Turn up the volume – Listening to hot dark sector phase transitions.

Seminar talk at Carleton University

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Deutsches Elektronen Synchrotron (DESY)

Based on work with Fatih Ertas and Felix Kahlhöfer
published in `[2109.06208]`, `JCAP01(2022)???`

January 17, 2022

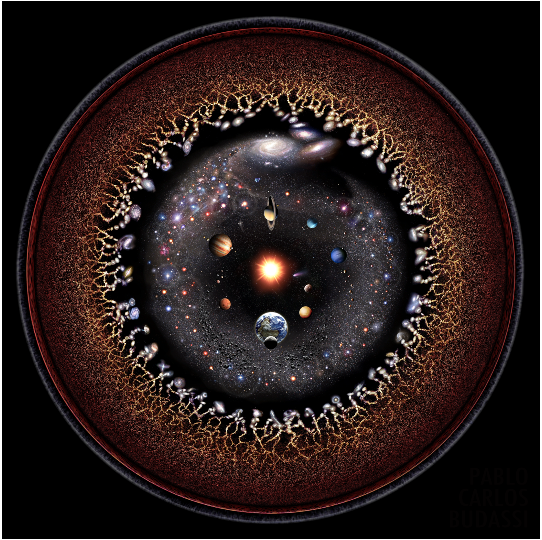
Outline of this talk.

1. Why dark sector phase transitions?
2. Thermal evolution of dark sectors after a phase transition
3. The dark photon model
4. Conclusions

[Camille Flammarion, 1888]

Why dark sector phase transitions?

What we know about our Universe.



[Pablo Carlos Budassi, 2020]

From CMB anisotropies: ΛCDM model

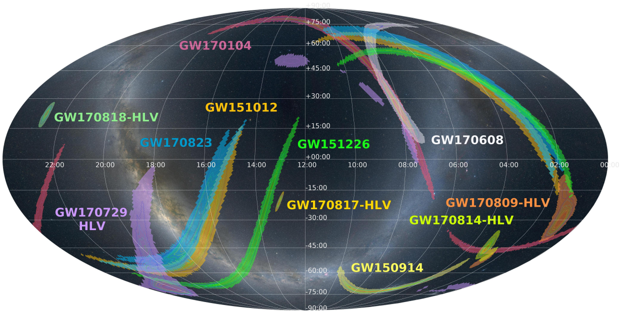
- Isotropic and homogeneous
- 13.8 billion years old
- Expands with a rate of about $68\,\mathrm{km\,s^{-1}\,Mpc^{-1}}$
- 95 % of today's energy content is dark!?

⇝ **What lies beyond the surface of last scattering?**

Gravitational waves as a “new” messenger.

- Observed 90 compact binary mergers since 2015
- Sensitivity will increase considerably with start of LISA, Einstein Telescope, etc.

⇝ **What will the stochastic gravitational wave background look like?**



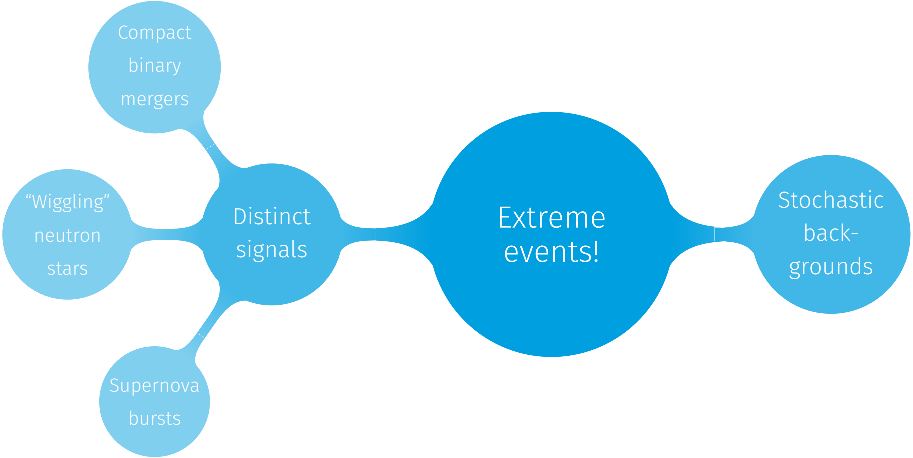
[LIGO, Virgo & KAGRA Collaboration, 2020]

Sources of gravitational radiation: Some examples.

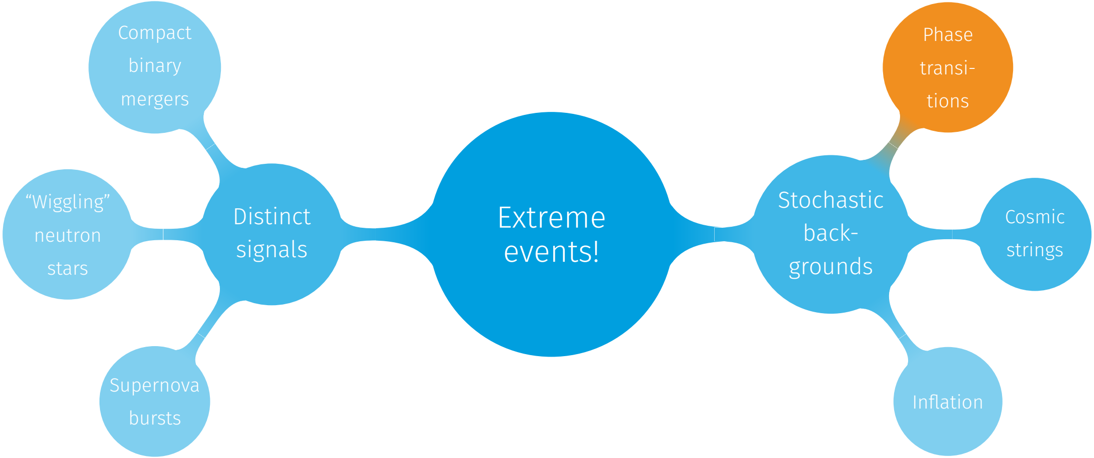
Extreme
events!

Sources of gravitational radiation: Some examples.

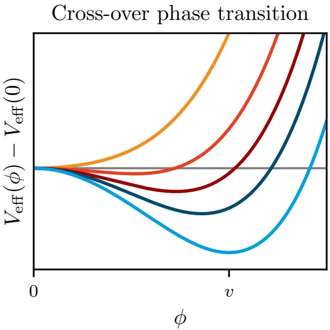
Sources of gravitational radiation: Some examples.

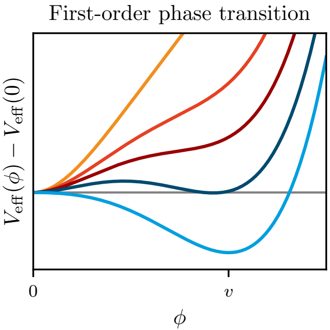


Sources of gravitational radiation: Some examples.



Cross-over and first-order phase transitions.



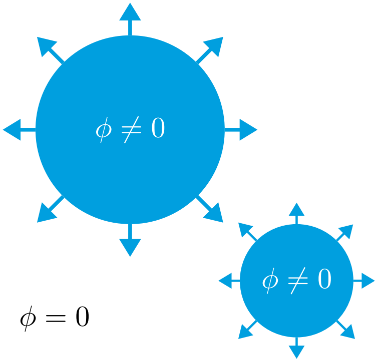


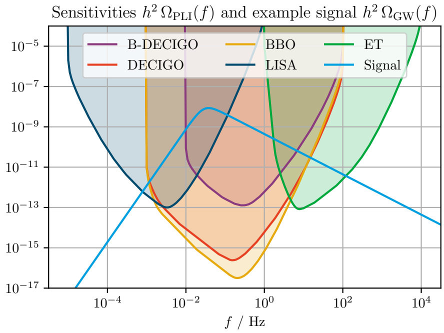
The scalar field "rolls down" from $\phi = 0$ to $\phi = v$, when the bath cools from high temperatures to low temperatures.

The scalar field tunnels to the true potential minimum ($\phi \neq 0$) to minimize its action ($\sim$ free energy).

Gravitational waves from first-order phase transitions.

Bubbles of the new phase nucleate, collide and perturb the surrounding plasma...





... giving rise to a stochastic gravitational wave background which could be observed in some years.

Objective of this talk.

We have seen that

- Primordial GWs could be used for "listening" beyond the CMB
- First-order phase transitions emit gravitational wave signals
- Majority of our Universe is "dark"

⇝ What kind of **dark sector** could produce observable GW signals?

Dark sector: particle bath without thermal contact to SM particles:

$$T_{\mathrm{DS}} = \xi \, T_{\mathrm{SM}}$$

Breitbach et al. [1811.11175] showed that cold ($\xi < 1$) dark sectors produce weak signals…

Objective of this talk.

We have seen that

- Primordial C
  "listening" b
- First-order p
  gravitationa
- Majority of c

⇝ What kind o
  produce observable GW signals?

ath without
articles:

SM

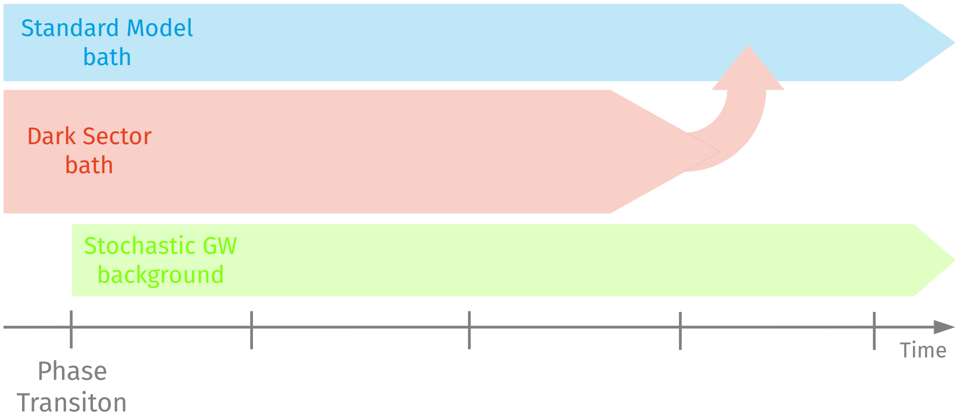
5] showed that
produce weak

Can hot $(\xi > 1)$ dark sector phase transitions emit observable GW signals?

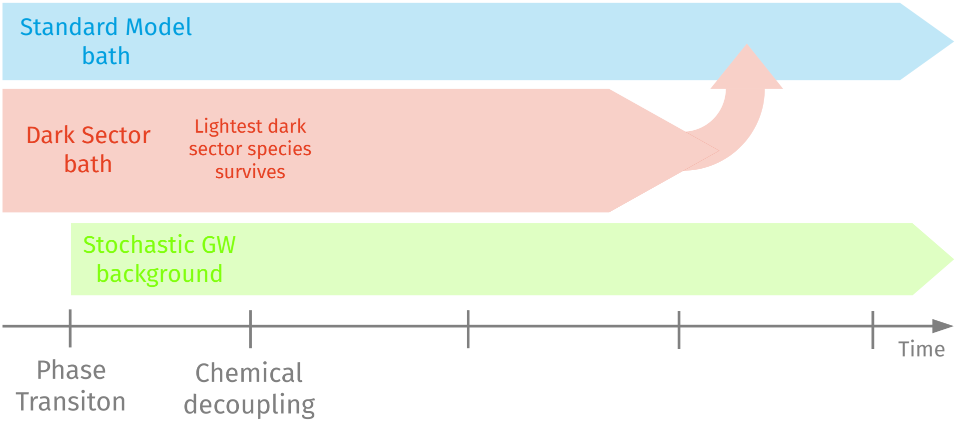
What happens when the dark sector finally decays to SM particles?

Thermal evolution of dark sectors after a phase transition.

Long-lived dark sector evolution after a phase transition.



Long-lived dark sector evolution after a phase transition.



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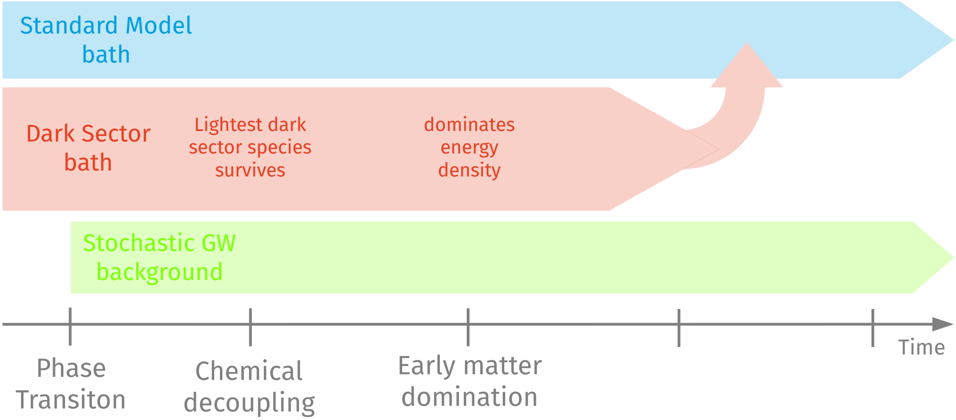

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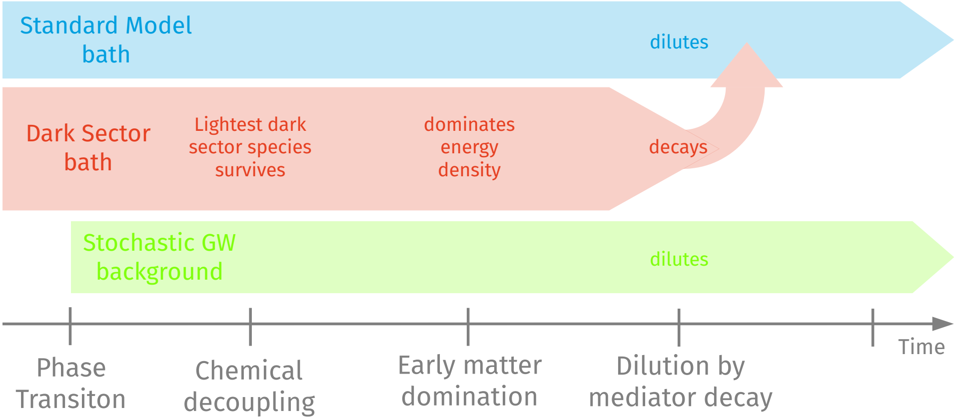

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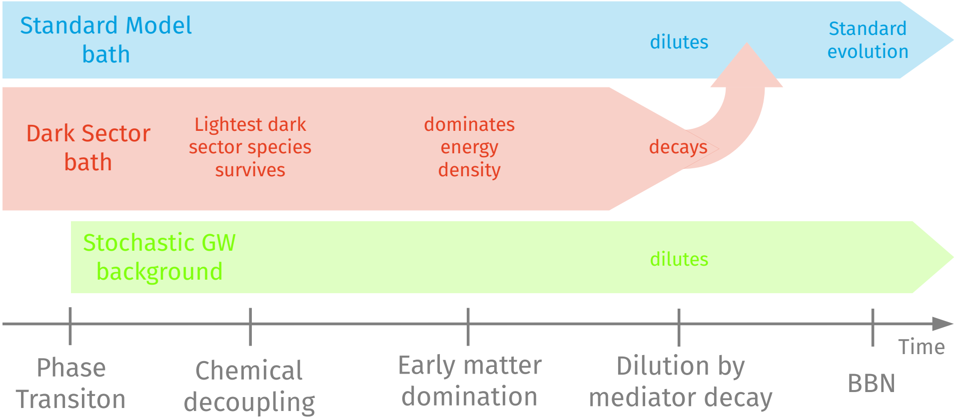

Describing the dark sector in thermal equilibrium.

For several dark sector species in thermal equilibrium: can define effective DOFs

$$\rho_{\mathsf{tot}}(T_{\mathsf{SM}}) = \left[g^{\mathsf{SM}}_{\mathsf{eff},\rho}(T_{\mathsf{SM}}) + g^{\mathsf{DS}}_{\mathsf{eff},\rho}(T_{\mathsf{SM}})\,\xi^4(T_{\mathsf{SM}})\right]\,\frac{\pi^2}{30}\,T^4_{\mathsf{SM}}$$

$$s_{\mathsf{tot}}(T_{\mathsf{SM}}) = \left[g^{\mathsf{SM}}_{\mathsf{eff},s}(T_{\mathsf{SM}}) + g^{\mathsf{DS}}_{\mathsf{eff},s}(T_{\mathsf{SM}})\,\xi^3(T_{\mathsf{SM}})\right]\,\frac{2\,\pi^2}{45}\,T^3_{\mathsf{SM}}$$

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$$s_{\text{tot}}(T_{\text{SM}}) = \left[g^{\text{SM}}_{\text{eff},s}(T_{\text{SM}}) + g^{\text{DS}}_{\text{eff},s}(T_{\text{SM}})\,\xi^3(T_{\text{SM}})\right] \frac{2\,\pi^2}{45}\,T^3_{\text{SM}}$$

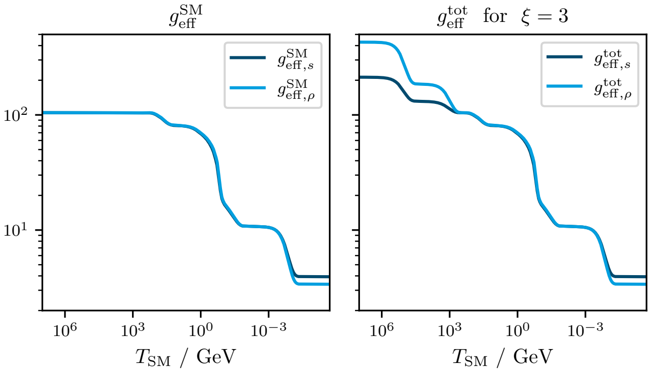
As entropy is conserved separately in the two baths, the temperature ratio follows

$$\xi(T_{\text{SM}}) = \tilde{\xi}\left(\frac{g^{\text{SM}}_{\text{eff},s}}{\tilde{g}^{\text{SM}}_{\text{eff},s}}\right)^{1/3}\left(\frac{\tilde{g}^{\text{DS}}_{\text{eff},s}}{g^{\text{DS}}_{\text{eff},s}}\right)^{1/3}$$

When SM particles annihilate ξ decreases, since the SM becomes hotter.

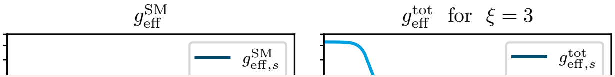
When dark sector DOF decrease ξ increases, since the DS becomes hotter.

Describing the dark sector in thermal equilibrium.



Example: Thermal evolution of a hot ($\xi = 3$) dark sector consisting of a dark photon ($m_{\text{DP}} = 10^6$ GeV) and a dark Higgs boson ($m_{\text{DH}} = 10^4$ GeV).

Describing the dark sector in thermal equilibrium.



This description breaks down when the dark species no longer follow their equilibrium (Bose-Einstein) distributions.

In our setup this occurs when the lightest dark sector species has a long lifetime.

Example: Thermal evolution of a hot ($\xi = 3$) dark sector consisting of a dark photon ($m_{\mathrm{DP}} = 10^6$ GeV) and a dark Higgs boson ($m_{\mathrm{DH}} = 10^4$ GeV).

The out-of-equilibrium decay of a dark mediator.

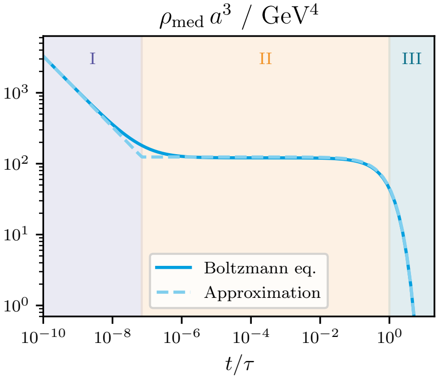
Evolution of the lightest dark sector state ("mediator") after chemical decoupling:

$$\dot{\rho}_{\text{med}} \simeq -3\,\zeta\,H\,\rho_{\text{med}} - \frac{\rho_{\text{med}}}{\tau}$$

with

$$\zeta = 1 + \frac{P_{\text{med}}}{\rho_{\text{med}}} = \begin{cases} 4/3 & \text{rel.} \\ 1 & \text{non-rel.} \end{cases}$$

Three phases: Relativistic, non-relativistic and decaying mediator



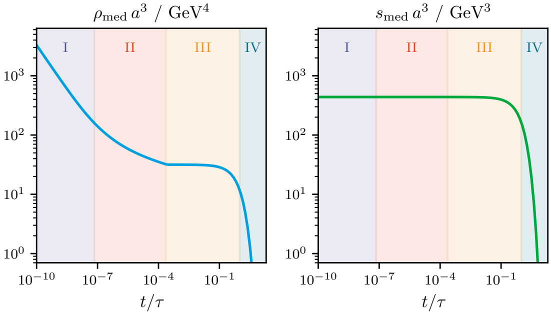
The out-of-equilibrium decay of a dark mediator.

Number-changing processes of the mediator lead to a “cannibalistic” phase with $\mu_{\text{med}} = 0$. Therefore, the unique function $\rho_{\text{med}}(s_{\text{med}})$ exists. We found:

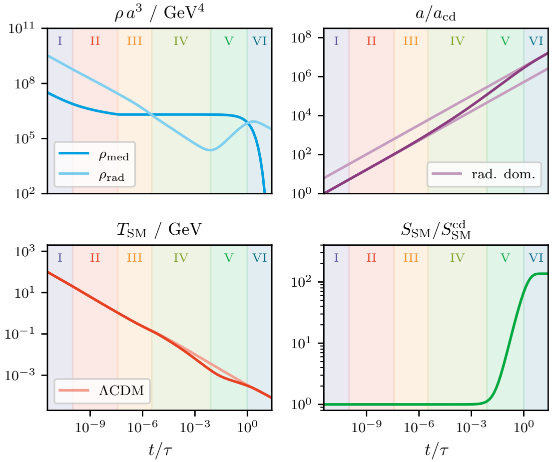
$$\zeta = \begin{cases} \dfrac{\mathrm{d}\ln \rho_{\text{med}}}{\mathrm{d}\ln s_{\text{med}}} & 3 \to 2 \text{ efficient} \\ 1 & 3 \to 2 \text{ inefficient} \end{cases}$$

During cannibalism, ζ goes smoothly from $4/3$ to 1.

Four phases: Relativistic, cannibalistic, non-relativistic and decaying mediator



The out-of-equilibrium decay of a dark mediator.



Energy densities $\rho_i(t)$ $\overset{\text{sets}}{\rightsquigarrow}$ Scale factor $a(t)$ $\overset{\text{sets}}{\rightsquigarrow}$ Temperatures $T_{\mathrm{SM/DS}}(t)$ $\overset{\text{set}}{\rightsquigarrow}$ Particle content $\overset{\text{sets}}{\rightsquigarrow}$ $\rho_i(t)$ $\overset{\text{sets}}{\rightsquigarrow}$ ...

Six phases:

- I Relativistic mediator
- II Cannibalistic mediator
- III Non-relativistic mediator
- IV Early matter domination
- V Entropy injection
- VI Mediator decay

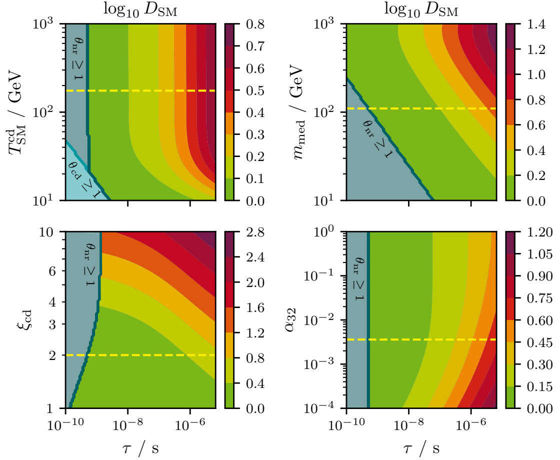
The entropy injection into the SM bath.

Dilution factor: [1811.03608v3]

$$D_{\text{SM}} \equiv \frac{S_{\text{SM}}^{\text{after decay}}}{S_{\text{SM}}^{\text{before decay}}}$$

depends on:

- SM temperature $T_{\text{SM}}^{\text{cd}}$ at chemical decoupling
- Mediator mass m_{med}
- Temperature ratio ξ_{cd} at chemical decoupling
- Effective $3 \to 2$ coupling α_{32}



Parametrization of the GW signal.

Assuming strong[1] phase transitions, the GW spectrum can be parameterized by

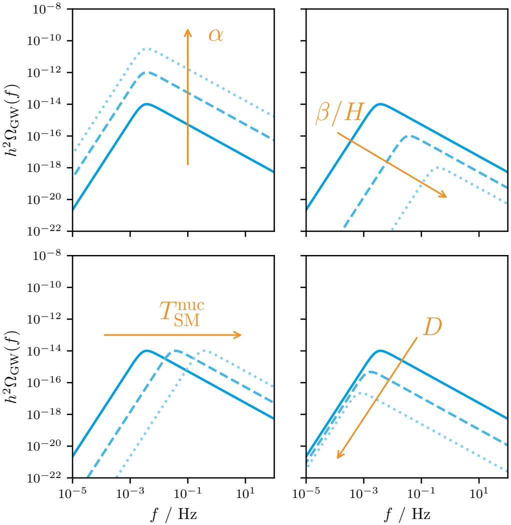
$$h^2\,\Omega_{\mathrm{GW}}(f) \simeq \frac{\mathcal{O}(10^{-6})}{D^{4/3}} \left(\frac{\alpha}{1+\alpha}\right)^2 \left(\frac{\beta}{H}\right)^{-2} \frac{3.8\,(f/f_{\mathrm{p}})^{2.8}}{1+2.8\,(f/f_{\mathrm{p}})^{3.8}}\,, \quad \text{where}$$

$$D \equiv \frac{g_{\mathrm{eff},s}^{\mathrm{SM,nuc}}}{g_{\mathrm{eff},s}^{\mathrm{tot,nuc}}}\, D_{\mathrm{SM}} \qquad \text{and} \qquad f_{\mathrm{p}} \simeq \frac{\mathcal{O}(10\,\mu\mathrm{Hz})}{D^{1/3}} \left(\frac{\beta}{H}\right) \left(\frac{T_{\mathrm{SM}}^{\mathrm{nuc}}}{100\,\mathrm{GeV}}\right)$$

⇝ GW spectrum fixed by the transition strength α, the inverse time scale β/H, the nucleation temperature $T_{\mathrm{SM}}^{\mathrm{n}}$ and the dilution factor D

[1] This is only to get an intuition, the actually performed calculations are more involved

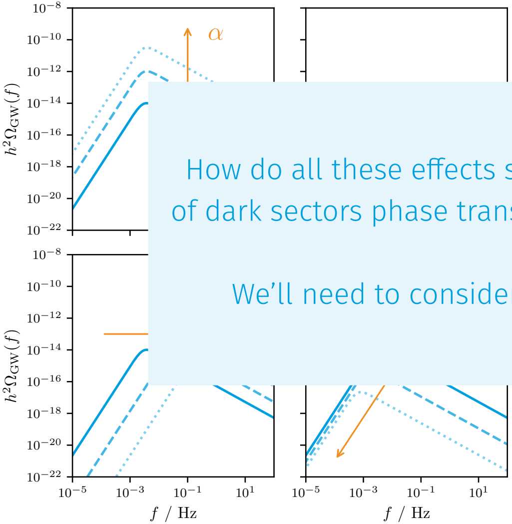
Parametrization of the GW signal.



For observable signals:

- Strong transitions, high α
- Slow transitions, low β/H
- Nucleation temperature $T_{\mathrm{SM}}^{\mathrm{nuc}} \propto f_{\mathrm{p}} \simeq f_{\mathrm{exp}}$
- Little dilution, low D

Parametrization of the GW signal.



How do all these effects sum up and what kind of dark sectors phase transitions are observable?

We'll need to consider a specific model!

signals:

sitions, high α

tions, low β/H

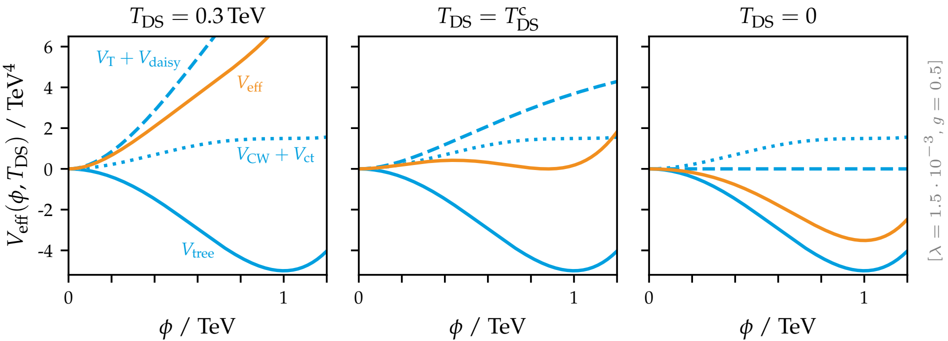
temperature

$f_{\rm exp}$

on, low D

The dark photon model.

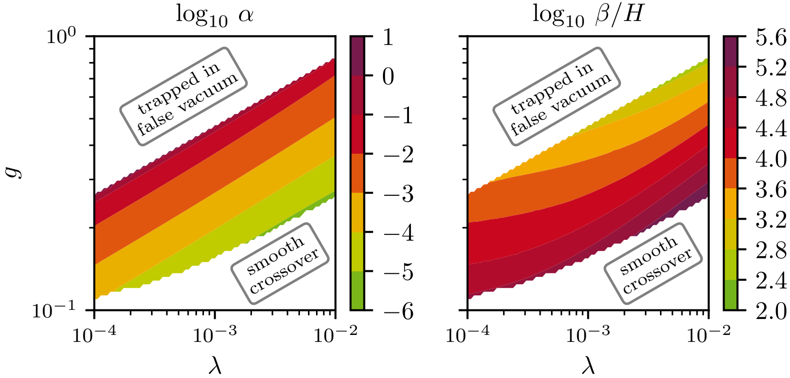
The dark photon model.



Add a $U(1)_D$ to the SM gauge groups. Its gauge boson, the "dark photon", gets massive when a "dark Higgs" obtains $\phi \neq 0$. Effective potential controlled by the tree-level VEV v, dark Higgs quartic coupling λ and gauge coupling g.

Strength and time scale of the transition.

Analyze the phase structure and determine the strength α and inverse time scale β/H. Vary quartic coupling λ and gauge coupling g to identify region of strong and slow transitions.



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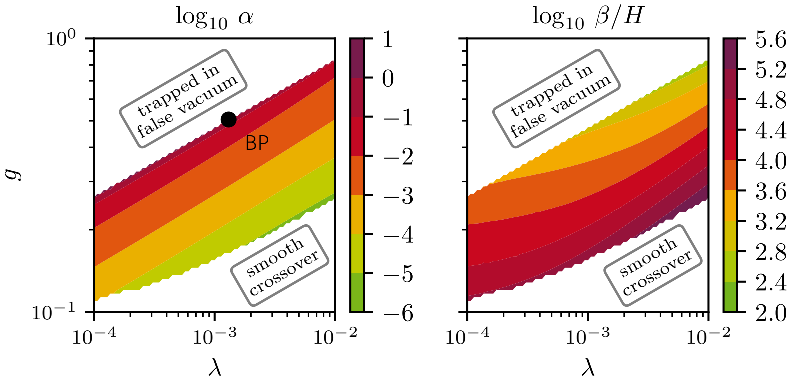
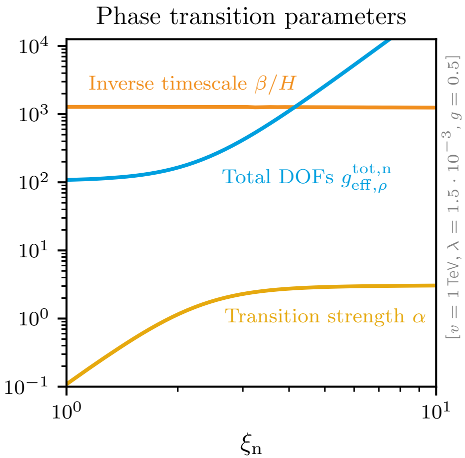
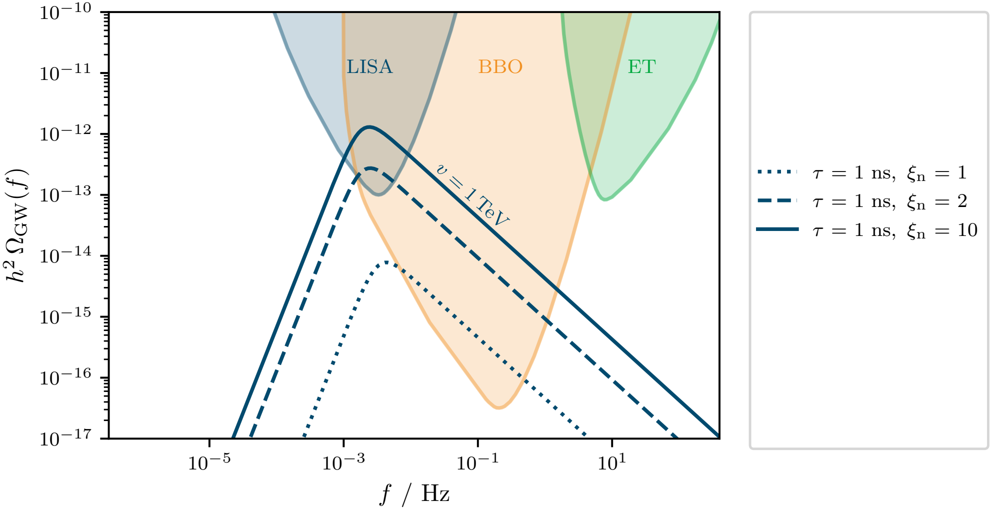


The temperature ratio's impact on α and β/H.

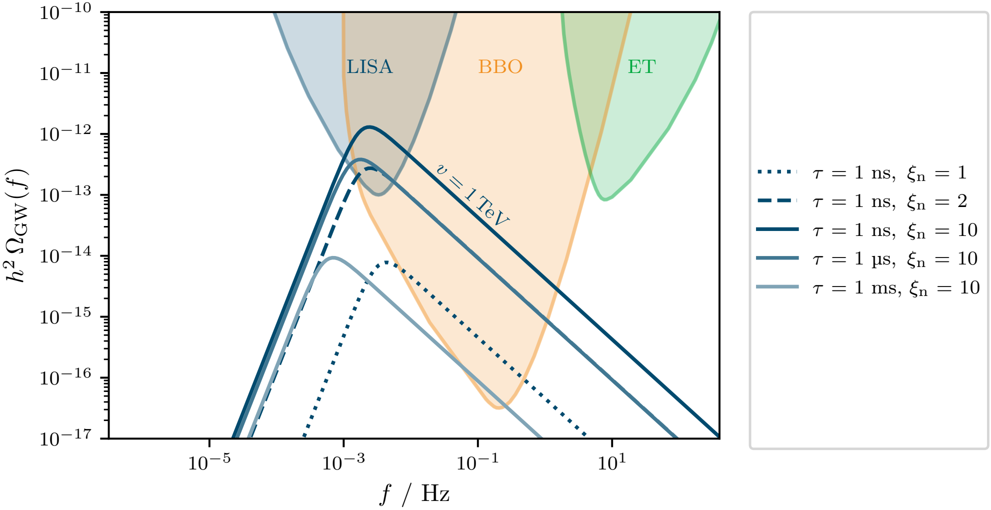


The transition strength α increases $\propto \xi_{\mathrm{n}}^4$, but only until the Universe is completely dominated by the dark sector. Then, the relative temperature difference becomes irrelevant. The inverse timescale is virtually independent of ξ_{n}.

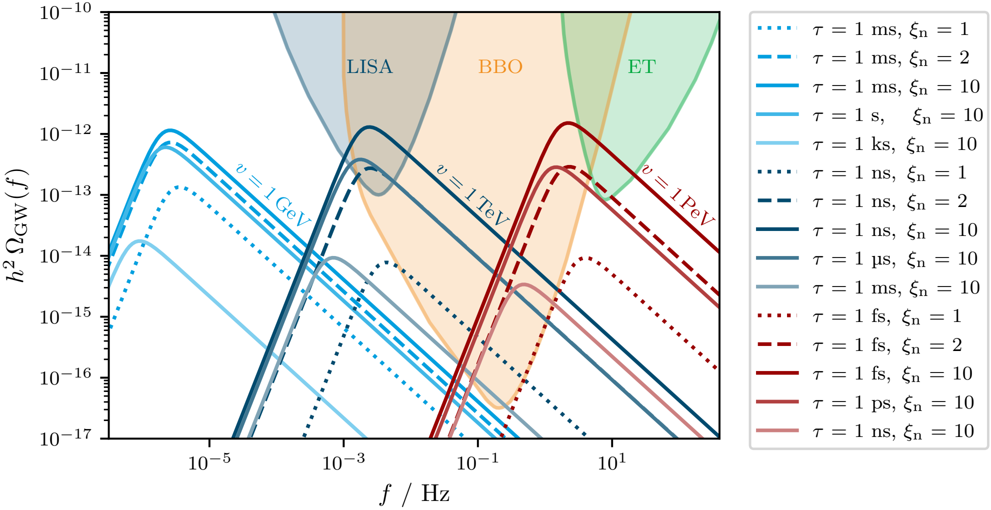
The temperature ratio's impact on the GW signal.



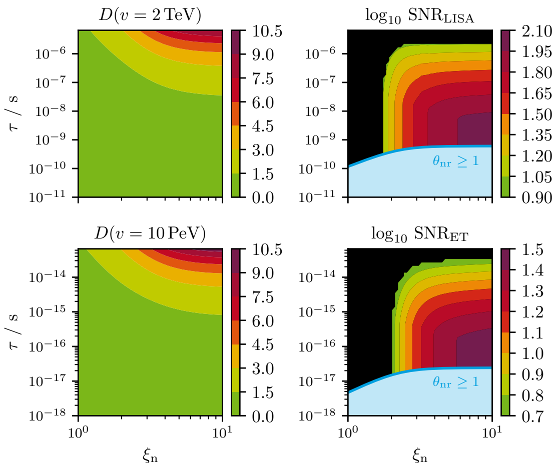
The dark Higgs lifetime's impact on the GW signal.



The vacuum expectation value's impact on the GW signal.



Benchmark point analysis.

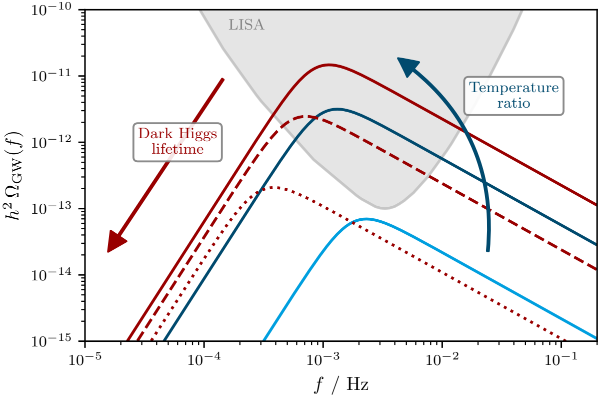


GWs observable by LISA and Einstein Telescope for $\xi_n > 2$, if dark sector is not too long-lived. Otherwise, signals are too diluted.

In blue region, the non-relativistic decay assumption breaks: Inverse decays become relevant, thermalizing the two baths.

Summary.

- Hot dark sectors can produce strong first-order phase transitions
- Parts of the $U(1)_D$ model parameter space will be testable by LISA and ET
- If the mediator species becomes too long-lived, its out-of-equilibrium decay will dilute the signal



Conclusions.

Conclusions and outlook.

- **Hot dark sector phase transitions** are experimentally testable!
- **Dilution** of GWs can be an important issue when dealing with dark sectors.
- **Our code** for model parameter scans, effective potentials, dilution factors, signal-to-noise ratios, etc. is available on github.
- **Bubble wall effects** $\rightsquigarrow$ further work is necessary.
- **DM candidate** could easily be added to the model...
- **IPTA** [2201.03980] just observed something that looks as if it could be a dark sector phase transition at BBN temperatures... ?!

Thank you very much for your attention!

Please feel free to ask me about anything, if you have questions.

Backup slides.

First-order phase transitions in thermal field theory.

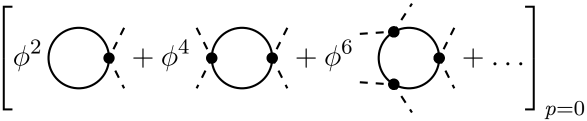
To demonstrate construction of $V_{\text{eff}}(\phi, T)$, take the toy-model Lagrangian...

$$\mathcal{L} = \frac{1}{2}\left(\partial_\mu \phi\right)\left(\partial^\mu \phi\right) - V_{\text{tree}}(\phi)$$

$$\text{with} \quad V_{\text{tree}}(\phi) = -\frac{1}{2}\mu^2\phi^2 + \frac{\lambda}{4}\phi^4$$

... and consider all 1-loop 1-PI graphs:

$$V_{\text{eff},\Phi}^{1-\text{loop}}(\phi) =$$



First-order phase transitions in thermal field theory.

And calculate 1-loop effective potential with $m^2(\phi) = \partial_\phi^2 V_{\text{tree}}(\phi) = -\mu^2 + 3\lambda\phi^2$

$$
\begin{aligned}
V_{\text{eff}}(\phi, T) =& \frac{1}{2}\int \frac{\mathrm{d}^4 k_{\text{E}}}{(2\pi)^4} \log\left[k_{\text{E}}^2 + m^2(\phi)\right] \\
=& \frac{T}{2}\sum_n \int_{\mathbf{k}} \log\left[\left(\frac{2\pi n}{T}\right)^2 + E_k^2\right] \\
=& \int_{\mathbf{k}} \left[\frac{E_k}{2} + T\log\left\{1 - e^{-E_k/T}\right\}\right] \\
=& V_{\text{CW}}(\phi) + V_{\text{T}}(\phi, T)
\end{aligned}
$$

with k_E^0 being $\frac{2\pi}{T}$-periodic

with $E_k = \sqrt{k^2 + m^2(\phi)}$

Interpretation: V_{tree} is the classical energy density contained in a background field ϕ, $V_{\text{CW}}(+V_{\text{T}})$ is the vacuum energy density of a quantum field living in this background, which is completely analogous to the zero-point energy of a harmonic oscillator (in a thermal bath)

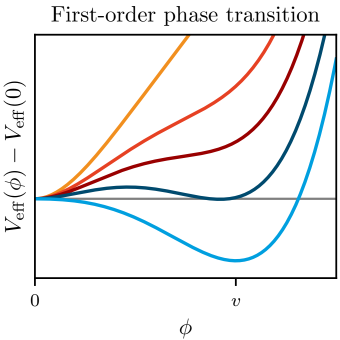
First-order phase transitions in thermal field theory.

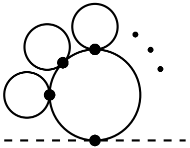
$$V_T = \int_{\mathbf{k}} T \log \left\{ 1 - e^{-E_k/T} \right\}$$
$$= -\frac{\pi^2 T^4}{90} + \frac{T^2 m^2(\phi)}{24} - \frac{T m^3(\phi)}{12\pi} + \dots$$

However, around T_c, V_{eff} is dominated by $>$ 1-loop effects. “Daisies” dominate:

$$V_{\text{daisy}} = -\frac{T}{12\pi} \left[\left(m^2(\phi) + \Pi(T) \right)^{3/2} - m^3(\phi) \right]$$

And cancel the potential barrier in V_{eff}. But: Transversal gauge boson component doesn’t acquire $\Pi(T)$. $\rightsquigarrow$ Gauge bosons can save potential barrier and thus FOPTs.





First-order phase transitions in thermal field theory.

Summary:

$$V_{\text{eff}}^{\text{1-loop}}(\phi, T) = V_{\text{tree}}(\phi) + V_{\text{CW}}(\phi) + V_{\text{ct}}(\phi) + V_{\text{T}}(\phi, T) + V_{\text{daisy}}(\phi, T)$$

- $V_{\text{CW}}(\phi) + V_{\text{ct}}(\phi)$: Coleman-Weinberg potential and its counter-terms
- $V_{\text{T}}(\phi, T)$: 1-loop thermal corrections
- $V_{\text{daisy}}(\phi, T)$: Daisy corrections, dominate at T_{c}

How to get a thermal FOPT?

- Need scalar charged under gauge group with massive gauge bosons
- Dominant $V_{\text{tree}} + V_{\text{CW}}$ contributions can always destroy potential barrier, though $\rightsquigarrow$ as in SM with too high m_h forbidding FOPT

First-order phase transitions in thermal field theory.

$$V_{\text{eff}}^{1-\text{loop}}(\phi,\,T) = V_{\text{tree}} + V_{\text{CW}} + V_{\text{ct}} + V_T + V_{\text{daisy}}$$

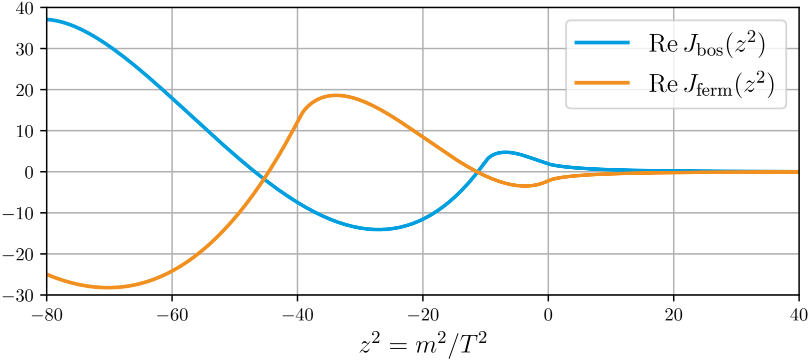
has the individual contributions

$$V_{\text{CW}}(\phi) = \sum_x \eta_x\, n_x\, \frac{m_x^4(\phi)}{64\,\pi^2}\left[\ln\frac{m_x^2(\phi)}{\Lambda^2} - C_a\right]\,,$$

$$V_T(\phi,\,T) = \frac{T^4}{2\,\pi^2}\sum_x \eta_x\, n_x\, J_{\eta_x}\left(\frac{m_x^2(\phi)}{T^2}\right)\,,$$

$$V_{\text{daisy}}(\phi,\,T) = -\frac{T}{12\,\pi}\sum_{\text{b}} n_b^{\text{L}}\left[\left(m^2(\phi)+\Pi(T)\right)_b^{3/2} - \left(m^2(\phi)\right)_b^{3/2}\right]$$

Thermal functions.



Bubble expansion.

Euclidean action of scalar field

$$S[\phi] = \int \mathrm{d}^4 x_\mathsf{E} \left[\frac{1}{2} \left(\frac{\partial \phi}{\partial \tau} \right)^2 + \frac{(\nabla \phi)^2}{2} + V_\mathsf{eff}(\phi) \right]$$

Minimizing for O(4)-case gives

$$\frac{\mathrm{d}^2 \phi}{\mathrm{d}\rho^2} + \frac{3}{\rho} \frac{\mathrm{d}\phi}{\mathrm{d}\rho} = V'_\mathsf{eff}(\phi)$$

At finite T and in real space:

$$\frac{\mathrm{d}^2 \phi}{\mathrm{d}r^2} + \frac{2}{r} \frac{\mathrm{d}\phi}{\mathrm{d}r} = V'_\mathsf{eff}(\phi, T)$$

Can be solved by overshoot-undershoot method

Bubble formation and thermal tunneling.

Nucleation rate: $\Gamma = \mathcal{A} e^{-S_4}$ with

$$S_4 = \int \frac{1}{2}\left(\frac{\mathrm{d}\phi}{\mathrm{d}\tau}\right)^2 + \frac{1}{2}(\nabla\phi)^2 + V_{\text{eff}}(\phi)\,\mathrm{d}^4 x_{\text{E}}$$

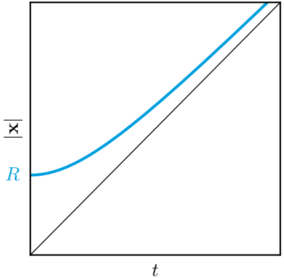
and $\mathcal{A} \sim T^4$. Extremalization yields KG equation with classical potential source:

$$\frac{\mathrm{d}^2\phi}{\mathrm{d}\tau^2} + \Delta\phi = \frac{\mathrm{d}\,V_{\text{eff}}}{\mathrm{d}\phi}$$

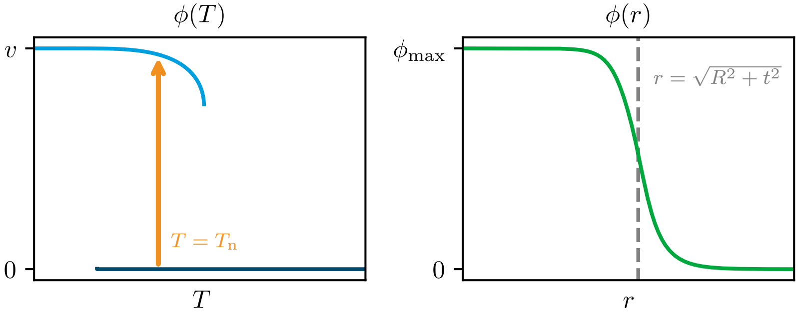
with b.c. $\phi(\rho \to \infty) \to 0$ and $\phi'(\rho = 0) = 0$ where $\rho \equiv \sqrt{\tau^2 + |\mathbf{x}|^2}$.
Solutions typically $O(4)$ symmetric:

$$\frac{\mathrm{d}^2\phi}{\mathrm{d}\rho^2} + \frac{3}{\rho}\frac{\mathrm{d}\phi}{\mathrm{d}\rho} = \frac{\mathrm{d}\,V_{\text{eff}}}{\mathrm{d}\phi}$$

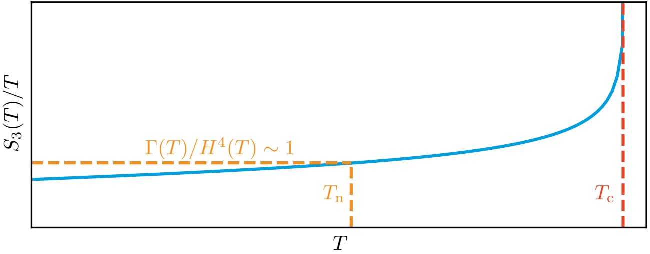
In 3-space: $r = |\mathbf{x}| = \sqrt{\rho^2 + c^2 t^2} \rightsquigarrow$
Nucleation and expansion with $v \to c$



Temperature dependence of potential minima and bubble profile.



Nucleation criterion.



The nucleation condition $\Gamma(T_\mathrm{n})\, H^{-4}(T_\mathrm{n}) = 1$ gives

$$\left.\frac{S_3(T)}{T}\right|_{T=T_\mathrm{n}} \sim 146 - 2\,\ln\left(\frac{g^\mathrm{tot}_{\mathrm{eff},\rho}(T_\mathrm{n})}{100}\right) - 4\,\ln\left(\frac{T_\mathrm{n}}{100\,\mathrm{GeV}}\right)$$

Can be solved by repeated evaluation of S_3/T and subsequent minimization.

GW parameter calculation.

Radiation energy density at nucleation

$$\rho_\mathrm{R} = \frac{\pi^2}{30}\left(g_{\mathrm{eff},\rho}^{\mathrm{SM,n}} + g_{\mathrm{eff},\rho}^{\mathrm{DS,n}}\,\xi^4\right)\left(T_\mathrm{SM}^\mathrm{n}\right)^4$$

Transition strength

$$\alpha^{(\mathrm{DS})} = \frac{1}{\rho_\mathrm{R}^{(\mathrm{DS})}}\left(-\Delta V + \frac{1}{4}\,T_\mathrm{DS}^\mathrm{n}\,\frac{\partial \Delta V}{\partial T}\bigg|_{T_\mathrm{DS}^\mathrm{n}}\right)$$

Inverse time scale

$$\frac{\beta}{H} = T_\mathrm{DS}^\mathrm{n}\,\frac{\mathrm{d}S_\mathrm{E}(T)}{\mathrm{d}T}\bigg|_{T_\mathrm{DS}^\mathrm{n}}$$

Critical transition strength for runaway bubbles ($\alpha^\mathrm{DS} > \alpha_\infty^\mathrm{DS}$)

$$\alpha_\infty^\mathrm{DS} = \frac{\left(T_\mathrm{DS}^\mathrm{n}\right)^2}{\rho_\mathrm{R}^\mathrm{DS}}\left(\sum_{i=\mathrm{bos}} n_i\,\frac{\Delta m_i^2}{24} + \sum_{i=\mathrm{fer}} n_i\,\frac{\Delta m_i^2}{48}\right)$$

SGWB model.

$$\Omega_{\mathsf{GW}}(f) = \frac{1}{\rho_c}\frac{\mathrm{d}\rho_{\mathsf{GW}}(f)}{\mathrm{d}\log f} \simeq \sum \mathcal{N}\,\Delta\left(\frac{\kappa\,\alpha}{1+\alpha}\right)^p \left(\frac{H}{\beta}\right)^q s(f)$$

	Scalar field Ω_ϕ	Sound waves Ω_{sw}	Turbulence Ω_{turb}
$\mathcal{N}$	1	$1.59 \cdot 10^{-1}$	$2.01 \cdot 10^{1}$
κ	κ_ϕ	κ_{sw}	$\varepsilon_{\mathsf{turb}}\kappa_{\mathsf{sw}}$
p	2	2	$\frac{3}{2}$
q	2	1	1
Δ	$\frac{0.11 v_w^3}{0.42 + v_w^2}$	v_w	v_w
f_{p}	$\frac{0.62\beta}{1.8 - 0.1 v_w + v_w^2}$	$\frac{2\beta}{\sqrt{3} v_w}$	$\frac{3.5\beta}{2 v_w}$
$s(f)$	$\frac{3.8(f/f_{\mathsf{p}})^{2.8}}{1 + 2.8(f/f_{\mathsf{p}})^{3.8}}$	$(f/f_{\mathsf{p}})^3 \left(\frac{7}{4 + 3(f/f_{\mathsf{p}})^2}\right)^{7/2}$	$\frac{(f/f_{\mathsf{p}})^3}{(1 + f/f_{\mathsf{p}})^{11/3}[1 + 8\pi(f/H)]}$

Redshift and dilution of the GW background.

After its emission, the GW signal gets red-shifted:

$$h^2\,\Omega_{\mathsf{GW}}(f) = \mathcal{R}h^2\;\Omega^{\mathsf{n}}_{\mathsf{GW}}\left(\frac{a_0}{a_{\mathsf{n}}}\,f\right)$$

Energy density:

$$\mathcal{R}h^2 \simeq \frac{2.4\cdot 10^{-5}}{D_{\mathsf{SM}}^{4/3}}\left(\frac{g_{\mathsf{eff},s}^{\mathsf{SM},0}}{g_{\mathsf{eff},s}^{\mathsf{SM,n}}}\right)^{4/3}\frac{g_{\mathsf{eff},\rho}^{\mathsf{tot,n}}}{2}$$

Frequency:

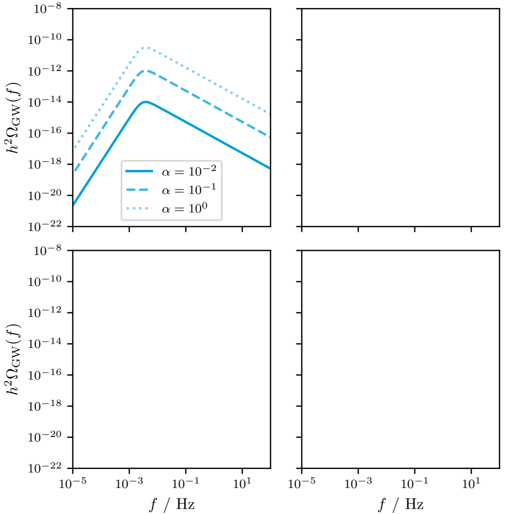
$$\frac{a_0}{a_{\mathsf{n}}} = D_{\mathsf{SM}}^{1/3}\left(\frac{g_{\mathsf{eff},s}^{\mathsf{SM,n}}}{g_{\mathsf{eff},s}^{\mathsf{SM},0}}\right)^{1/3}\frac{T^{\mathsf{n}}_{\mathsf{SM}}}{T^{0}_{\mathsf{SM}}}$$

Parametrization of the GW signal.

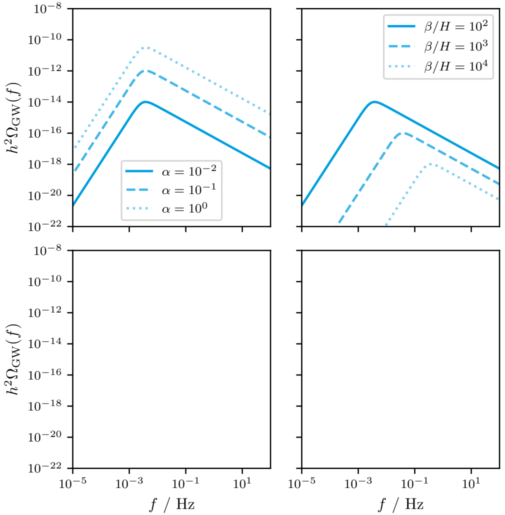
Transition strength:

$$\alpha = \frac{\Delta\theta}{\rho_{\mathrm{rad}}^{\mathrm{n}}}$$

relates the latent heat (trace of the energy momentum tensor between the two phases) $\Delta\theta$ of the transition with the energy density $\rho_{\mathrm{rad}}^{\mathrm{n}}$ of the surrounding heat bath. For fixed $T_{\mathrm{DS}}^{\mathrm{n}}$: $\rho_{\mathrm{rad}}^{\mathrm{n}} \propto \xi_{\mathrm{n}}^{-4}$. The transition strength thus grows $\propto \xi_{\mathrm{n}}^{4}$!

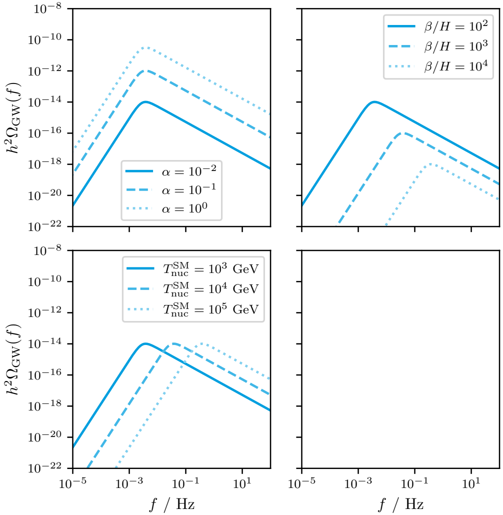


Inverse time scale:
The computation of β/H is complicated, but shows no relevant dependence of the temperature ratio between the sectors. Larger β/H indicate fast transitions. In that case, many small bubbles collide, resulting in weak signals at high frequencies.



Nucleation temperature:

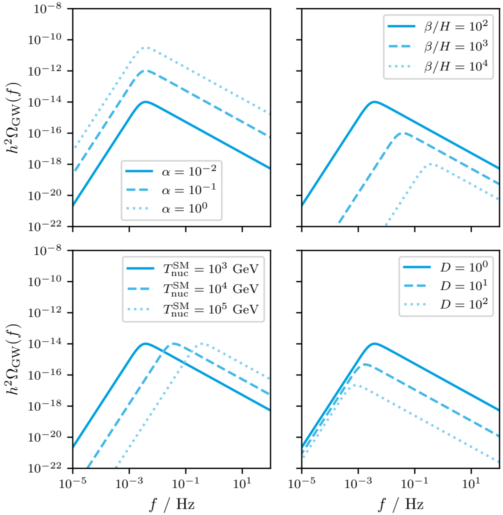
Keeping $T^{\mathrm{n}}_{\mathrm{DS}}$ fixed, a larger temperature ratio ξ_{n} at nucleation leads to a lower $T^{\mathrm{n}}_{\mathrm{SM}}$. This corresponds to lower peak frequencies.



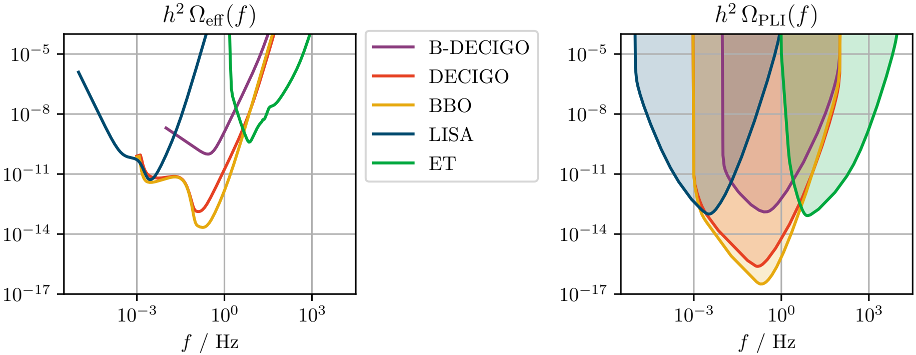
Parametrization of the GW signal.

Dilution:

The redshift to lower frequencies and signals strengths increases with the dilution factor. D grows with the temperature ratio ξ_{n}, as more energy is injected into the SM bath from the dark sector. Unlike D_{SM}, D saturates for high temperature ratios.



Experimental sensitivities.



Effective degrees of freedom.

$$g^x_{\text{eff},\rho}(T_x) \equiv \frac{\rho_x(T_x)}{\rho^{\text{rel}}_{\text{bos}}(T_x)\big|_{g=1}} = g_x \frac{15}{\pi^4} \int_{z_x}^{\infty} \mathrm{d}u_x \, \frac{u_x^2 \sqrt{u_x^2 - z_x^2}}{e^{u_x} \pm 1} \,,$$

$$g^x_{\text{eff},P}(T_x) \equiv \frac{P_x(T_x)}{P^{\text{rel}}_{\text{bos}}(T_x)\big|_{g=1}} = g_x \frac{15}{\pi^4} \int_{z_x}^{\infty} \mathrm{d}u_x \, \frac{\left(u_x^2 - z_x^2\right)^{3/2}}{e^{u_x} \pm 1} \,,$$

$$g^x_{\text{eff},s}(T_x) = \frac{3\, g^x_{\text{eff},\rho}(T_x) + g^x_{\text{eff},P}(T_x)}{4} \,,$$

where $u_x = \sqrt{m_x^2 + p^2}/T_x$ and $z_x = m_x/T_x$. Sum over all SM and DS species:

$$g^{\text{tot}}_{\text{eff},\rho} = g^{\text{SM}}_{\text{eff},\rho}(T_{\text{SM}}) + g^{\text{DS}}_{\text{eff},\rho}(T_{\text{SM}})\, \xi^4(T_{\text{SM}})$$

$$g^{\text{tot}}_{\text{eff},s} = g^{\text{SM}}_{\text{eff},s}(T_{\text{SM}}) + g^{\text{DS}}_{\text{eff},s}(T_{\text{SM}})\, \xi^3(T_{\text{SM}})$$

Mediator cannibalism.

Conserved comoving mediator entropy $s_{\text{med}}\, a^3 = \text{const}$ gives

$$\frac{\mathrm{d} \ln s_{\text{med}}}{\mathrm{d}t} = \frac{\mathrm{d} \ln s_{\text{med}}}{\mathrm{d} \ln \rho_{\text{med}}} \frac{\dot{\rho}_{\text{med}}}{\rho_{\text{med}}} = -3\, H(t)\ ,$$

from which follows that

$$\dot{\rho}_{\text{med}} = -3 \frac{\mathrm{d} \ln \rho_{\text{med}}}{\mathrm{d} \ln s_{\text{med}}} H(t)\, \rho_{\text{med}}(t)\ .$$

For $\mu_{\text{med}} = 0$, one can find function $\rho_{\text{med}}(s_{\text{med}})$, independent of particle species:

$$\frac{\mathrm{d} \ln \rho_{\text{med}}}{\mathrm{d} \ln s_{\text{med}}} = \frac{\mathrm{d}\rho_{\text{med}}}{\mathrm{d}s_{\text{med}}} \frac{s_{\text{med}}}{\rho_{\text{med}}} = \frac{\mathrm{d}\bar{\rho}_{\text{med}}}{\mathrm{d}\bar{s}_{\text{med}}} \frac{\bar{s}_{\text{med}}}{\bar{\rho}_{\text{med}}} = \frac{\mathrm{d} \ln \bar{\rho}_{\text{med}}}{\mathrm{d} \ln \bar{s}_{\text{med}}} = \frac{\mathrm{d} \ln \bar{\rho}}{\mathrm{d} \ln \bar{s}}$$

with $\bar{s}_{\text{med}} \equiv 2\,\pi^2\, s_{\text{med}}/(g_{\text{med}}\, T_{\text{DS}}^3)$ and $\bar{\rho}_{\text{med}} \equiv 2\,\pi^2\, \rho_{\text{med}}/(g_{\text{med}}\, T_{\text{DS}}^4)$.

Mediator cannibalism.

That yields

$$\dot{\rho}_{\mathrm{med}} \simeq -3\,\zeta\,H\,\rho_{\mathrm{med}} - \rho_{\mathrm{med}}/\tau$$

with

$$\zeta(t) = \begin{cases} \dfrac{\mathrm{d}\ln\bar{\rho}}{\mathrm{d}\ln\bar{s}}(\rho_{\mathrm{med}}) & \text{for} \quad \Gamma_{\mathrm{nc}}(t) \geq H(t) \\ 4/3 & \text{for} \quad \Gamma_{\mathrm{nc}}(t) < H(t), \quad t < \tilde{t} \\ 1 & \text{for} \quad \Gamma_{\mathrm{nc}}(t) < H(t), \quad t \geq \tilde{t} \end{cases},$$

where $\tilde{t} \simeq 7.0\,t_{\mathrm{cd}}\,(T_{\mathrm{DS}}^{\mathrm{cd}}/m_{\mathrm{med}})^2$ denotes the time when the mediator gets non-relativistic. Number changing process rate is approximated by

$$\Gamma_{\mathrm{nc}} \simeq \Gamma_{32} \simeq \langle\sigma_{32}\,v^2\rangle\,n_{\mathrm{med}}^2$$

Mediator cannibalism.

The averaged cross section reads

$$\langle \sigma_{32}\, v^2 \rangle = \frac{25\sqrt{5}\,\pi^2}{5184}\,\frac{\alpha_{32}^3}{m_{\text{med}}^5} + \mathcal{O}\left(\frac{T_{\text{DS}}}{m_{\text{med}}}\right).$$

where

$$(4\,\pi\,\alpha_{32})^3 \equiv \left(\frac{\kappa_3}{m}\right)^2 \left[\left(\frac{\kappa_3}{m}\right)^2 + 3\,\kappa_4\right]^2$$

for a potential $V(\phi) = \frac{m^2}{2}\,\phi^2 + \frac{\kappa_3}{3!}\,\phi^3 + \frac{\kappa_4}{4!}\,\phi^4$. In our model: $\alpha_{32} = 2.3\,\lambda$.

Coupled set of ODEs underlying the entropy injection.

$$\bar{a}' = \frac{\bar{a}}{\theta_{\rm H}} \sqrt{r + \frac{r^{\rm cd}_{\rm mat}}{\bar{a}^3} + \frac{r^{\rm cd}_{\rm rad}}{\bar{a}^4} \frac{\gamma}{\gamma_{\rm cd}} \frac{\mathcal{S}}{\mathcal{G}^{1/3}}} \,,$$

$$\mathcal{S}' = \frac{r\,\bar{a}^4}{r^{\rm cd}_{\rm rad}} \mathcal{G}^{1/3} \gamma_{\rm cd} \,,$$

$$r' = -r - 3\,\frac{\bar{a}'}{\bar{a}} \zeta\, r \,,$$

$$\mathcal{G}' = -\frac{3}{4} \frac{T^{\rm cd}_{\rm SM}\,\mathcal{G}\,\hat{\mathcal{G}}}{\mathcal{S}^{3/4}\,\bar{a}} \frac{4\,\mathcal{S}\,\bar{a}' - \mathcal{S}'\,\bar{a}}{T^{\rm cd}_{\rm SM}\,\hat{\mathcal{G}}\,\mathcal{S}^{1/4} + 3\,\mathcal{G}^{4/3}\bar{a}} \,,$$

$$\gamma' = \hat{\gamma}\, T^{\rm cd}_{\rm SM} \frac{3\,\mathcal{G}\,\bar{a}\,\mathcal{S}' - 12\,\mathcal{G}\,\bar{a}'\,\mathcal{S} - 4\,\mathcal{G}'\,\bar{a}\,\mathcal{S}}{12\,\mathcal{G}^{4/3}\,\mathcal{S}^{3/4}\,\bar{a}^2} \,.$$

with initial condition $\bar{a}_{\rm cd} = \mathcal{S}_{\rm cd} = r_{\rm cd} = \mathcal{G}_{\rm cd} = 1$ and $\gamma_{\rm cd}$.

- Normalized scale factor $\bar{a} = a/a_{\rm cd}$
- Characteristic time scale $\theta_{\rm H} = \sqrt{3\,m^2_{\rm Pl}\,\rho^{\rm cd}_{\rm med}/\tau^2}$
- Normalized mediator energy density $r = \rho_{\rm med}/\rho^{\rm cd}_{\rm med}$
- Normalized initial DM density $r^{\rm cd}_{\rm mat} = \rho^{\rm cd}_{\rm DM}/\rho^{\rm cd}_{\rm med}$
- Normalized initial radiation energy density $r^{\rm cd}_{\rm rad} = \rho^{\rm cd}_{\rm rad}/\rho^{\rm cd}_{\rm med}$
- Normalized DOFs $\gamma = g^{\rm SM}_{{\rm eff},\rho}/g^{\rm SM}_{{\rm eff},s}$
- Normalized DOFs $\mathcal{G} = g^{\rm SM}_{{\rm eff},s}/g^{\rm SM,cd}_{{\rm eff},s}$
- Normalized SM entropy $\mathcal{S} = \left(S_{\rm SM}/S^{\rm cd}_{\rm SM}\right)^{4/3}$

The $U(1)_D$ model in detail.

Lagrangian:

$$\mathcal{L} \supset |D_\mu \Phi|^2 + |D_\mu H|^2 - \frac{1}{4} B'_{\mu\nu} B'^{\mu\nu} - \frac{\epsilon}{2} B'_{\mu\nu} B^{\mu\nu} - V(\Phi, H) ,$$

$$D_\mu \Phi = \left(\partial_\mu + i\, g\, B'_\mu\right) \Phi ,$$

$$V_{\text{tree}}(\Phi, H) = -\mu^2 \Phi^* \Phi + \lambda (\Phi^* \Phi)^2 - \mu_H^2 H^\dagger H + \lambda_H (H^\dagger H)^2 + \lambda_p (\Phi^* \Phi)(H^\dagger H) .$$

Mass spectrum:

$$m^2_{(h,\phi)}(h,\phi) = \begin{pmatrix} -\mu_H^2 + 3\lambda_H h^2 + \frac{\lambda_p}{2}\phi^2 & \lambda_p h \phi \\ \lambda_p h \phi & -\mu^2 + 3\lambda \phi^2 + \frac{\lambda_p}{2} h^2 \end{pmatrix} ,$$

$$m^2_{G^0, G^+}(h,\phi) = -\mu_H^2 + \lambda_H h^2 + \frac{\lambda_p}{2}\phi^2 ,$$

$$m^2_{\varphi}(h,\phi) = -\mu^2 + \lambda \phi^2 + \frac{\lambda_p}{2} h^2 .$$

The $\mathsf{U}(1)_{\mathsf{D}}$ model in detail.

For $\lambda_p, \epsilon \to 0$ and $\mu^2 = \lambda\, v^2$, the field-dependent dark Higgs and dark photon masses are given by

$$m_{\mathsf{DP}} = g\,\phi \overset{T=0}{=} g\,v\,, \qquad\qquad m_{\mathsf{DH}} = \sqrt{2\,\lambda}\,\phi \overset{T=0}{=} \sqrt{2\,\lambda}\,v\,.$$

The corresponding Debye masses are

$$\Pi_\Phi(T_{\mathsf{DS}}) = \left(\frac{\lambda}{3} + \frac{g^2}{4}\right) T_{\mathsf{DS}}^2\,, \qquad\qquad \Pi^{\mathsf{L}}_{A'}(T_{\mathsf{DS}}) = \frac{g^2}{3}\, T_{\mathsf{DS}}^2\,.$$

- Quartic dark Higgs coupling: λ
- $U(1)_{\mathsf{D}}$ gauge coupling: g
- Dark Higgs lifetime: τ
- Dark Higgs VEV: $v = \frac{\mu}{\sqrt{\lambda}}$
- Temperature ratio: $\xi_{\mathsf{n}} = \left.\frac{T_{\mathsf{DS}}}{T_{\mathsf{SM}}}\right|_{\mathsf{n}}$

Signal-to-noise ratios for LISA and the ET.

Compute the overlap of the signals $h^2\,\Omega_{\mathsf{GW}}(f)$ and expected sensitivities $h^2\,\Omega_{\mathsf{obs}}(f)$ and weight it with the duration of the observation t_{obs} to obtain a signal-to-noise measure:

$$\rho^2 = t_{\mathsf{obs}} \int_{f_{\mathsf{min}}}^{f_{\mathsf{max}}} \mathrm{d}f \left[\frac{h^2\,\Omega_{\mathsf{GW}}(f)}{h^2\,\Omega_{\mathsf{obs}}(f)} \right]^2$$

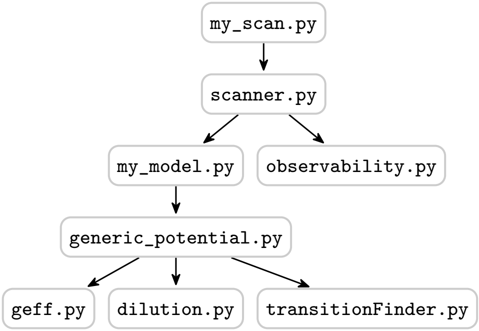
If ρ exceeds a certain threshold value for a given signal, the signal is observable.

To analyze the impact of ξ_{n} and τ on the observability of the signals produced by our model, consider the benchmark points

Benchmark point	λ	g	v
LISA	$1.5 \cdot 10^{-3}$	0.5	2 TeV
ET	$1.5 \cdot 10^{-3}$	0.5	10 PeV

Structure of `TransitionListener`.

Structure:



Example output: