

Turn up the volume — Listening to hot dark sector phase transitions.

Kosmologietag at Bielefeld University

Carlo Tasillo,
Deutsches Elektronen Synchrotron (DESY)

Presented results based on [JCAP02\(2022\)014](#) (together with F. Ertas and F. Kahlhöfer) and ongoing work with T. Bringmann, F. Depta, T. Konstandin and K. Schmidt-Hoberg.

May 5, 2022



Outline of this talk.

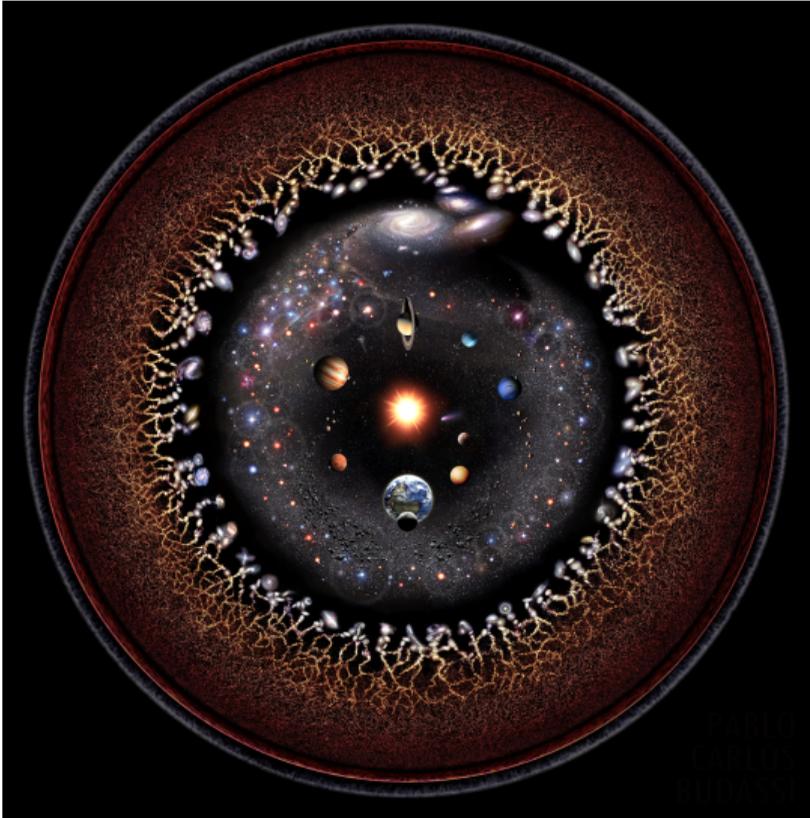
- 1 Context and motivation
- 2 Thermal evolution of dark sectors after a phase transition
- 3 The dark photon model
- 4 Conclusions



[Camille Flammarion, 1888]

Context and Motivation.

What we know about our Universe.



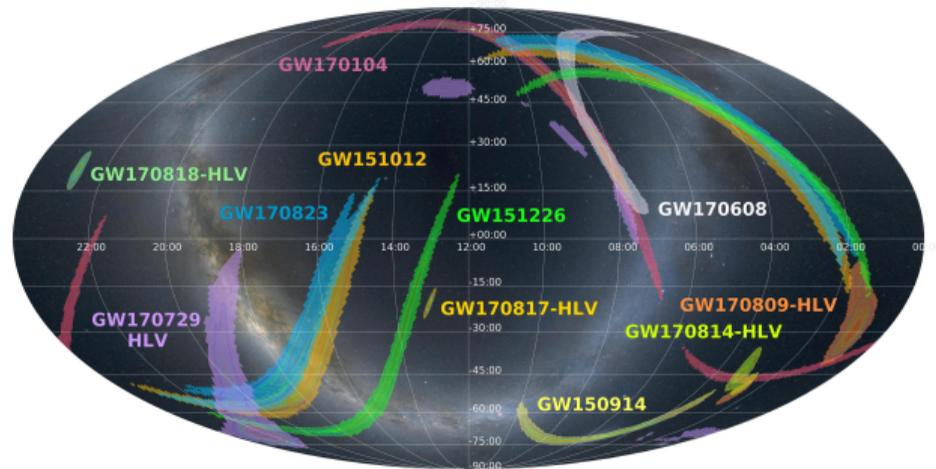
[Pablo Carlos Budassi, 2020]

The CMB anisotropies tell us that 95 % of today's energy content of the Universe is dark... [Planck, 2018]

→ **What lies beyond the surface of last scattering?**

Gravitational waves as a “new” messenger.

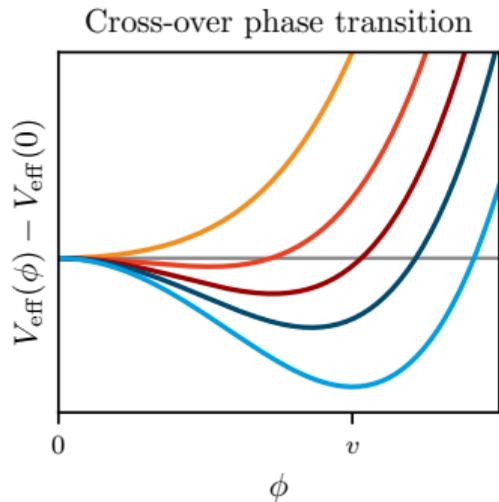
- G_N is tiny!
- >90 compact binary mergers since 2015 and counting
- LISA, Einstein Telescope, etc. will increase sensitivity considerably



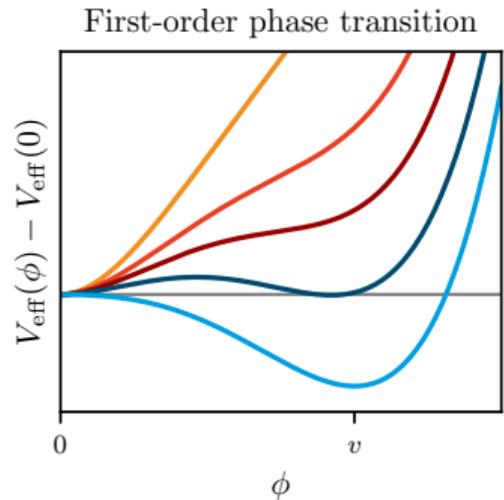
[LIGO, Virgo & KAGRA Collaboration, 2020]

→ **What will the stochastic gravitational wave background look like?**

Cross-over and first-order phase transitions.



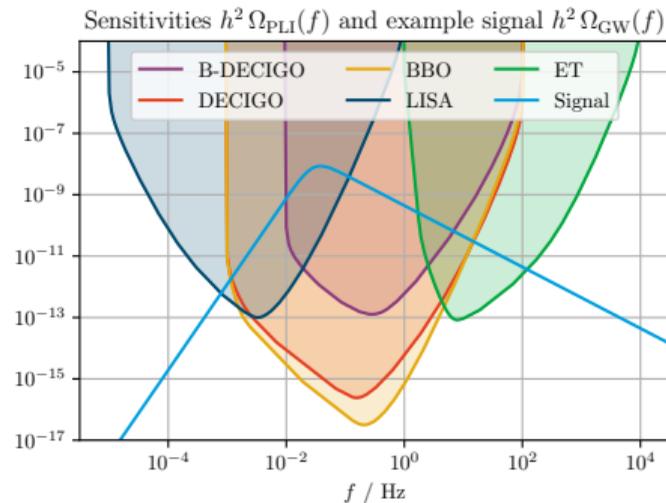
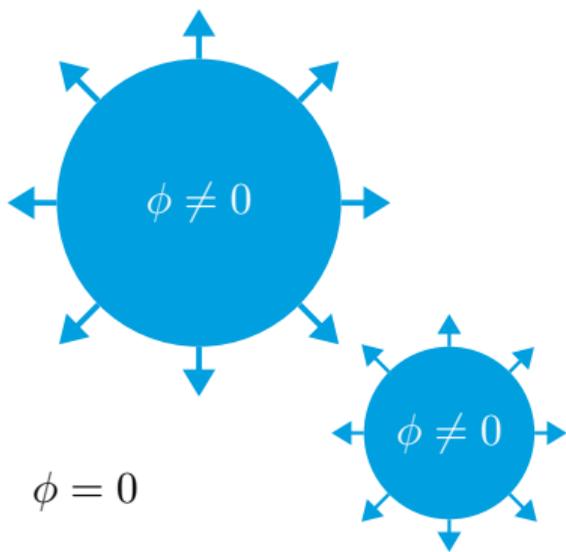
The scalar field “rolls down” from $\phi = 0$ to $\phi = v$, when the bath cools from **high temperatures** to **low temperatures**.



The scalar field tunnels to the true potential minimum ($\phi \neq 0$) to minimize its action.

Gravitational waves from first-order phase transitions.

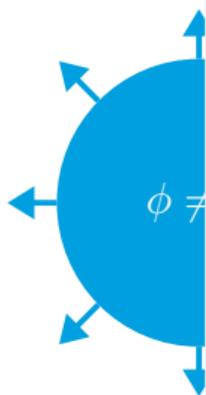
Bubbles of the new phase nucleate, collide and perturb the surrounding plasma...



... giving rise to a stochastic gravitational wave background which could hopefully be observed soon.

Gravitational waves from first-order phase transitions.

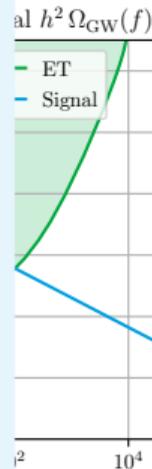
Bubbles of the new phase nucleate, collide and percolate through the plasma...



$\phi = 0$

- + Majority of our Universe is “dark”
- + First-order phase transitions emit gravitational wave signals
- + Primordial GWs could be used for “listening” beyond the CMB

- = What kind of **dark sectors** could produce primordial GW signals?



Gravitational waves could hopefully be

Thermal evolution of dark sectors after a phase transition.

The dark sector temperature ratio.

Dark sector: particle bath without thermal contact to SM particles:

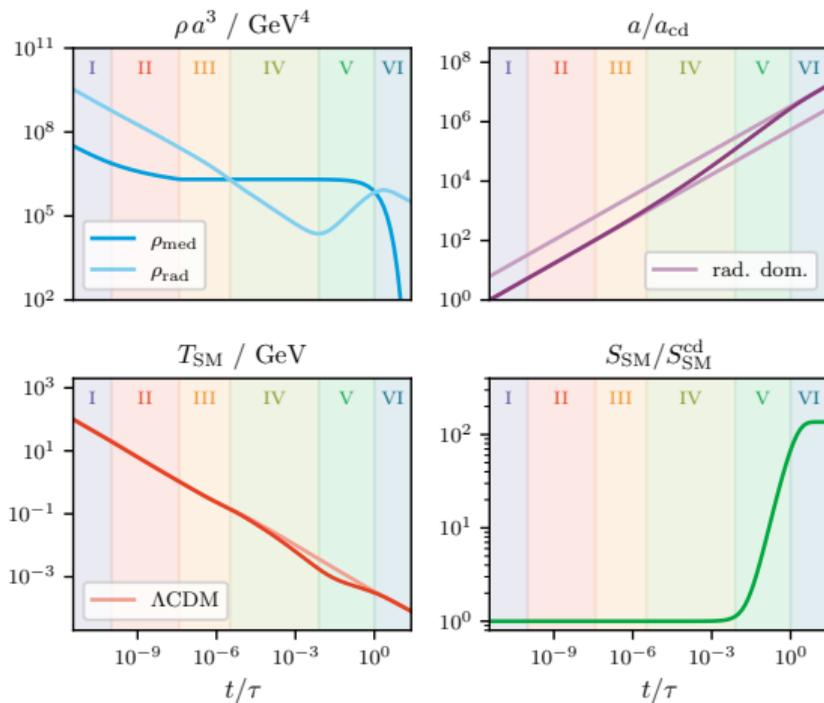
$$T_{\text{DS}} = \xi T_{\text{SM}}$$

We have no reason to assume $\xi = 1$.
Through ΔN_{eff} constraints, we conclude that light secluded dark sectors must be **cold** ($\xi < 1$) since BBN.

Breitbach et al. showed that **cold** dark sector phase transitions produce weak GW signals [1811.11175]...

↪ Can **hot** ($\xi > 1$) dark sector phase transitions before BBN emit observable GW signals? What happens when the dark sector thermalizes and decays to SM particles?

The out-of-equilibrium decay of a dark mediator.



Energy densities $\rho_i(t)$ \rightsquigarrow Scale factor $a(t)$ \rightsquigarrow Temperatures $T_{\text{SM}/\text{DS}}(t)$ \rightsquigarrow Particle content $\rightsquigarrow \rho_i(t)$ \rightsquigarrow ...

Six phases:

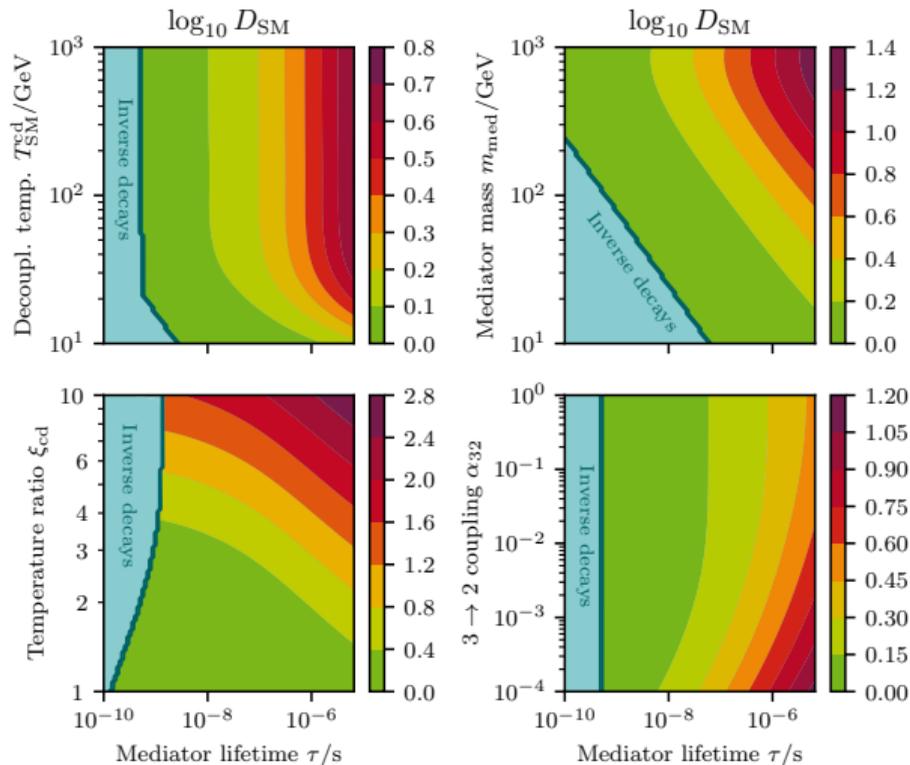
- I Relativistic mediator
- II Cannibalistic mediator
- III Non-relativistic mediator
- IV Early matter domination
- V Entropy injection
- VI Mediator decay

The entropy injection into the SM bath.

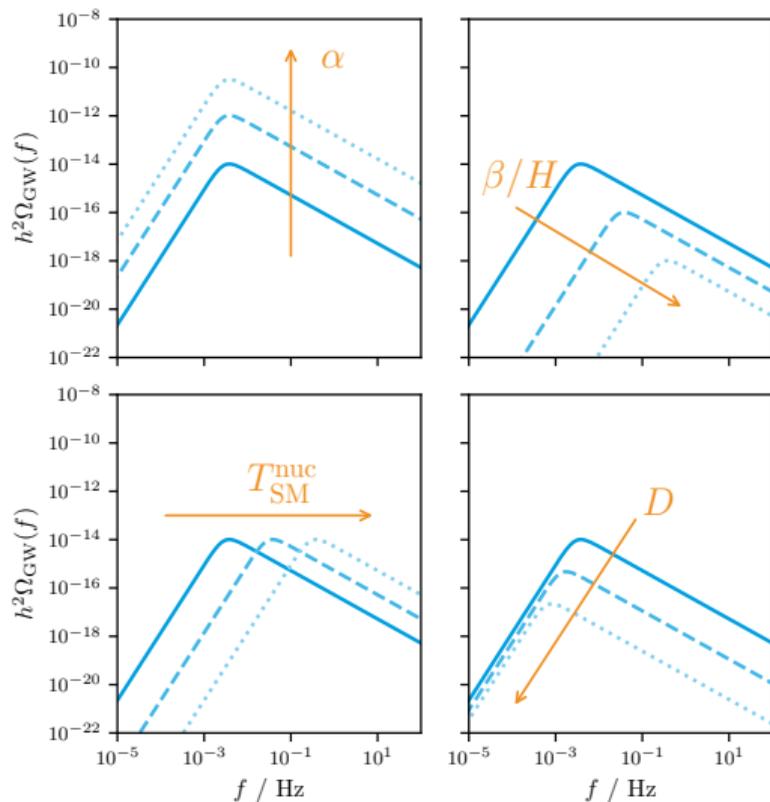
The extra expansion of the Universe with respect to Λ CDM's radiation domination can be parameterized by the dilution factor

$$D_{\text{SM}} \equiv \frac{S_{\text{SM}}^{\text{after decay}}}{S_{\text{SM}}^{\text{before decay}}}$$

[Cirelli et al. 1811.03608]



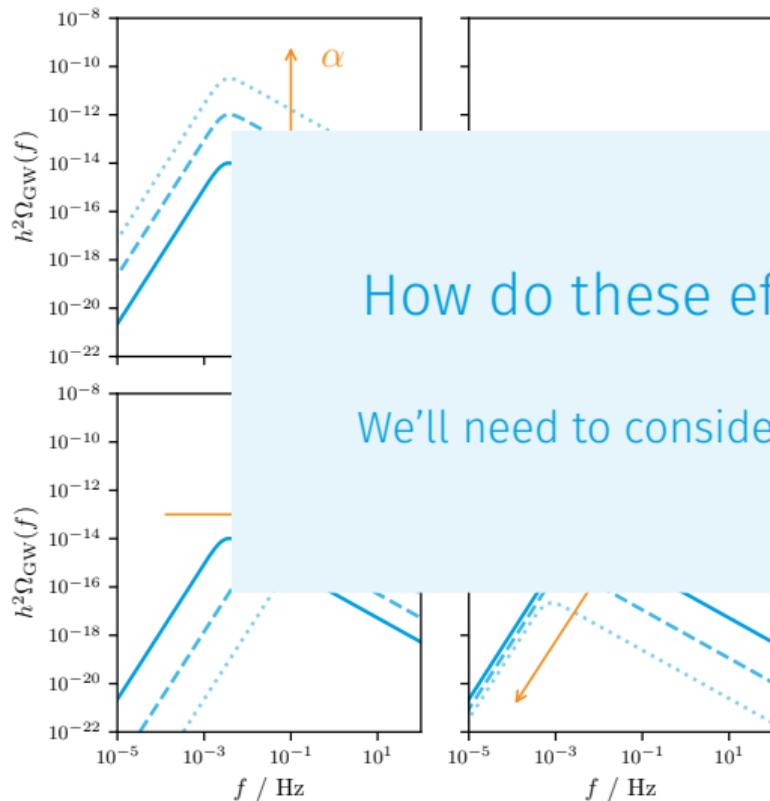
Parametrization of the GW signal.



For observable signals:

- Strong transitions, high α
- Slow transitions, low β/H
- Nucleation temperature
 $T_{\text{SM}}^{\text{nuc}} \propto f_p \simeq f_{\text{exp}}$
- Little dilution, low $D \propto D_{\text{SM}}$

Parametrization of the GW signal.



How do these effects sum up?

We'll need to consider a specific model!

signals:

situations, high α

situations, low β/H

temperature

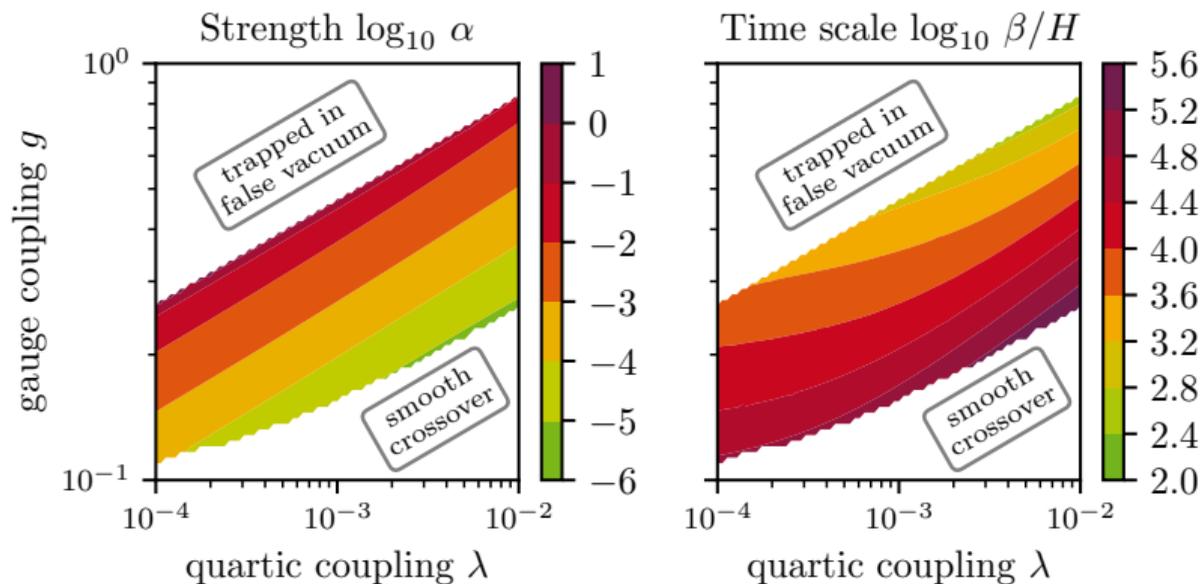
f_{exp}

on, low $D \propto D_{\text{SM}}$

The dark photon model.

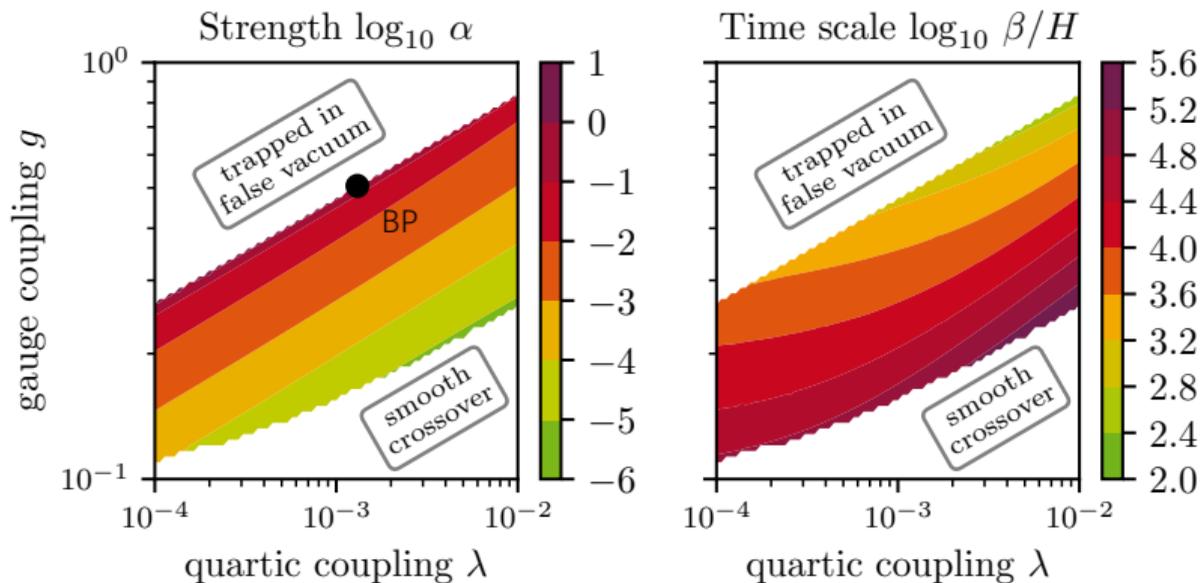
Strength and time scale of the transition.

Analyze the phase structure and determine the strength α and inverse time scale β/H . Vary quartic coupling λ and gauge coupling g to identify region of strong and slow transitions.

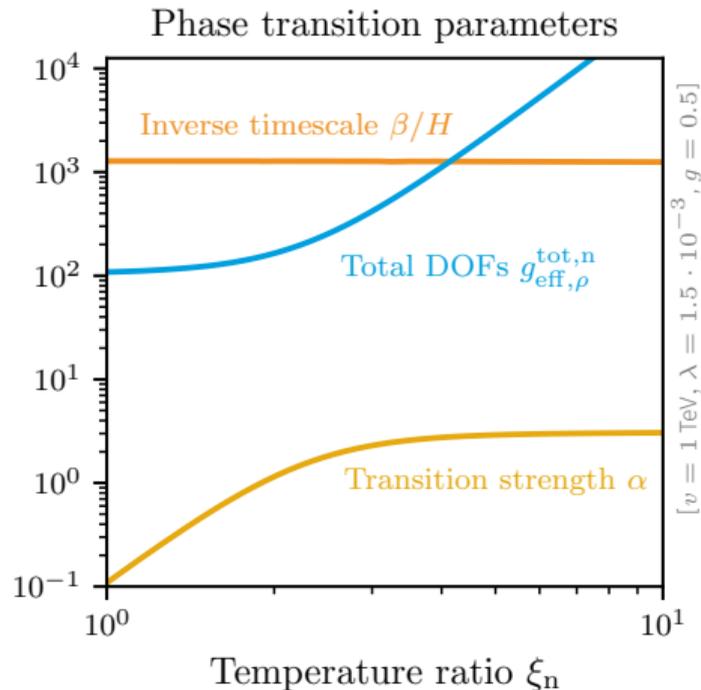


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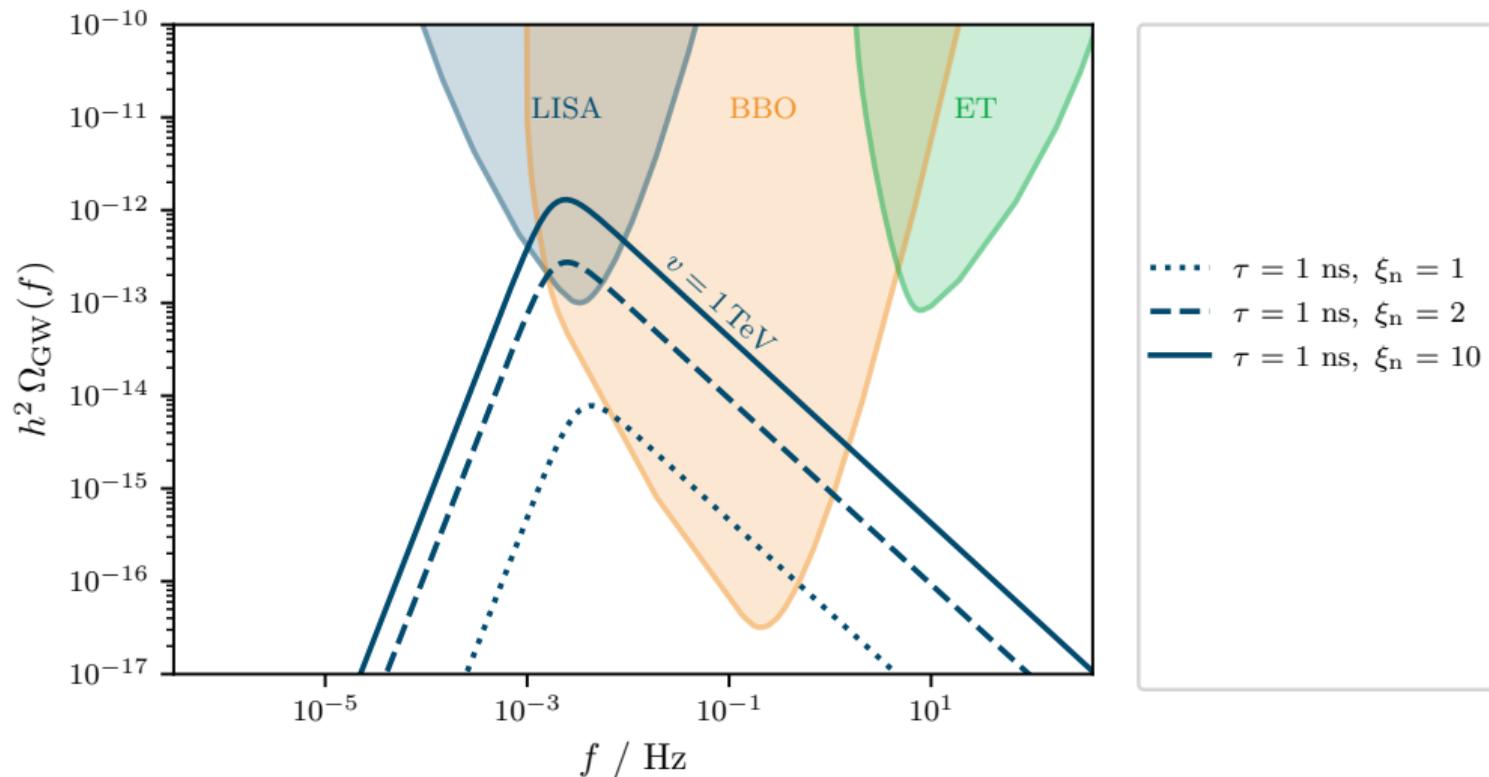


The temperature ratio's impact on α and β/H .

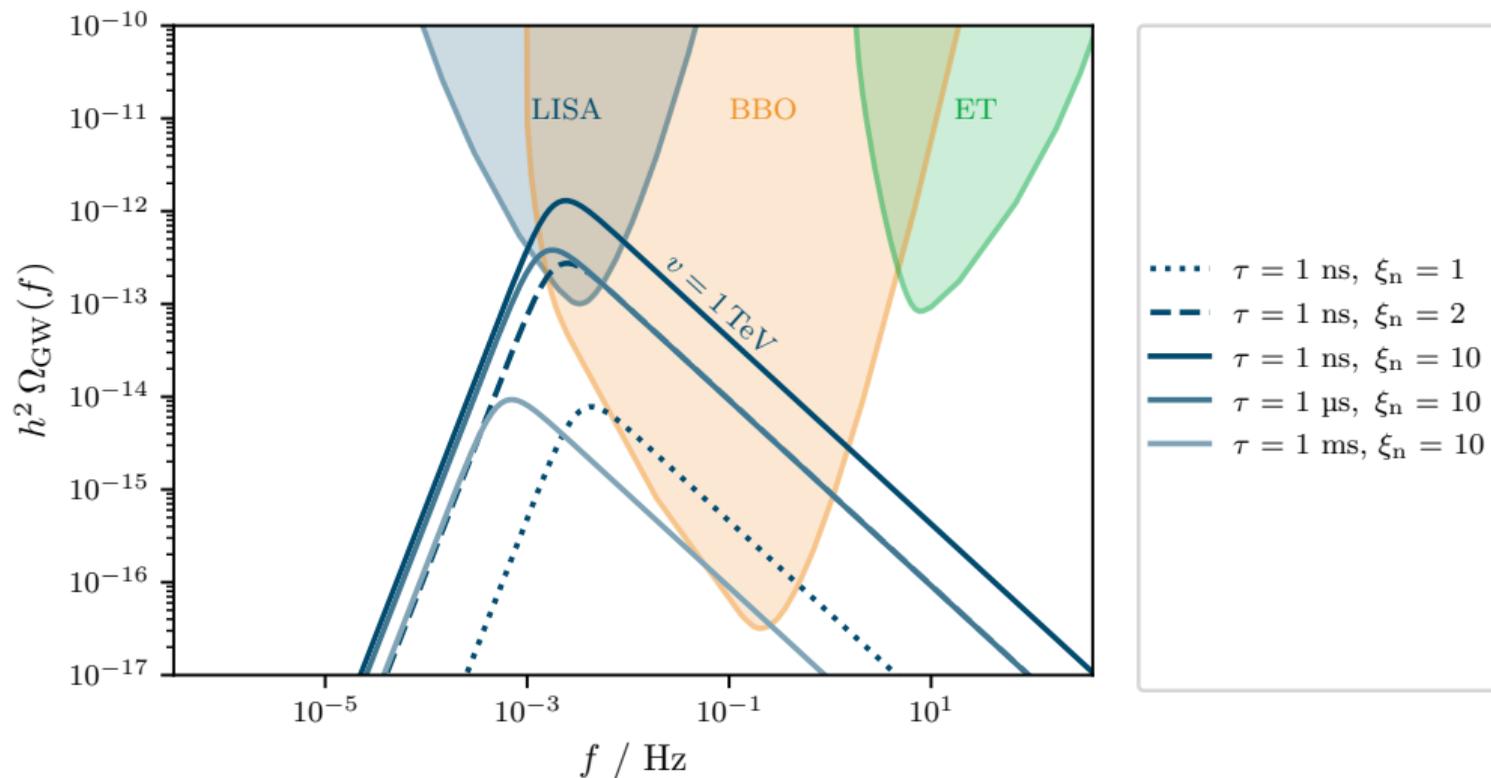


The transition strength α increases $\propto \xi_n^4$, but only until the Universe is completely dominated by the dark sector at $\xi_n \approx 3$. Then, the temperature ratio becomes irrelevant. The inverse timescale is virtually independent of ξ_n .

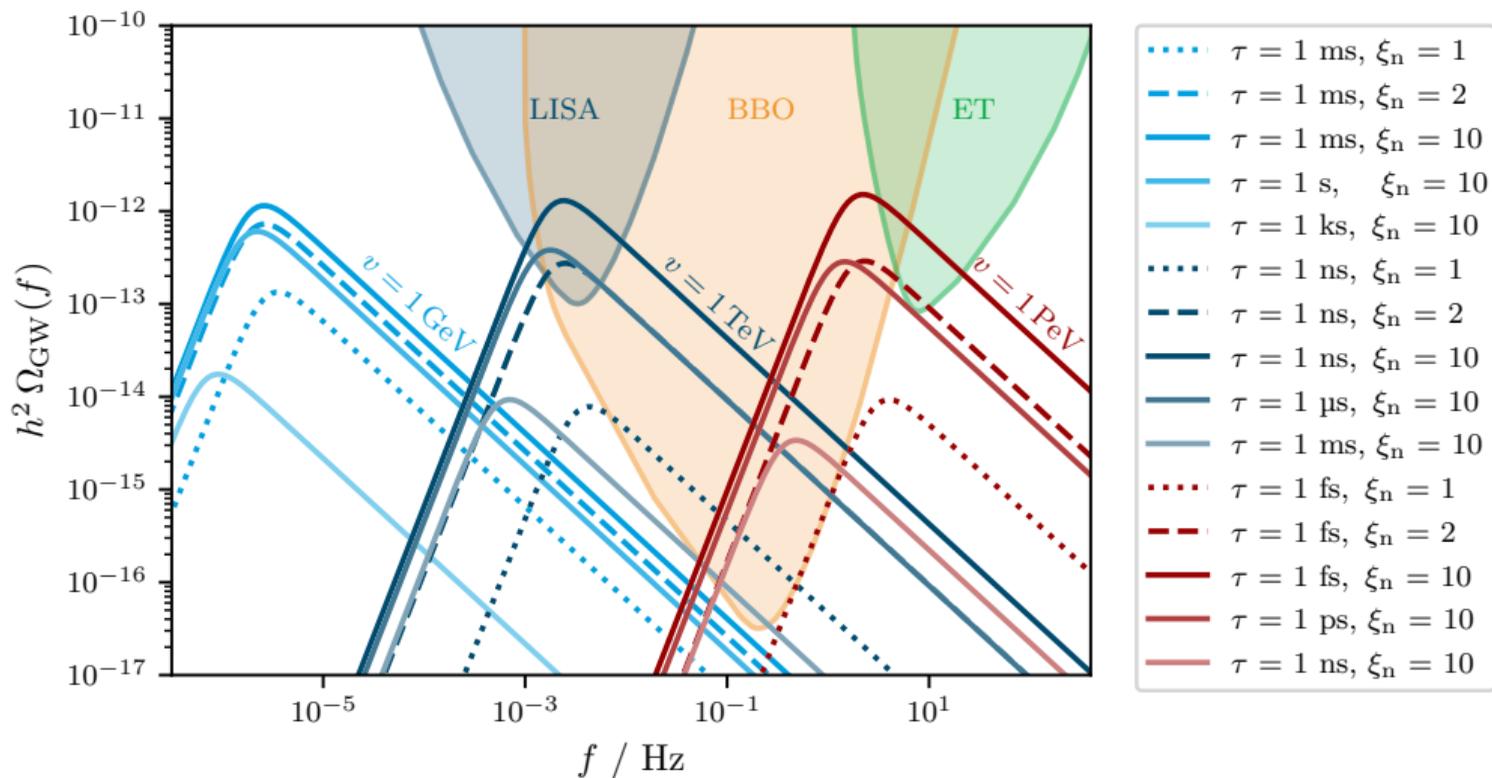
The temperature ratio's impact on the GW signal.



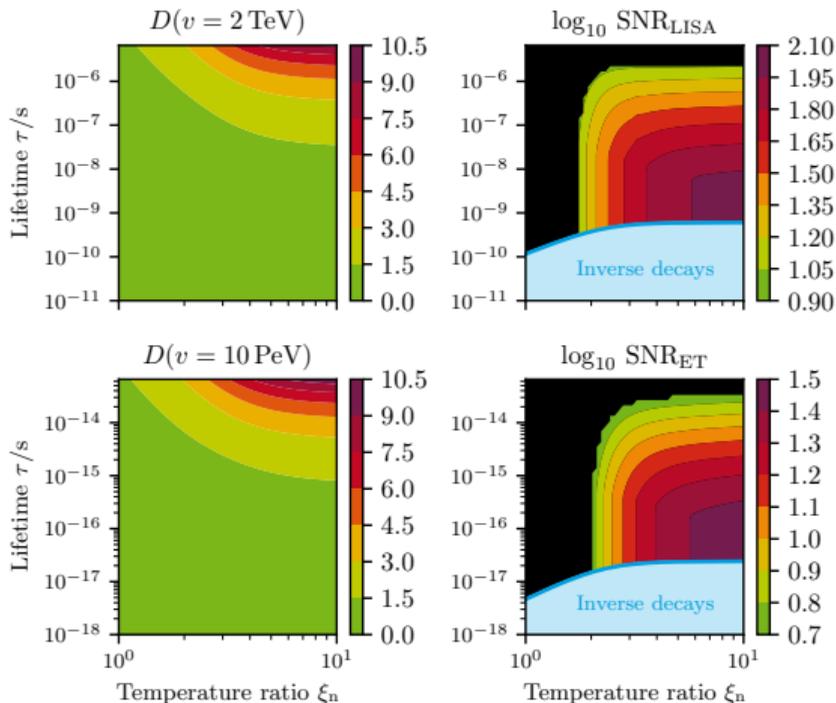
The dark Higgs lifetime's impact on the GW signal.



The vacuum expectation value's impact on the GW signal.



Benchmark point analysis.

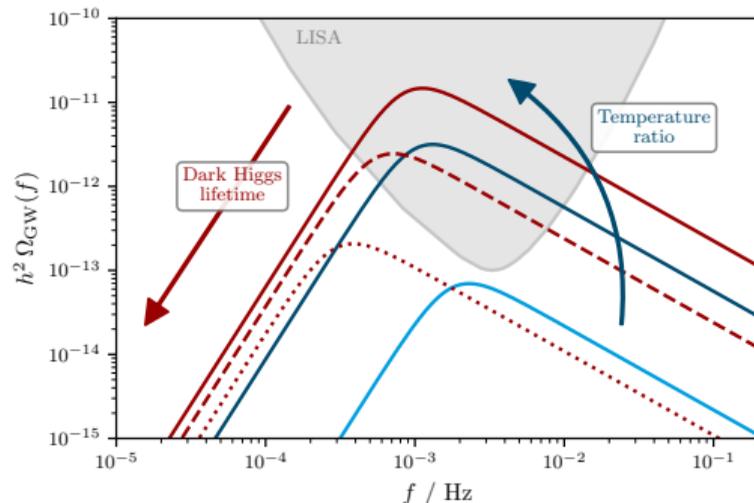


GWs observable by LISA and Einstein Telescope for $\xi_n > 2$, if dark sector is not too long-lived. Otherwise, signals are too diluted.

In blue region, the non-relativistic decay assumption breaks: Inverse decays become relevant, thermalizing the two baths.

Conclusion and outlook.

- **The temperature ratio** of a dark sector to the SM is important!
- **Dilution** of GWs can become an issue for decaying dark sectors.
- **Pulsar Timing Arrays** [2009.05596, 2107.12112, 2201.03980] could have observed an MeV temperature phase transition... See Kai Schmitz' talk later today!



And by the way: Our code `TransitionListener` for model parameter scans, effective potentials, dilution factors, signal-to-noise ratios, etc. is available on GitHub!

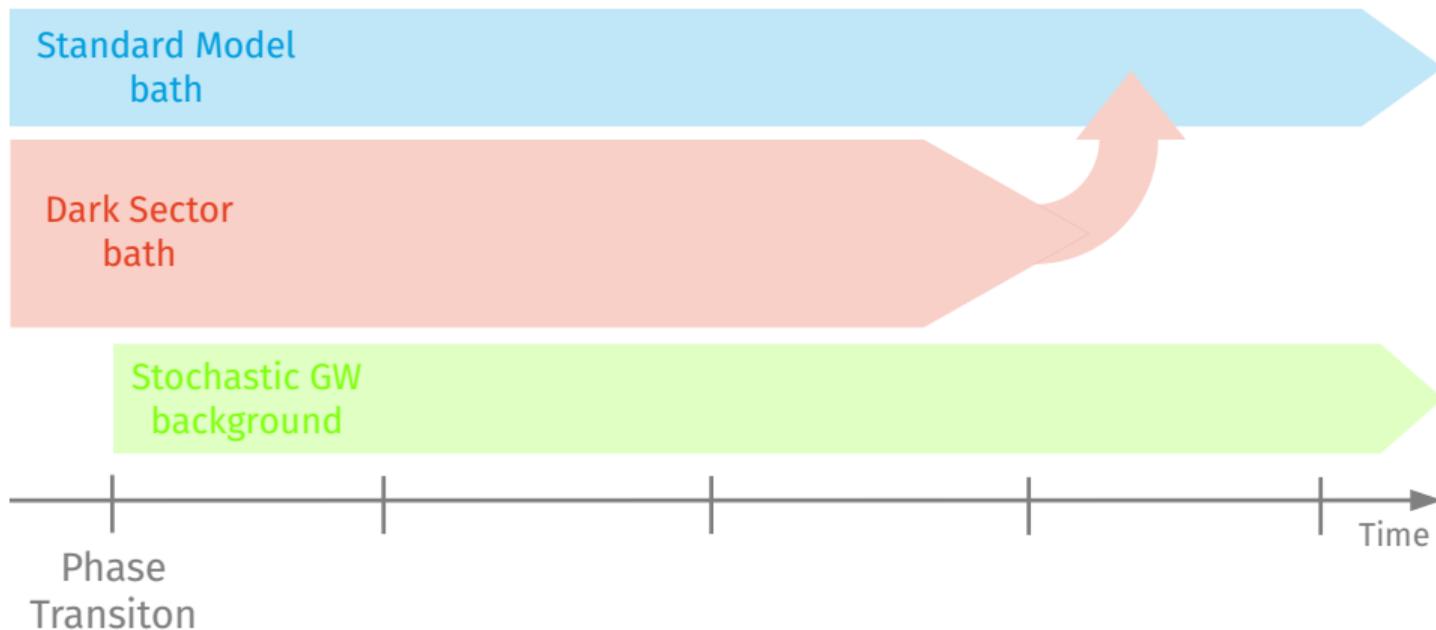
**Thank you very
much for your
attention!**

Please feel free to ask me
about anything, if you have
questions.

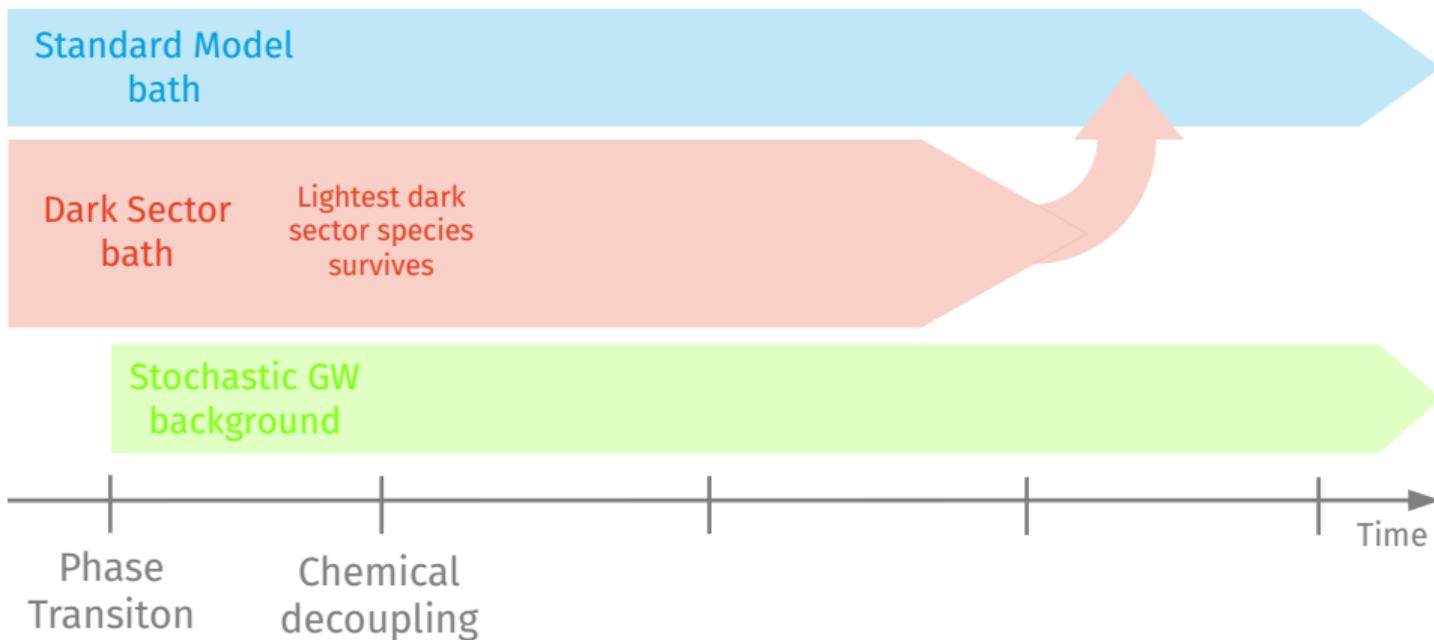


Backup slides.

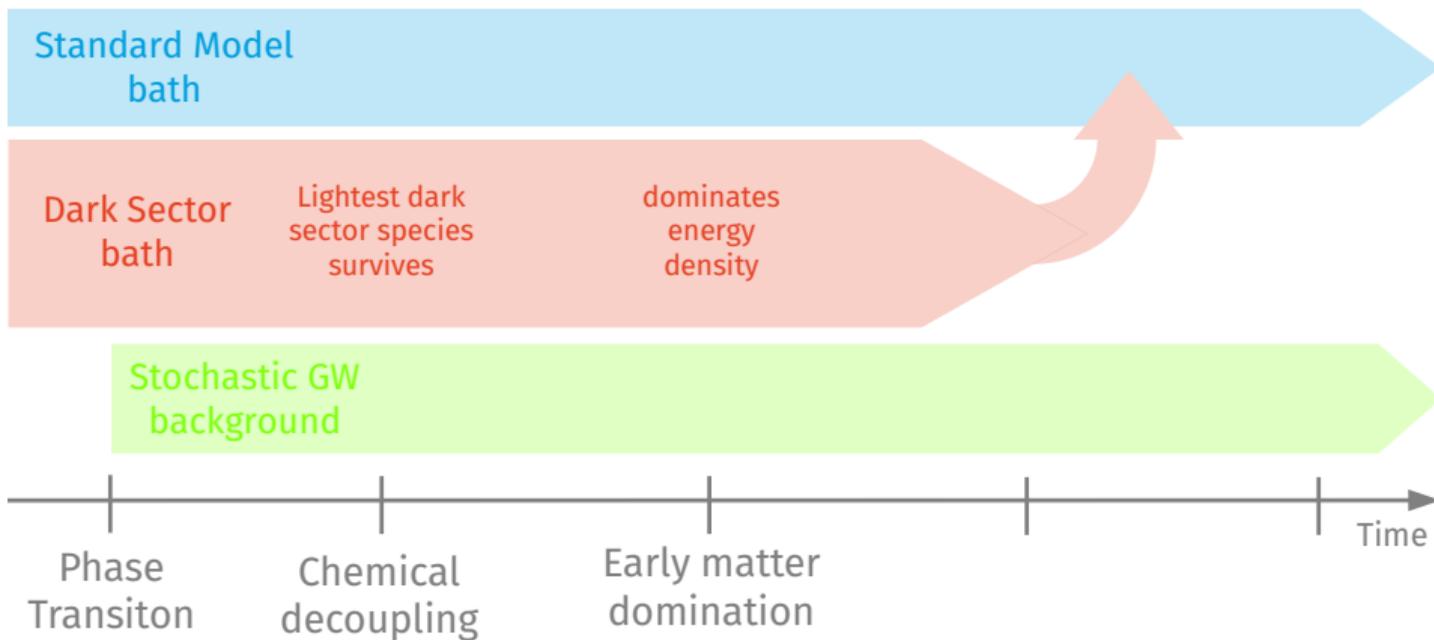
Long-lived dark sector evolution after a phase transition.



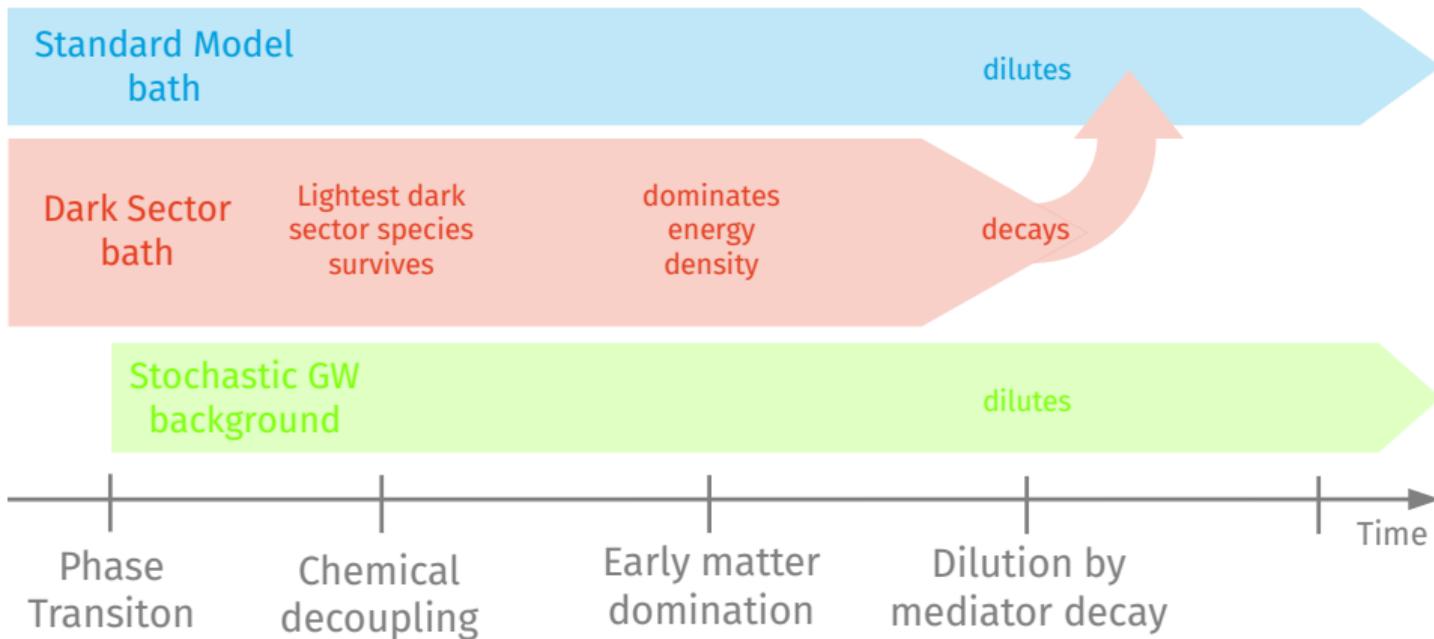
Long-lived dark sector evolution after a phase transition.



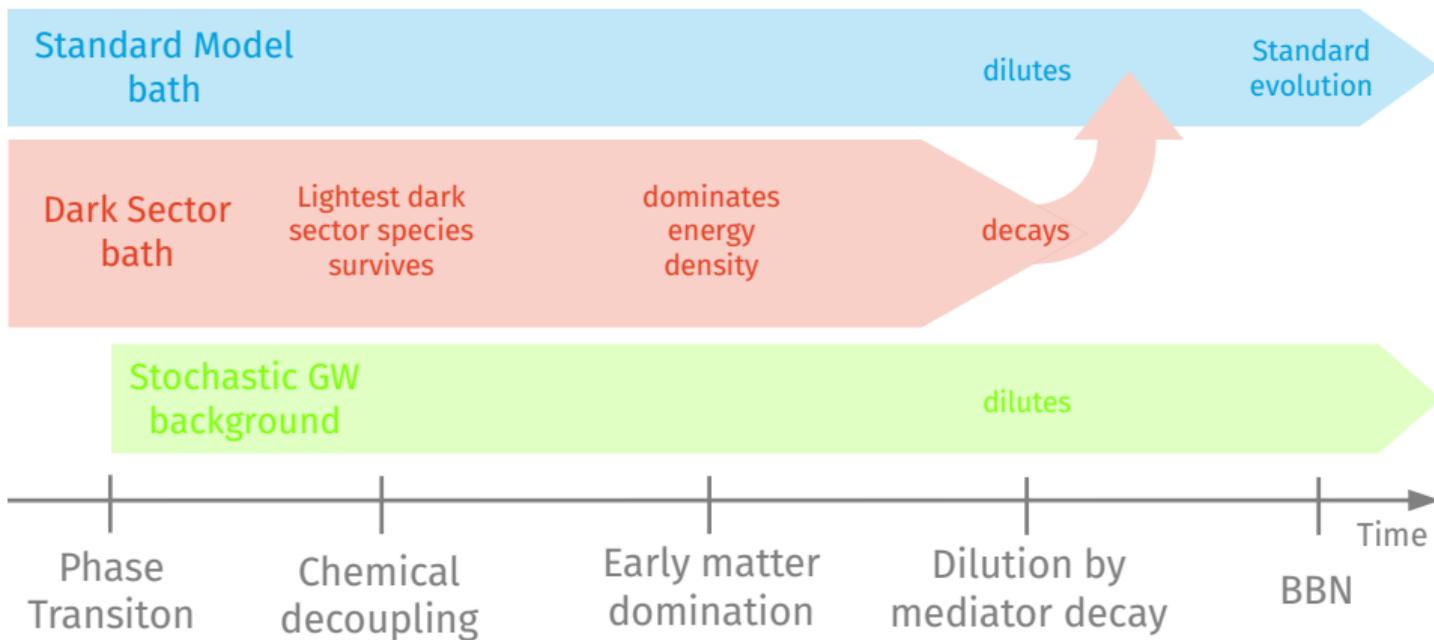
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Long-lived dark sector evolution after a phase transition.



Long-lived dark sector evolution after a phase transition.



Sources of gravitational radiation: Some examples.

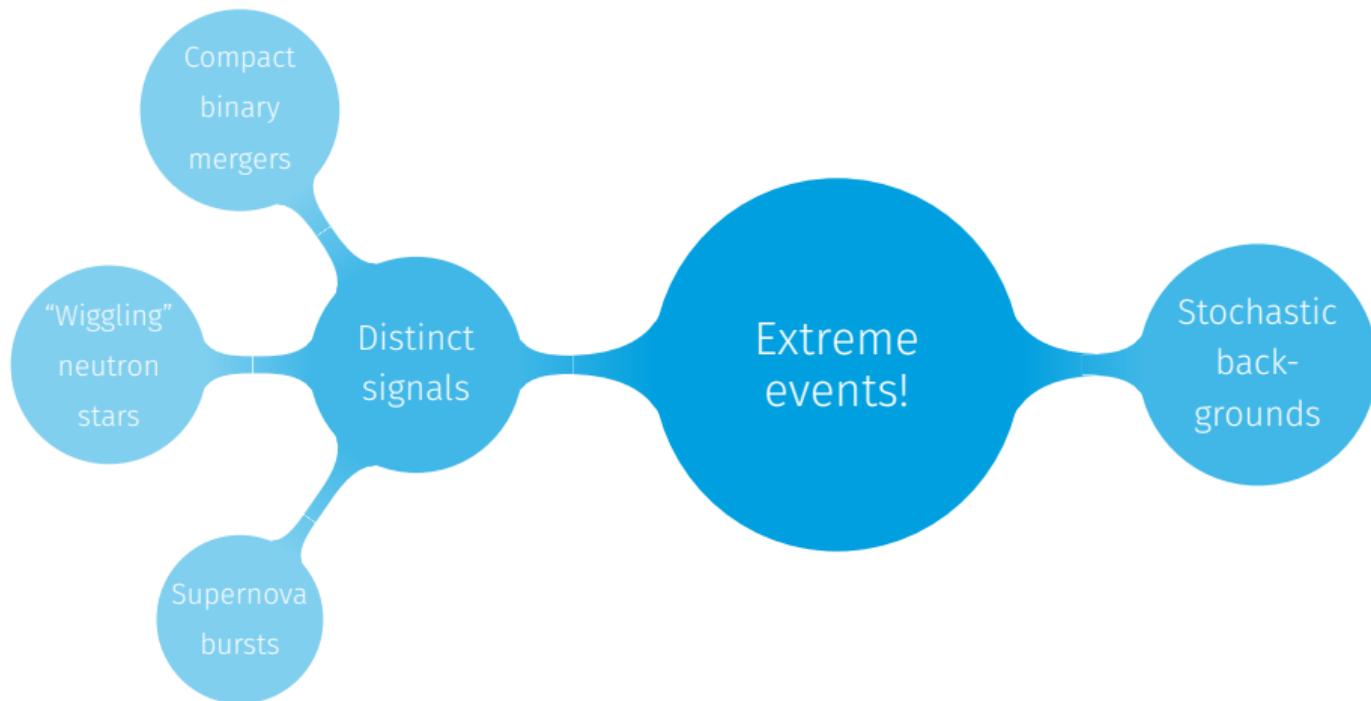


Extreme
events!

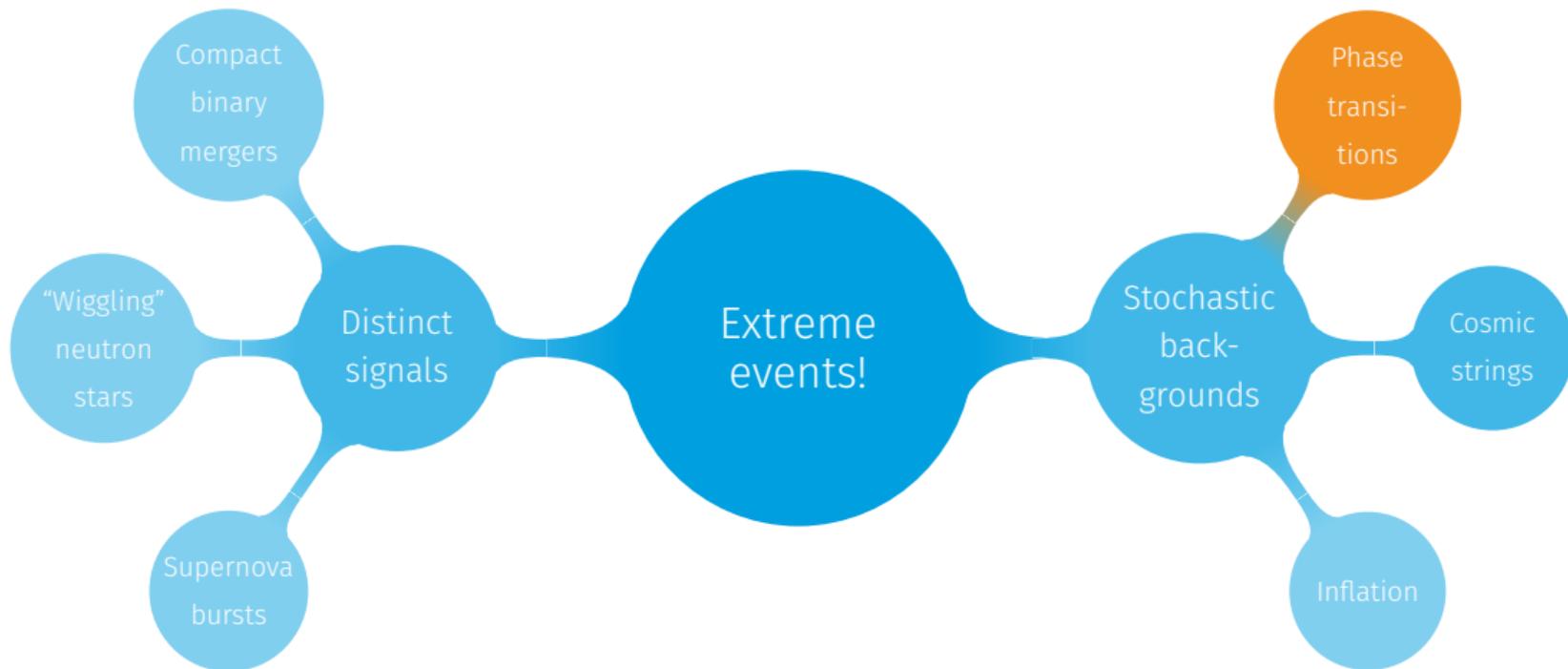
Sources of gravitational radiation: Some examples.



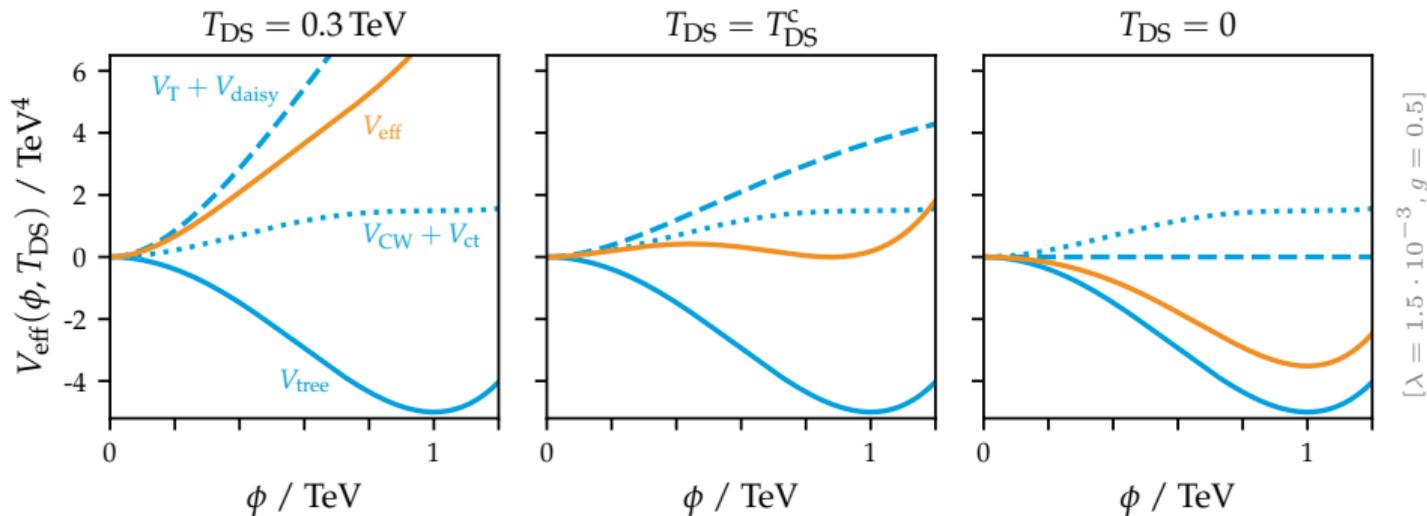
Sources of gravitational radiation: Some examples.



Sources of gravitational radiation: Some examples.



The dark photon model.



Add a $U(1)_D$ to the SM gauge groups. Its gauge boson, the “dark photon”, gets massive when a “dark Higgs” obtains $\phi \neq 0$. Effective potential controlled by the tree-level VEV v , dark Higgs quartic coupling λ and gauge coupling g .

Parametrization of the GW signal.

Assuming strong¹ phase transitions, the GW spectrum can be parameterized by

$$h^2 \Omega_{\text{GW}}(f) \simeq \frac{\mathcal{O}(10^{-6})}{D^{4/3}} \left(\frac{\alpha}{1+\alpha} \right)^2 \left(\frac{\beta}{H} \right)^{-2} \frac{3.8 (f/f_p)^{2.8}}{1 + 2.8 (f/f_p)^{3.8}}, \quad \text{where}$$

$$D \equiv \frac{g_{\text{eff},s}^{\text{SM,nuc}}}{g_{\text{eff},s}^{\text{tot,nuc}}} D_{\text{SM}} \quad \text{and} \quad f_p \simeq \frac{\mathcal{O}(10 \mu\text{Hz})}{D^{1/3}} \left(\frac{\beta}{H} \right) \left(\frac{T_{\text{SM}}^{\text{nuc}}}{100 \text{ GeV}} \right)$$

↪ GW spectrum fixed by the transition strength α , the inverse time scale β/H , the nucleation temperature T_{SM}^{n} and the dilution factor D

¹This is only to get an intuition, the actually performed calculations are more involved

The out-of-equilibrium decay of a dark mediator.

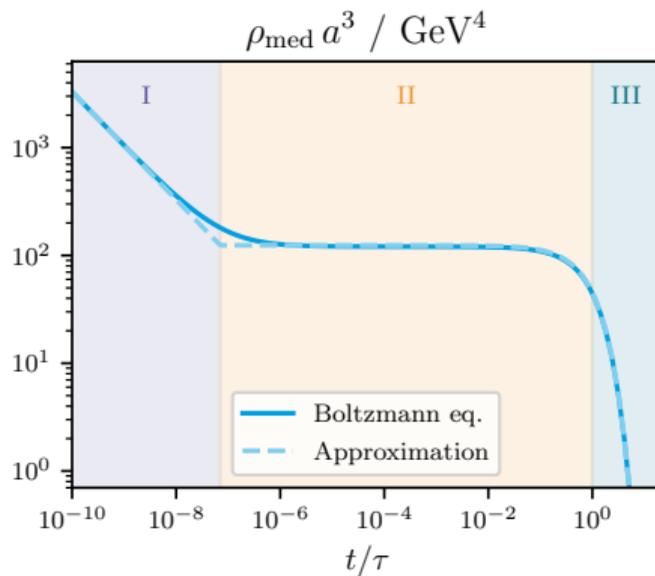
Evolution of the lightest dark sector state (“mediator”) after chemical decoupling:

$$\dot{\rho}_{\text{med}} \simeq -3 \zeta H \rho_{\text{med}} - \frac{\rho_{\text{med}}}{\tau}$$

with

$$\zeta = 1 + \frac{P_{\text{med}}}{\rho_{\text{med}}} = \begin{cases} 4/3 & \text{rel.} \\ 1 & \text{non-rel.} \end{cases}$$

Three phases: Relativistic, non-relativistic and decaying mediator



The out-of-equilibrium decay of a dark mediator.

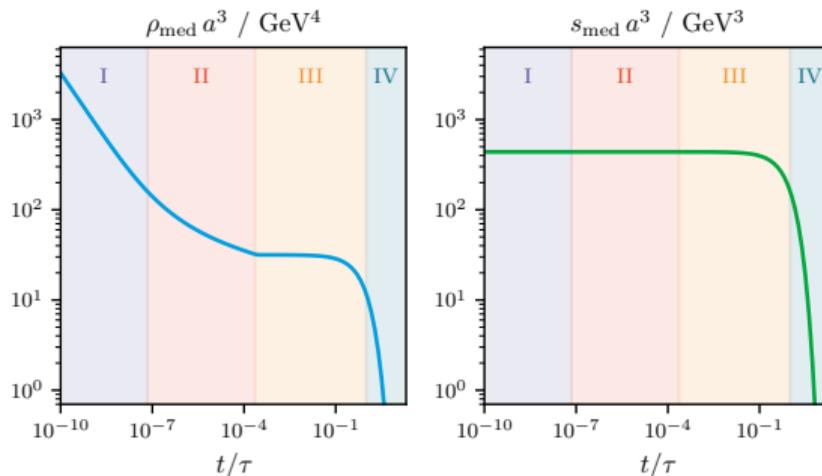
Number-changing processes of the mediator lead to a “cannibalistic” phase with $\mu_{\text{med}} = 0$. Therefore, the unique function $\rho_{\text{med}}(s_{\text{med}})$ exists.

We found:

$$\zeta = \begin{cases} \frac{d \ln \rho_{\text{med}}}{d \ln s_{\text{med}}} & 3 \rightarrow 2 \text{ efficient} \\ 1 & 3 \rightarrow 2 \text{ inefficient} \end{cases}$$

During cannibalism, ζ goes smoothly from $4/3$ to 1 .

Four phases: Relativistic, cannibalistic, non-relativistic and decaying mediator



Describing the dark sector in thermal equilibrium.

For several dark sector species in thermal equilibrium: can define effective DOFs

$$\rho_{\text{tot}}(T_{\text{SM}}) = \left[g_{\text{eff},\rho}^{\text{SM}}(T_{\text{SM}}) + g_{\text{eff},\rho}^{\text{DS}}(T_{\text{SM}}) \xi^4(T_{\text{SM}}) \right] \frac{\pi^2}{30} T_{\text{SM}}^4$$
$$s_{\text{tot}}(T_{\text{SM}}) = \left[g_{\text{eff},s}^{\text{SM}}(T_{\text{SM}}) + g_{\text{eff},s}^{\text{DS}}(T_{\text{SM}}) \xi^3(T_{\text{SM}}) \right] \frac{2\pi^2}{45} T_{\text{SM}}^3$$

Describing the dark sector in thermal equilibrium.

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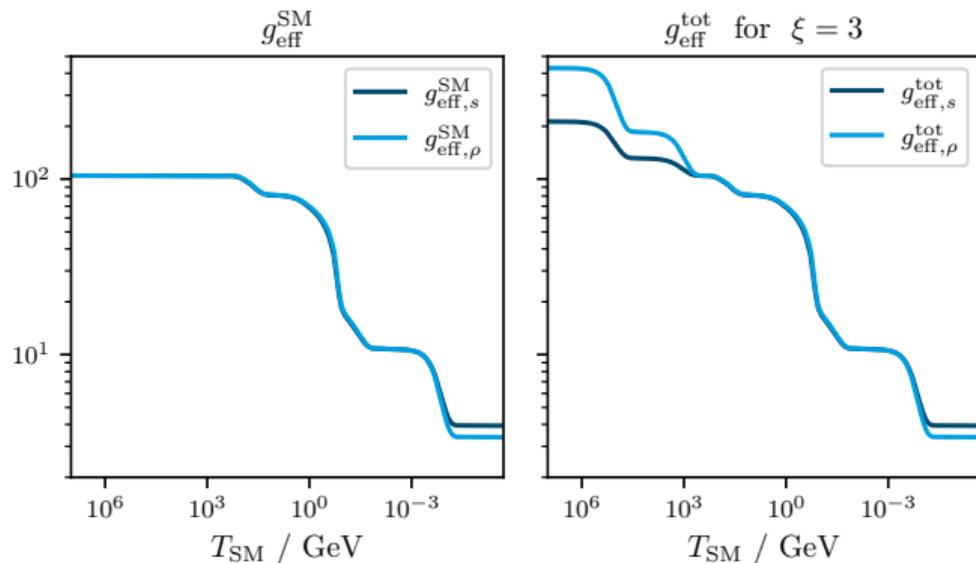
As entropy is conserved separately in the two baths, the temperature ratio follows

$$\xi(T_{\text{SM}}) = \tilde{\xi} \left(\frac{g_{\text{eff},s}^{\text{SM}}}{\tilde{g}_{\text{eff},s}^{\text{SM}}} \right)^{1/3} \left(\frac{\tilde{g}_{\text{eff},s}^{\text{DS}}}{g_{\text{eff},s}^{\text{DS}}} \right)^{1/3}$$

When SM particles annihilate ξ decreases, since the SM becomes hotter.

When dark sector DOF decrease ξ increases, since the DS becomes hotter.

Describing the dark sector in thermal equilibrium.



Example: Thermal evolution of a hot ($\xi = 3$) dark sector consisting of a dark photon ($m_{\text{DP}} = 10^6$ GeV) and a dark Higgs boson ($m_{\text{DH}} = 10^4$ GeV).

Describing the dark sector in thermal equilibrium.



This description breaks down when the dark species no longer follow their equilibrium (Bose-Einstein) distributions.

In our setup this occurs when the lightest dark sector species has a long lifetime.

Example: Thermal evolution of a hot ($\xi = 3$) dark sector consisting of a dark photon ($m_{\text{DP}} = 10^6$ GeV) and a dark Higgs boson ($m_{\text{DH}} = 10^4$ GeV).

First-order phase transitions in thermal field theory.

To demonstrate construction of $V_{\text{eff}}(\phi, T)$, take the toy-model Lagrangian...

$$\mathcal{L} = \frac{1}{2} (\partial_\mu \phi) (\partial^\mu \phi) - V_{\text{tree}}(\phi)$$

$$\text{with } V_{\text{tree}}(\phi) = -\frac{1}{2}\mu^2\phi^2 + \frac{\lambda}{4}\phi^4$$

... and consider all 1-loop 1-PI graphs:

$$V_{\text{eff},\Phi}^{1\text{-loop}}(\phi) = \left[\phi^2 \text{ (circle with 1 vertex) } + \phi^4 \text{ (circle with 2 vertices) } + \phi^6 \text{ (circle with 3 vertices) } + \dots \right]_{p=0}$$

First-order phase transitions in thermal field theory.

And calculate 1-loop effective potential with $m^2(\phi) = \partial_\phi^2 V_{\text{tree}}(\phi) = -\mu^2 + 3\lambda\phi^2$

$$\begin{aligned} V_{\text{eff}}(\phi, T) &= \frac{1}{2} \int \frac{d^4 k_E}{(2\pi)^4} \log [k_E^2 + m^2(\phi)] && \text{with } k_E^0 \text{ being } \frac{2\pi}{T}\text{-periodic} \\ &= \frac{T}{2} \sum_n \int_{\mathbf{k}} \log \left[\left(\frac{2\pi n}{T} \right)^2 + E_k^2 \right] && \text{with } E_k = \sqrt{k^2 + m^2(\phi)} \\ &= \int_{\mathbf{k}} \left[\frac{E_k}{2} + T \log \left\{ 1 - e^{-E_k/T} \right\} \right] \\ &= V_{\text{CW}}(\phi) + V_{\text{T}}(\phi, T) \end{aligned}$$

Interpretation: V_{tree} is the classical energy density contained in a background field ϕ , $V_{\text{CW}}(+V_{\text{T}})$ is the vacuum energy density of a quantum field living in this background, which is completely analogous to the zero-point energy of a harmonic oscillator (in a thermal bath)

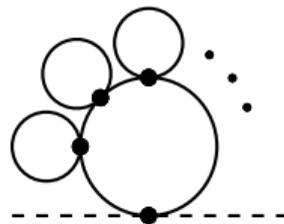
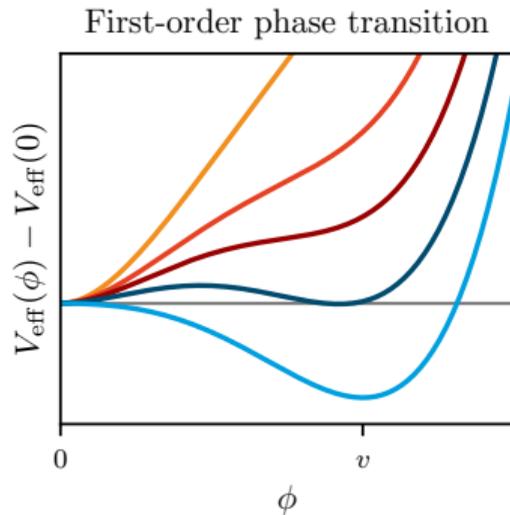
First-order phase transitions in thermal field theory.

$$\begin{aligned} V_T &= \int_{\mathbf{k}} T \log \left\{ 1 - e^{-E_k/T} \right\} \\ &= -\frac{\pi^2 T^4}{90} + \frac{T^2 m^2(\phi)}{24} - \frac{T m^3(\phi)}{12\pi} + \dots \end{aligned}$$

However, around T_c , V_{eff} is dominated by > 1 -loop effects. “Daisies” dominate:

$$V_{\text{daisy}} = -\frac{T}{12\pi} \left[(m^2(\phi) + \Pi(T))^{3/2} - m^3(\phi) \right]$$

And cancel the potential barrier in V_{eff} . But: Transversal gauge boson component doesn't acquire $\Pi(T)$. \rightsquigarrow Gauge bosons can save potential barrier and thus FOPTs.



First-order phase transitions in thermal field theory.

Summary:

$$V_{\text{eff}}^{1\text{-loop}}(\phi, T) = V_{\text{tree}}(\phi) + V_{\text{CW}}(\phi) + V_{\text{ct}}(\phi) + V_{\text{T}}(\phi, T) + V_{\text{daisy}}(\phi, T)$$

Coleman-Weinberg
potential and its
counter-terms

1-loop thermal
corrections

Daisy corrections,
dominate at T_c

How to get a thermal FOPT?

- Need scalar charged under gauge group with massive gauge bosons
- Dominant $V_{\text{tree}} + V_{\text{CW}}$ contributions can always destroy potential barrier, though \rightsquigarrow as in SM with too high m_h forbidding FOPT

First-order phase transitions in thermal field theory.

$$V_{\text{eff}}^{1\text{-loop}}(\phi, T) = V_{\text{tree}} + V_{\text{CW}} + V_{\text{ct}} + V_T + V_{\text{daisy}}$$

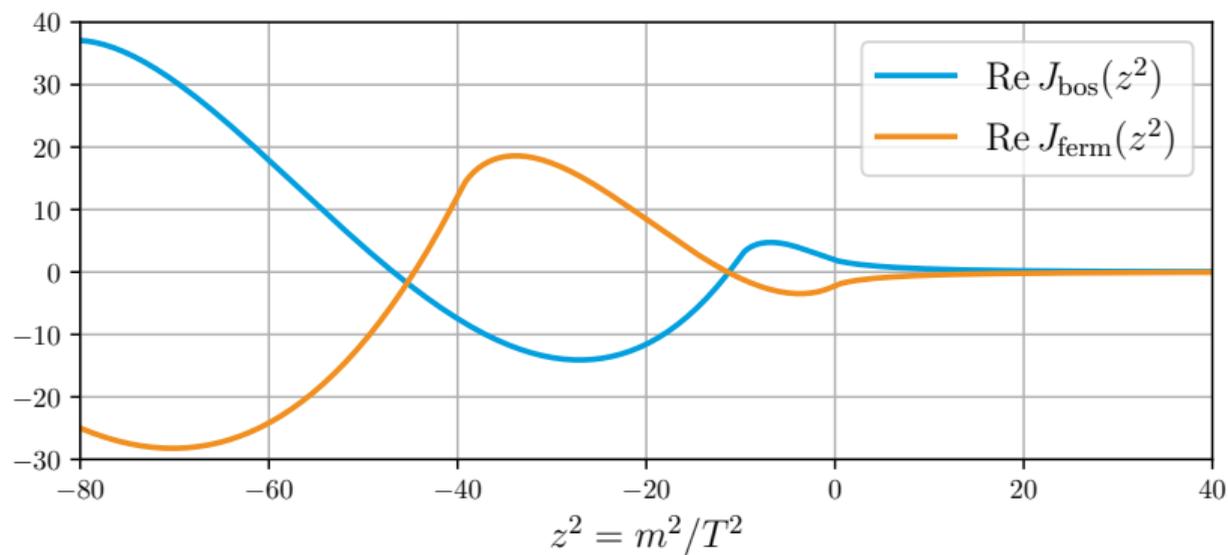
has the individual contributions

$$V_{\text{CW}}(\phi) = \sum_x \eta_x n_x \frac{m_x^4(\phi)}{64 \pi^2} \left[\ln \frac{m_x^2(\phi)}{\Lambda^2} - C_a \right],$$

$$V_T(\phi, T) = \frac{T^4}{2 \pi^2} \sum_x \eta_x n_x J_{\eta_x} \left(\frac{m_x^2(\phi)}{T^2} \right),$$

$$V_{\text{daisy}}(\phi, T) = -\frac{T}{12 \pi} \sum_b n_b^{\perp} \left[(m^2(\phi) + \Pi(T))_b^{3/2} - (m^2(\phi))_b^{3/2} \right]$$

Thermal functions.



Bubble expansion.

Euclidean action of scalar field

$$S[\phi] = \int d^4x_E \left[\frac{1}{2} \left(\frac{\partial \phi}{\partial \tau} \right)^2 + \frac{(\nabla \phi)^2}{2} + V_{\text{eff}}(\phi) \right]$$

Minimizing for O(4)-case gives

$$\frac{d^2 \phi}{d\rho^2} + \frac{3}{\rho} \frac{d\phi}{d\rho} = V'_{\text{eff}}(\phi)$$

At finite T and in real space:

$$\frac{d^2 \phi}{dr^2} + \frac{2}{r} \frac{d\phi}{dr} = V'_{\text{eff}}(\phi, T)$$

Can be solved by overshoot-undershoot method

Bubble formation and thermal tunneling.

Nucleation rate: $\Gamma = \mathcal{A}e^{-S_4}$ with

$$S_4 = \int \frac{1}{2} \left(\frac{d\phi}{d\tau} \right)^2 + \frac{1}{2} (\nabla\phi)^2 + V_{\text{eff}}(\phi) d^4x_E$$

and $\mathcal{A} \sim T^4$. Extremalization yields KG equation with classical potential source:

$$\frac{d^2\phi}{d\tau^2} + \Delta\phi = \frac{dV_{\text{eff}}}{d\phi}$$

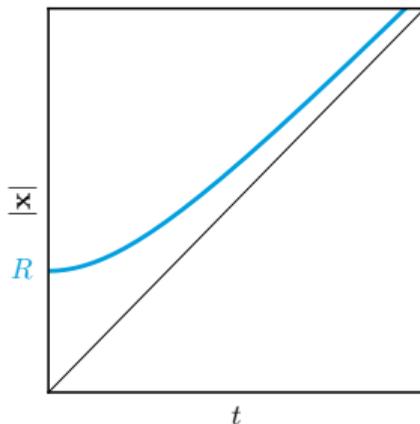
with b.c. $\phi(\rho \rightarrow \infty) \rightarrow 0$ and

$\phi'(\rho = 0) = 0$ where $\rho \equiv \sqrt{\tau^2 + |\mathbf{x}|^2}$.

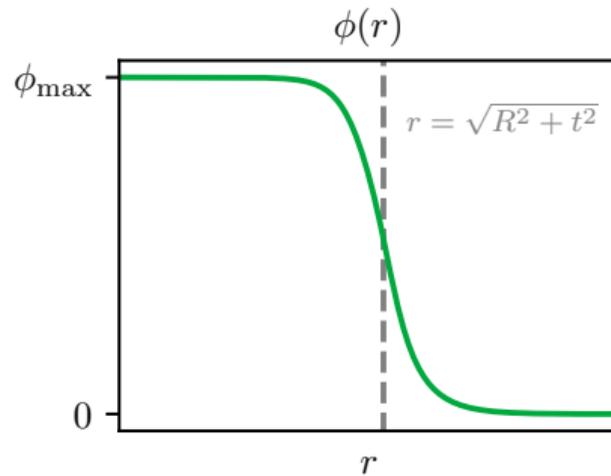
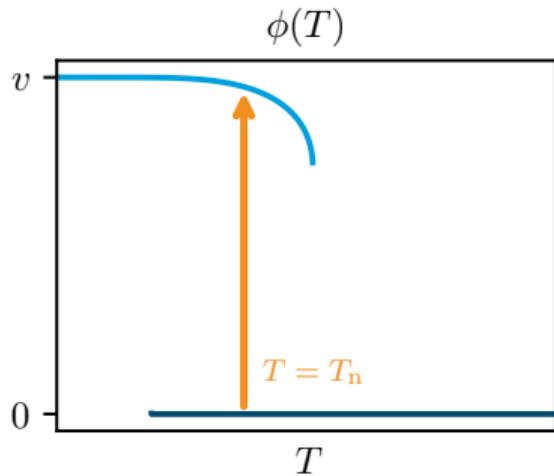
Solutions typically $O(4)$ symmetric:

$$\frac{d^2\phi}{d\rho^2} + \frac{3}{\rho} \frac{d\phi}{d\rho} = \frac{dV_{\text{eff}}}{d\phi}$$

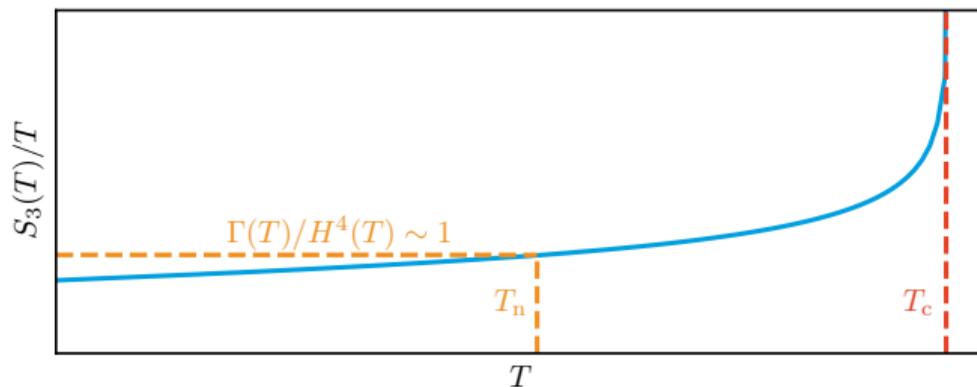
In 3-space: $r = |\mathbf{x}| = \sqrt{\rho^2 - c^2t^2} \rightsquigarrow$
Nucleation and expansion with $v \rightarrow c$



Temperature dependence of potential minima and bubble profile.



Nucleation criterion.



The nucleation condition $\Gamma(T_n) H^{-4}(T_n) = 1$ gives

$$\left. \frac{S_3(T)}{T} \right|_{T=T_n} \sim 146 - 2 \ln \left(\frac{g_{\text{eff},\rho}^{\text{tot}}(T_n)}{100} \right) - 4 \ln \left(\frac{T_n}{100 \text{ GeV}} \right)$$

Can be solved by repeated evaluation of S_3/T and subsequent minimization.

GW parameter calculation.

Radiation energy density at nucleation

$$\rho_R = \frac{\pi^2}{30} \left(g_{\text{eff},\rho}^{\text{SM},n} + g_{\text{eff},\rho}^{\text{DS},n} \xi^4 \right) (T_{\text{SM}}^n)^4$$

Transition strength

$$\alpha^{(\text{DS})} = \frac{1}{\rho_R^{(\text{DS})}} \left(-\Delta V + \frac{1}{4} T_{\text{DS}}^n \left. \frac{\partial \Delta V}{\partial T} \right|_{T_{\text{DS}}^n} \right)$$

Inverse time scale

$$\frac{\beta}{H} = T_{\text{DS}}^n \left. \frac{dS_E(T)}{dT} \right|_{T_{\text{DS}}^n}$$

Critical transition strength for runaway bubbles ($\alpha^{\text{DS}} > \alpha_{\infty}^{\text{DS}}$)

$$\alpha_{\infty}^{\text{DS}} = \frac{(T_{\text{DS}}^n)^2}{\rho_R^{\text{DS}}} \left(\sum_{i=\text{bos}} n_i \frac{\Delta m_i^2}{24} + \sum_{i=\text{fer}} n_i \frac{\Delta m_i^2}{48} \right)$$

$$\Omega_{\text{GW}}(f) = \frac{1}{\rho_c} \frac{d\rho_{\text{GW}}(f)}{d \log f} \simeq \sum \mathcal{N} \Delta \left(\frac{\kappa \alpha}{1 + \alpha} \right)^p \left(\frac{H}{\beta} \right)^q s(f)$$

	Scalar field Ω_ϕ	Sound waves Ω_{SW}	Turbulence Ω_{turb}
\mathcal{N}	1	$1.59 \cdot 10^{-1}$	$2.01 \cdot 10^1$
κ	κ_ϕ	κ_{SW}	$\varepsilon_{\text{turb}} \kappa_{\text{SW}}$
p	2	2	$\frac{3}{2}$
q	2	1	1
Δ	$\frac{0.11 v_w^3}{0.42 + v_w^2}$	v_w	v_w
f_p	$\frac{0.62 \beta}{1.8 - 0.1 v_w + v_w^2}$	$\frac{2\beta}{\sqrt{3} v_w}$	$\frac{3.5\beta}{2 v_w}$
$s(f)$	$\frac{3.8(f/f_p)^{2.8}}{1 + 2.8(f/f_p)^{3.8}}$	$(f/f_p)^3 \left(\frac{7}{4 + 3(f/f_p)^2} \right)^{7/2}$	$\frac{(f/f_p)^3}{(1 + f/f_p)^{11/3} [1 + 8\pi(f/H)]}$

Redshift and dilution of the GW background.

After its emission, the GW signal gets red-shifted:

$$h^2 \Omega_{\text{GW}}(f) = \mathcal{R}h^2 \Omega_{\text{GW}}^n \left(\frac{a_0}{a_n} f \right)$$

Energy density:

$$\mathcal{R}h^2 \simeq \frac{2.4 \cdot 10^{-5}}{D_{\text{SM}}^{4/3}} \left(\frac{g_{\text{eff},s}^{\text{SM},0}}{g_{\text{eff},s}^{\text{SM},n}} \right)^{4/3} \frac{g_{\text{eff},\rho}^{\text{tot},n}}{2}$$

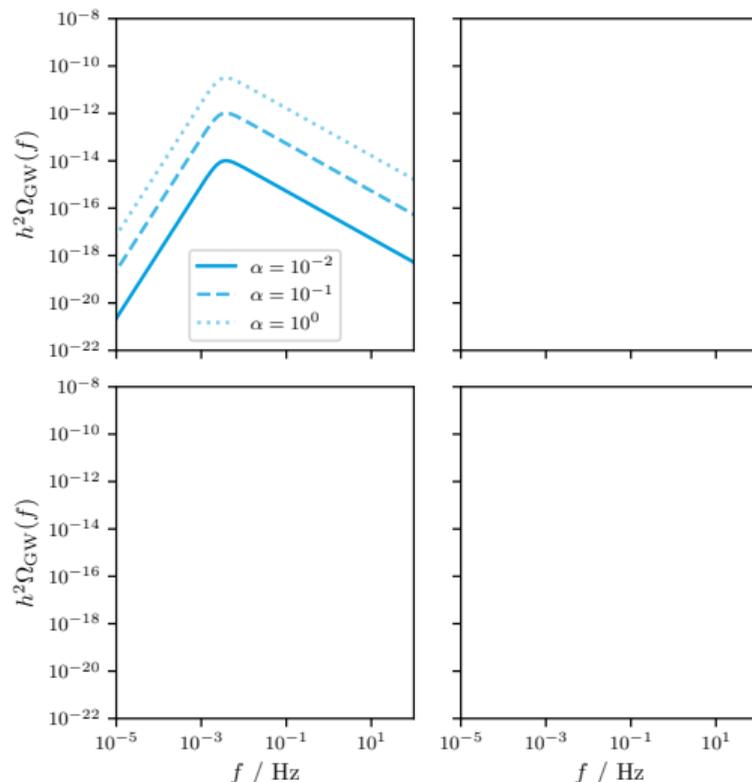
Frequency:

$$\frac{a_0}{a_n} = D_{\text{SM}}^{1/3} \left(\frac{g_{\text{eff},s}^{\text{SM},n}}{g_{\text{eff},s}^{\text{SM},0}} \right)^{1/3} \frac{T_{\text{SM}}^n}{T_{\text{SM}}^0}$$

Transition strength:

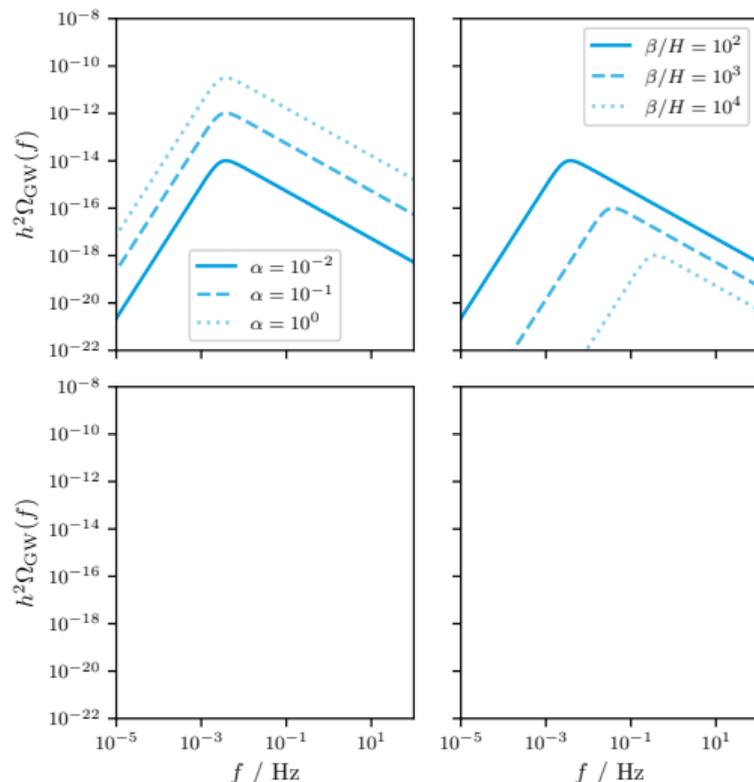
$$\alpha = \frac{\Delta\theta}{\rho_{\text{rad}}^n}$$

relates the latent heat (trace of the energy momentum tensor between the two phases) $\Delta\theta$ of the transition with the energy density ρ_{rad}^n of the surrounding heat bath. For fixed T_{DS}^n : $\rho_{\text{rad}}^n \propto \xi_n^{-4}$. The transition strength thus grows $\propto \xi_n^4$.



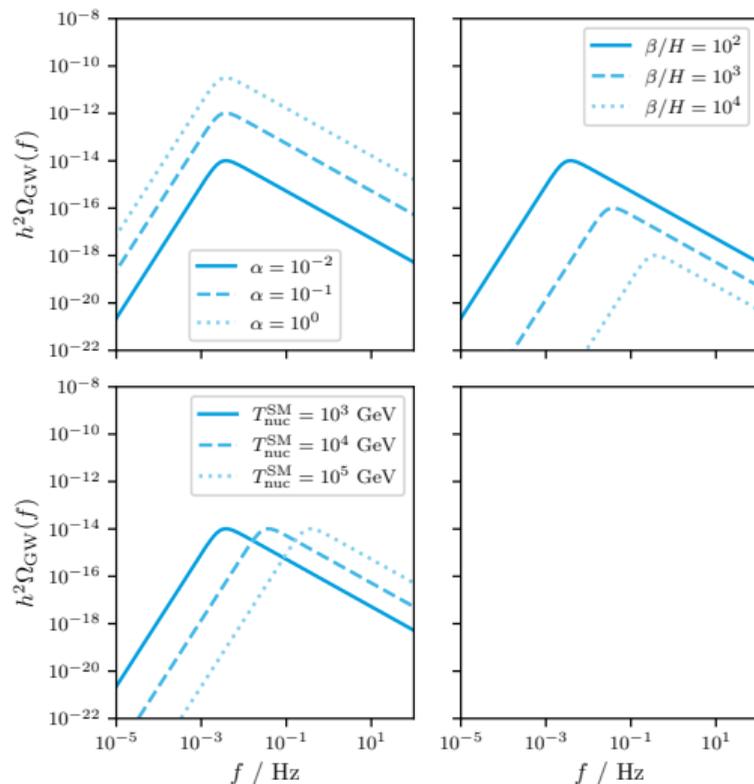
Inverse time scale:

The computation of β/H is complicated, but shows no relevant dependence of the temperature ratio between the sectors. Larger β/H indicate fast transitions. In that case, many small bubbles collide, resulting in weak signals at high frequencies.



Nucleation temperature:

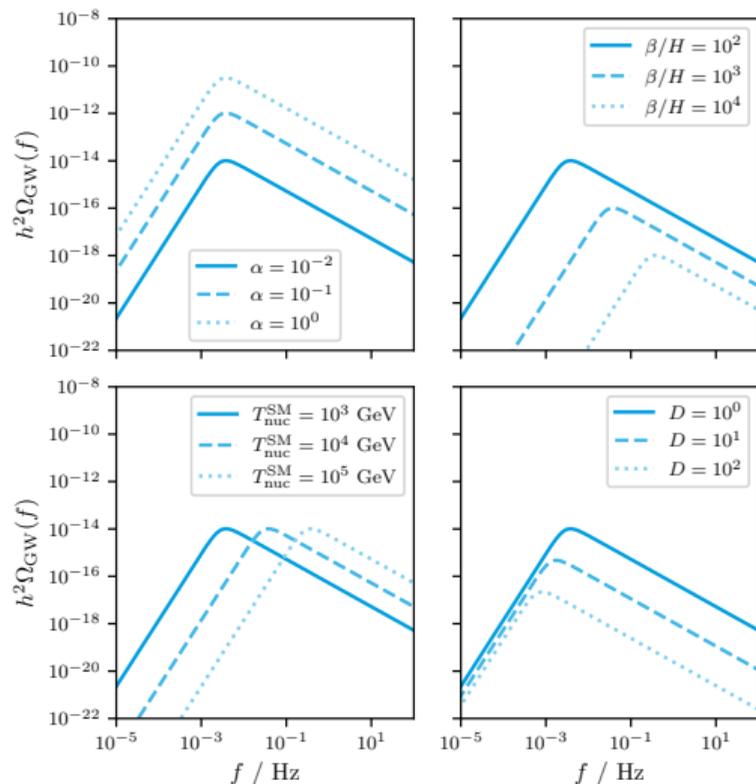
Keeping T_{DS}^{n} fixed, a larger temperature ratio ξ_{n} at nucleation leads to a lower T_{SM}^{n} . This corresponds to lower peak frequencies.



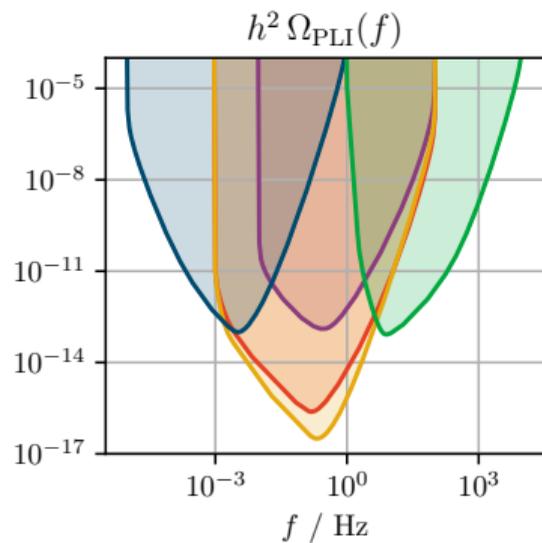
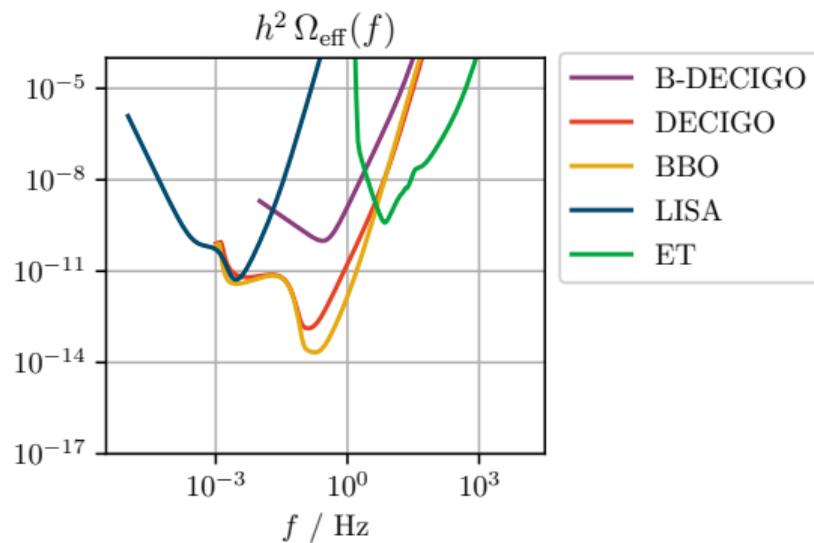
Parametrization of the GW signal.

Dilution:

The redshift to lower frequencies and signals strengths increases with the dilution factor. D grows with the temperature ratio ξ_n , as more energy is injected into the SM bath from the dark sector. Unlike D_{SM} , D saturates for high temperature ratios.



Experimental sensitivities.



Effective degrees of freedom.

$$g_{\text{eff},\rho}^x(T_x) \equiv \frac{\rho_x(T_x)}{\rho_{\text{bos}}^{\text{rel}}(T_x)|_{g=1}} = g_x \frac{15}{\pi^4} \int_{z_x}^{\infty} du_x \frac{u_x^2 \sqrt{u_x^2 - z_x^2}}{e^{u_x} \pm 1},$$

$$g_{\text{eff},P}^x(T_x) \equiv \frac{P_x(T_x)}{P_{\text{bos}}^{\text{rel}}(T_x)|_{g=1}} = g_x \frac{15}{\pi^4} \int_{z_x}^{\infty} du_x \frac{(u_x^2 - z_x^2)^{3/2}}{e^{u_x} \pm 1},$$

$$g_{\text{eff},s}^x(T_x) = \frac{3 g_{\text{eff},\rho}^x(T_x) + g_{\text{eff},P}^x(T_x)}{4},$$

where $u_x = \sqrt{m_x^2 + p^2}/T_x$ and $z_x = m_x/T_x$. Sum over all SM and DS species:

$$g_{\text{eff},\rho}^{\text{tot}} = g_{\text{eff},\rho}^{\text{SM}}(T_{\text{SM}}) + g_{\text{eff},\rho}^{\text{DS}}(T_{\text{SM}}) \xi^4(T_{\text{SM}})$$

$$g_{\text{eff},s}^{\text{tot}} = g_{\text{eff},s}^{\text{SM}}(T_{\text{SM}}) + g_{\text{eff},s}^{\text{DS}}(T_{\text{SM}}) \xi^3(T_{\text{SM}})$$

Mediator cannibalism.

Conserved comoving mediator entropy $s_{\text{med}} a^3 = \text{const}$ gives

$$\frac{d \ln s_{\text{med}}}{dt} = \frac{d \ln s_{\text{med}}}{d \ln \rho_{\text{med}}} \frac{\dot{\rho}_{\text{med}}}{\rho_{\text{med}}} = -3 H(t) ,$$

from which follows that

$$\dot{\rho}_{\text{med}} = -3 \frac{d \ln \rho_{\text{med}}}{d \ln s_{\text{med}}} H(t) \rho_{\text{med}}(t) .$$

For $\mu_{\text{med}} = 0$, one can find function $\rho_{\text{med}}(s_{\text{med}})$, independent of particle species:

$$\frac{d \ln \rho_{\text{med}}}{d \ln s_{\text{med}}} = \frac{d \rho_{\text{med}}}{d s_{\text{med}}} \frac{s_{\text{med}}}{\rho_{\text{med}}} = \frac{d \bar{\rho}_{\text{med}}}{d \bar{s}_{\text{med}}} \frac{\bar{s}_{\text{med}}}{\bar{\rho}_{\text{med}}} = \frac{d \ln \bar{\rho}_{\text{med}}}{d \ln \bar{s}_{\text{med}}} = \frac{d \ln \bar{\rho}}{d \ln \bar{s}}$$

with $\bar{s}_{\text{med}} \equiv 2 \pi^2 s_{\text{med}} / (g_{\text{med}} T_{\text{DS}}^3)$ and $\bar{\rho}_{\text{med}} \equiv 2 \pi^2 \rho_{\text{med}} / (g_{\text{med}} T_{\text{DS}}^4)$.

Mediator cannibalism.

That yields

$$\dot{\rho}_{\text{med}} \simeq -3 \zeta H \rho_{\text{med}} - \rho_{\text{med}}/\tau$$

with

$$\zeta(t) = \begin{cases} \frac{d \ln \bar{\rho}}{d \ln \bar{s}}(\rho_{\text{med}}) & \text{for } \Gamma_{\text{nc}}(t) \geq H(t) \\ 4/3 & \text{for } \Gamma_{\text{nc}}(t) < H(t), \quad t < \tilde{t} , \\ 1 & \text{for } \Gamma_{\text{nc}}(t) < H(t), \quad t \geq \tilde{t} \end{cases}$$

where $\tilde{t} \simeq 7.0 t_{\text{cd}} (T_{\text{DS}}^{\text{cd}}/m_{\text{med}})^2$ denotes the time when the mediator gets non-relativistic. Number changing process rate is approximated by

$$\Gamma_{\text{nc}} \simeq \Gamma_{32} \simeq \langle \sigma_{32} v^2 \rangle n_{\text{med}}^2$$

The averaged cross section reads

$$\langle \sigma_{32} v^2 \rangle = \frac{25 \sqrt{5} \pi^2}{5184} \frac{\alpha_{32}^3}{m_{\text{med}}^5} + \mathcal{O} \left(\frac{T_{\text{DS}}}{m_{\text{med}}} \right).$$

where

$$(4 \pi \alpha_{32})^3 \equiv \left(\frac{\kappa_3}{m} \right)^2 \left[\left(\frac{\kappa_3}{m} \right)^2 + 3 \kappa_4 \right]^2$$

for a potential $V(\phi) = \frac{m^2}{2} \phi^2 + \frac{\kappa_3}{3!} \phi^3 + \frac{\kappa_4}{4!} \phi^4$. In our model: $\alpha_{32} = 2.3 \lambda$.

Coupled set of ODEs underlying the entropy injection.

$$\begin{aligned}\bar{a}' &= \frac{\bar{a}}{\theta_H} \sqrt{r + \frac{r_{\text{mat}}^{\text{cd}}}{\bar{a}^3} + \frac{r_{\text{rad}}^{\text{cd}}}{\bar{a}^4} \frac{\gamma}{\gamma_{\text{cd}}} \frac{\mathcal{S}}{\mathcal{G}^{1/3}}}, \\ \mathcal{S}' &= \frac{r \bar{a}^4}{r_{\text{rad}}^{\text{cd}}} \mathcal{G}^{1/3} \gamma_{\text{cd}}, \\ r' &= -r - 3 \frac{\bar{a}'}{\bar{a}} \zeta r, \\ \mathcal{G}' &= -\frac{3}{4} \frac{T_{\text{SM}}^{\text{cd}} \mathcal{G} \hat{\mathcal{G}}}{\mathcal{S}^{3/4} \bar{a}} \frac{4 \mathcal{S} \bar{a}' - \mathcal{S}' \bar{a}}{T_{\text{SM}}^{\text{cd}} \hat{\mathcal{G}} \mathcal{S}^{1/4} + 3 \mathcal{G}^{4/3} \bar{a}}, \\ \gamma' &= \hat{\gamma} T_{\text{SM}}^{\text{cd}} \frac{3 \mathcal{G} \bar{a} \mathcal{S}' - 12 \mathcal{G} \bar{a}' \mathcal{S} - 4 \mathcal{G}' \bar{a} \mathcal{S}}{12 \mathcal{G}^{4/3} \mathcal{S}^{3/4} \bar{a}^2}.\end{aligned}$$

with initial condition $\bar{a}_{\text{cd}} = \mathcal{S}_{\text{cd}} = r_{\text{cd}} = \mathcal{G}_{\text{cd}} = 1$ and γ_{cd} .

- Normalized scale factor $\bar{a} = a/a_{\text{cd}}$
- Characteristic time scale $\theta_H = \sqrt{3 m_{\text{Pl}}^2 \rho_{\text{med}}^{\text{cd}}/\tau^2}$
- Normalized mediator energy density
 $r = \rho_{\text{med}}/\rho_{\text{med}}^{\text{cd}}$
- Normalized initial DM density $r_{\text{mat}}^{\text{cd}} = \rho_{\text{DM}}^{\text{cd}}/\rho_{\text{med}}^{\text{cd}}$
- Normalized initial radiation energy density
 $r_{\text{rad}}^{\text{cd}} = \rho_{\text{rad}}^{\text{cd}}/\rho_{\text{med}}^{\text{cd}}$
- Normalized DOFs $\gamma = g_{\text{eff},\rho}^{\text{SM}}/g_{\text{eff},s}^{\text{SM}}$
- Normalized DOFs $\mathcal{G} = g_{\text{eff},s}^{\text{SM}}/g_{\text{eff},s}^{\text{SM,cd}}$
- Normalized SM entropy $\mathcal{S} = \left(S_{\text{SM}}/S_{\text{SM}}^{\text{cd}}\right)^{4/3}$

The $U(1)_D$ model in detail.

Lagrangian:

$$\mathcal{L} \supset |D_\mu \Phi|^2 + |D_\mu H|^2 - \frac{1}{4} B'_{\mu\nu} B'^{\mu\nu} - \frac{\epsilon}{2} B'_{\mu\nu} B^{\mu\nu} - V(\Phi, H) ,$$

$$D_\mu \Phi = (\partial_\mu + i g B'_\mu) \Phi ,$$

$$V_{\text{tree}}(\Phi, H) = -\mu^2 \Phi^* \Phi + \lambda (\Phi^* \Phi)^2 - \mu_H^2 H^\dagger H + \lambda_H (H^\dagger H)^2 + \lambda_p (\Phi^* \Phi) (H^\dagger H) .$$

Mass spectrum:

$$m_{(h, \phi)}^2(h, \phi) = \begin{pmatrix} -\mu_H^2 + 3 \lambda_H h^2 + \frac{\lambda_p}{2} \phi^2 & \lambda_p h \phi \\ \lambda_p h \phi & -\mu^2 + 3 \lambda \phi^2 + \frac{\lambda_p}{2} h^2 \end{pmatrix} ,$$

$$m_{G^0, G^+}^2(h, \phi) = -\mu_H^2 + \lambda_H h^2 + \frac{\lambda_p}{2} \phi^2 ,$$

$$m_\varphi^2(h, \phi) = -\mu^2 + \lambda \phi^2 + \frac{\lambda_p}{2} h^2 .$$

The $U(1)_D$ model in detail.

For $\lambda_p, \epsilon \rightarrow 0$ and $\mu^2 = \lambda v^2$, the field-dependent dark Higgs and dark photon masses are given by

$$m_{\text{DP}} = g \phi \stackrel{T=0}{=} g v ,$$

$$m_{\text{DH}} = \sqrt{2\lambda} \phi \stackrel{T=0}{=} \sqrt{2\lambda} v .$$

The corresponding Debye masses are

$$\Pi_{\Phi}(T_{\text{DS}}) = \left(\frac{\lambda}{3} + \frac{g^2}{4} \right) T_{\text{DS}}^2 ,$$

$$\Pi_{A'}^L(T_{\text{DS}}) = \frac{g^2}{3} T_{\text{DS}}^2 .$$

- Quartic dark Higgs coupling: λ
- $U(1)_D$ gauge coupling: g
- Dark Higgs lifetime: τ
- Dark Higgs VEV: $v = \frac{\mu}{\sqrt{\lambda}}$
- Temperature ratio: $\xi_n = \left. \frac{T_{\text{DS}}}{T_{\text{SM}}} \right|_n$

Signal-to-noise ratios for LISA and the ET.

Compute the overlap of the signals $h^2 \Omega_{\text{GW}}(f)$ and expected sensitivities $h^2 \Omega_{\text{obs}}(f)$ and weight it with the duration of the observation t_{obs} to obtain a signal-to-noise measure:

$$\rho^2 = t_{\text{obs}} \int_{f_{\text{min}}}^{f_{\text{max}}} df \left[\frac{h^2 \Omega_{\text{GW}}(f)}{h^2 \Omega_{\text{obs}}(f)} \right]^2$$

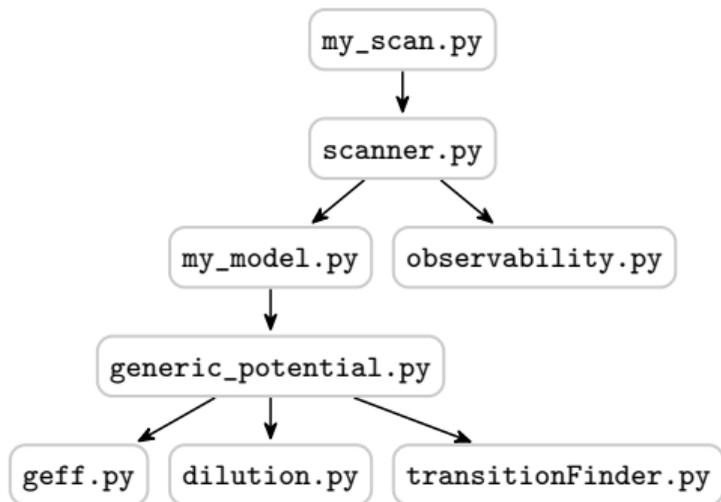
If ρ exceeds a certain threshold value for a given signal, the signal is observable.

To analyze the impact of ξ_n and τ on the observability of the signals produced by our model, consider the benchmark points

Benchmark point	λ	g	v
LISA	$1.5 \cdot 10^{-3}$	0.5	2 TeV
ET	$1.5 \cdot 10^{-3}$	0.5	10 PeV

Structure of TransitionListener.

Structure:



Example output:

