties of the GWB amplitude from SMBHBs



The Astrophysical Journal  
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2023 August 1



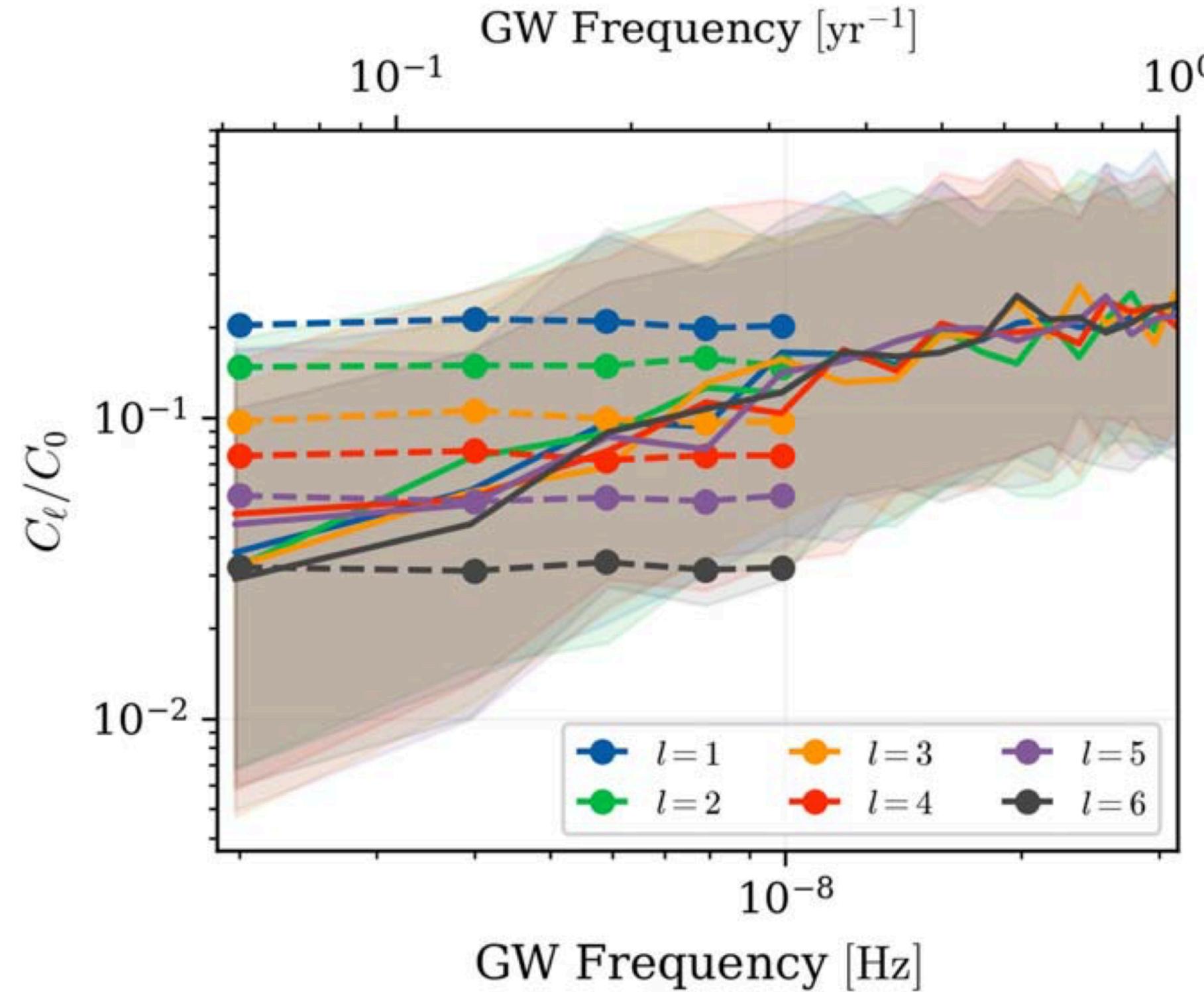
# Continuous waves in the NANOGrav 15yr data set



**Figure 1.** Savage-Dickey Bayes factors for the CW+CURN model vs. the CURN model as a function of frequency (black). Also shown are Bayes factors when excluding PSR J1713+0747 (red, only computed for  $f_{\text{GW}} > 24$  nHz) and Bayes factors based on a resampled posterior that takes into account the presence of HD correlations in the common red noise process, i.e., CW+HD vs. HD (orange, only computed for  $2.1$  nHz  $< f_{\text{GW}} < 5.9$  nHz). Shaded regions show the  $1\sigma$  uncertainties.



# NANOGrav 15yr data limits on anisotropy

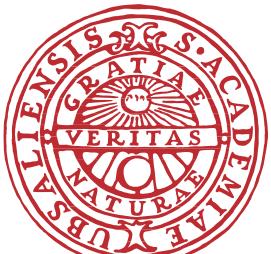


**Figure 11.** Normalized spherical-harmonic coefficients  $C_l/C_0$  of the gravitational-wave sky as produced by simulated populations of SMBHBs, filtered by consistency with the 15 yr isotropic gravitational-wave background estimation (Agazie et al. 2023b). The different colors correspond to individual harmonics from  $l = 1$  to  $l = 6$ . The solid lines represent the median realization of the median samples, and the shaded regions represent the 68% confidence intervals across all samples' median realizations. The circles connected by dashed lines represent the Bayesian upper limits as in Figure 5.

## Abstract

The North American Nanohertz Observatory for Gravitational Waves (NANOGrav) has reported evidence for the presence of an isotropic nanohertz gravitational-wave background (GWB) in its 15 yr data set. However, if the GWB is produced by a population of inspiraling supermassive black hole binary (SMBHB) systems, then the background is predicted to be anisotropic, depending on the distribution of these systems in the local Universe and the statistical properties of the SMBHB population. In this work, we search for anisotropy in the GWB using multiple methods and bases to describe the distribution of the GWB power on the sky. We do not find significant evidence of anisotropy. By modeling the angular power distribution as a sum over spherical harmonics (where the coefficients are not bound to always generate positive power everywhere), we find that the Bayesian 95% upper limit on the level of dipole anisotropy is  $(C_{l=1}/C_{l=0}) < 27\%$ . This is similar to the upper limit derived under the constraint of positive power everywhere, indicating that the dipole may be close to the data-informed regime. By contrast, the constraints on anisotropy at higher spherical-harmonic multipoles are strongly prior dominated. We also derive conservative estimates on the anisotropy expected from a random distribution of SMBHB systems using astrophysical simulations conditioned on the isotropic GWB inferred in the 15 yr data set and show that this data set has sufficient sensitivity to probe a large fraction of the predicted level of anisotropy. We end by highlighting the opportunities and challenges in searching for anisotropy in pulsar timing array data.

The Astrophysical Journal Letters, 956:L3 (15pp), 2023 October 10



# Different models for the GW spectrum from a FOPT

	IR	UV	References
Envelope	3	-1	[16, 27]
Bulk flow	1	-3	[17, 28]
Scalar lattice	3	-1.5	[38]

	IR	Intermediate	UV	References
Sound shell	9	1	-3	[22, 23]
Scalar + fluid lattice	—	1	-3	[18, 20, 21, 29]
Hybrid	[2,4]	[-1,0]	[-4,-3]	[30]
Higgsless	3	1	-3	This work

JCAP02(2023)011, Jinno,  
Konstandin, Rubira, Stomberg

Contribution from bubble wall collisions

